GRADE 7 MATH LEARNING GUIDE

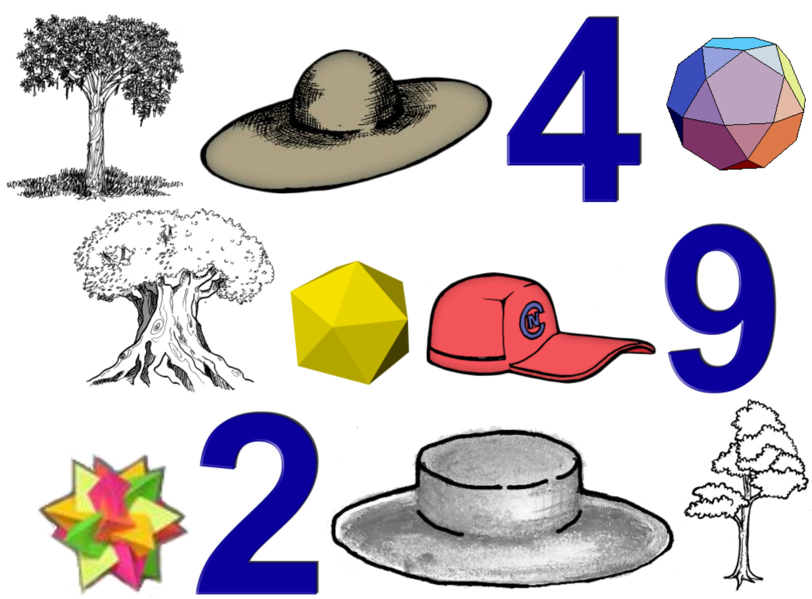
Lesson I: SETS: AN INTRODUCTION **Time: 1.5 hours**
Pre-requisite Concepts: Whole numbers

About the Lesson:
This is an introductory lesson on sets. A clear understanding of the concepts in this lesson will help you easily grasp number properties and enable you to quickly identify multiple solutions involving sets of numbers.

Objectives:
In this lesson, you are expected to:
1. Describe and illustrate
   a. well-defined sets;
   b. subsets;
   c. universal set, and;
   d. the null set.
2. Use Venn Diagrams to represent sets and subsets.

Lesson Proper:
A.
I. Activity

Below are some objects. Group them as you see fit and label each group.



Answer the following questions:
a. How many groups are there?
b. Does each object belong to a group?
c. Is there an object that belongs to more than one group? Which one?

The groups are called sets for as long as the objects in the group share a characteristic and are thus, well defined.

Problem: Consider the set consisting of whole numbers from 1 to 200. Let this be set U. Form smaller sets consisting of elements of U that share a different characteristic. For example, let E be the set of all even numbers from 1 to 200.

Can you form three more such sets? How many elements are there in each of these sets? Do any of these sets have any elements in common?

Did you think of a set with no element?

Important Terms to Remember

The following are terms that you must remember from this point on.

1. A **set** is a well-definedgroup of objects, called **elements** that share a common characteristic. For example, 3 of the objects above belong to the set of head covering or simply hats (ladies hat, baseball cap, hard hat).
2. The set *F* is a **subset** of set *A* if all elements of *F* are also elements of *A*. For example, the even numbers 2, 4 and 12 all belong to the set of whole numbers. Therefore, the even numbers 2, 4, and 12 form a subset of the set of whole numbers. *F* is a **proper subset** of *A* if *F* does not contain all elements of *A*.
3. The **universal set***U* is the set that contains all objects under consideration.
4. The **null set**$\varnothing$ is an empty set. The null set is a subset of any set.
5. The **cardinality of a set *A*** is the number of elements contained in *A*.

Notations and Symbols

In this section, you will learn some of the notations and symbols pertaining to sets.

1. Uppercase letters will be used to name sets and lowercase letters will be used to refer to any element of a set. For example, let H be the set of all objects on page 1 that cover or protect the head. We write

   H = {ladies hat, baseball cap, hard hat}

   This is the listing or roster method of naming the elements of a set.

   Another way of writing the elements of a set is with the use of a descriptor. This is the rule method. For example,*H* = {x| x covers and protects the head}. This is read as "the set H contains the element x such that x covers and protects the head."

2. The symbol $\varnothing$ or { } will be used to refer to an empty set.
3. If *F* is a subset of *A,* then we write $F \subseteq A$. We also say that *A* contains the set *F* and write it as $A \supseteq F$. If *F* is a proper subset of *A,* then we write $F \subset A$.
4. The cardinality of a set *A* is written as n(*A*).

II. Questions to Ponder (Post-Activity Discussion)

Let us answer the questions posed in the opening activity.

1. How many sets are there?
   There is the set of head covers (hats), the set of trees, the set of even numbers, and the set of polyhedra. But, there is also a set of round objects and a set of pointy objects. There are 6 well-defined sets.

2. Does each object belong to a set?*Yes.*

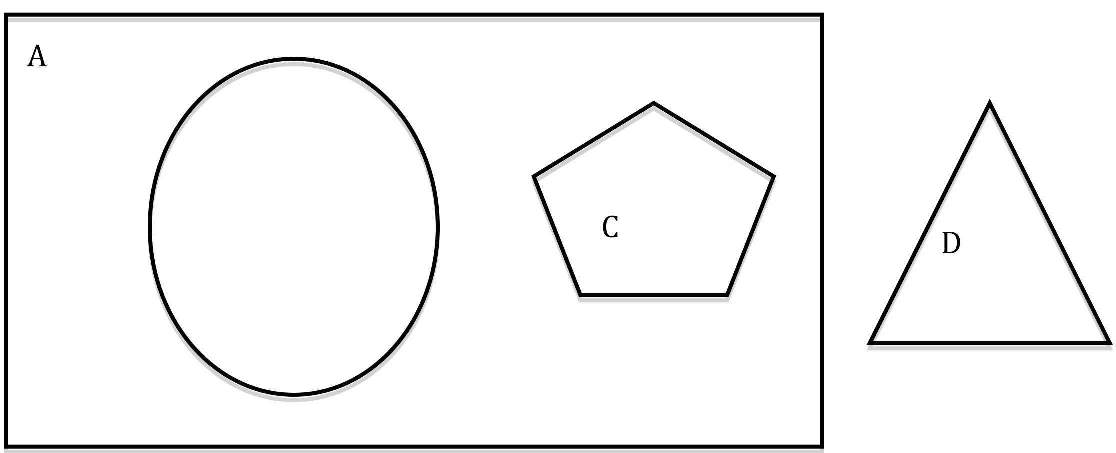
3. Is there an object that belongs to more than one set? Which ones?
   All the hats belong to the set of round objects. The pine trees and two of the polyhedra belong to the set of pointy objects.

III. Exercises
Do the following exercises.

1. Give 3 examples of well-defined sets.
2. Name two subsets of the set of whole numbers using both the listing method and the rule method.
3. Let *B* = [1, 3, 5, 7, 9}. List <u>all</u> the possible subsets of *B*.
4. Answer this question: How many subsets does a set of n elements have?

B. Venn Diagrams
Sets and subsets may be represented using <u>Venn Diagrams</u>. These are diagrams that make use of geometric shapes to show relationships between sets.

Consider the Venn diagram below. Let the universal set U be all the elements in sets A, B, C and D.



Each shape represents a set. Note that although there are no elements shown inside each shape, we can surmise how the sets are related to each other.Notice that set B is inside set A. This indicates that all elements in B are contained in A. The same with set C. Set D, however, is separate from A, B, C. What does it mean?

Exercises
Draw a Venn diagram to show the relationships between the following pairs or groups of sets:

1. E = {2, 4, 8, 16, 32}

F = {2, 32}

2. V is the set of all odd numbers
   W = {5, 15, 25, 35, 45, 55,….}
3. R = {x| x is a factor of 24}
   S = { }
   T = {7, 9, 11}

Summary

In this lesson, you learned about sets, subsets, the universal set, the null set and the cardinality of the set. You also learned to use the Venn diagram to show relationships between sets.

Lesson 2.1: Union and Intersection of Sets **Time: 1.5 hours**

Pre-requisite Concepts: Whole Numbers, definition of sets, Venn diagrams

About the Lesson:

After learning some introductory concepts about sets, a lesson on set operations follows. The student will learn how to combine sets (union) and how to determine the elements common to 2 or 3 sets (intersection).

Objectives:

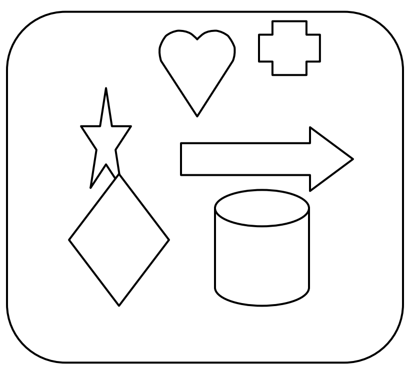
In this lesson, you are expected to:

1. Describe and define
    a. union of sets;
    b. intersection of sets.
2. Perform the set operations
    a. union of sets;
    b. intersection of sets.

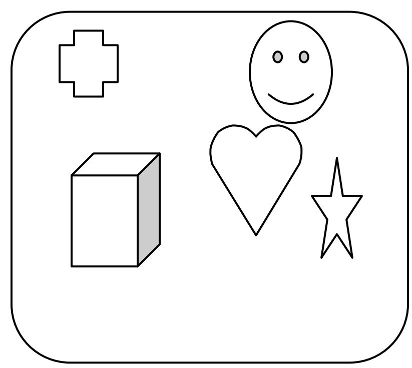
` 3. Use Venn diagrams to represent the union and intersection of sets.

Lesson Proper:

I. Activities



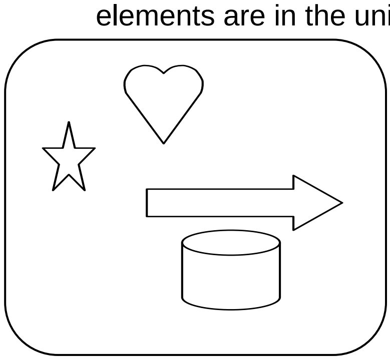
A



B

Answer the following questions:

1. Which of the following shows the union of set A and set B? How many elements are in the union of A and B?



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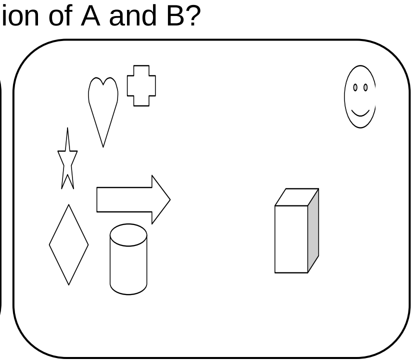

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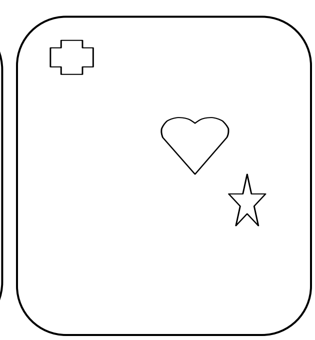
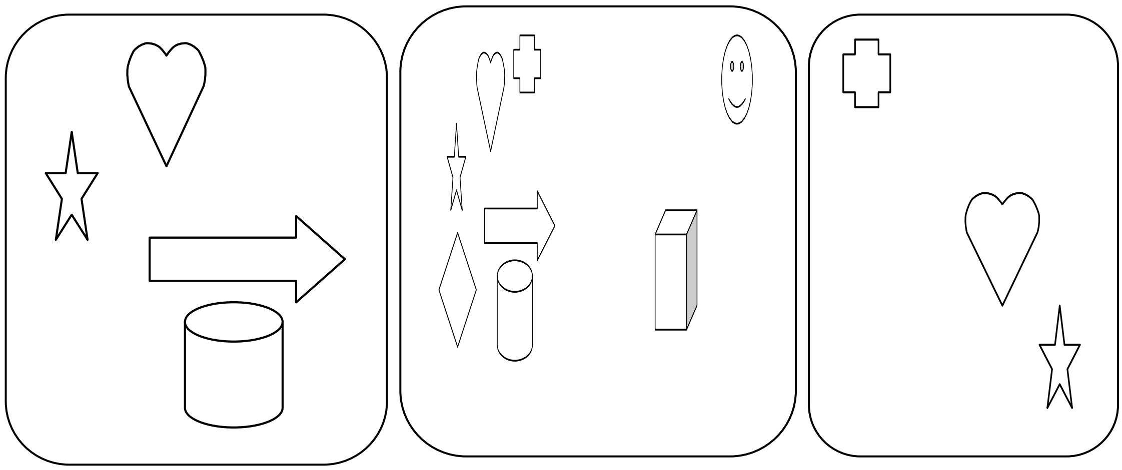


3

2. Which of the following shows the intersection of set A and set B? How many elements are there in the intersection of A and B?



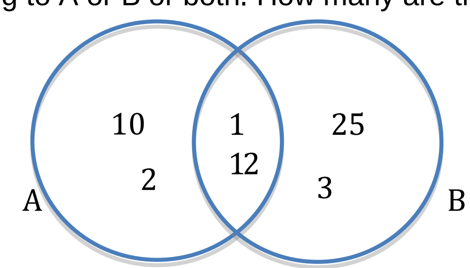
1 **2** **3**

Here’s another activity:
Let

$V = \{2x \mid x \in \mathrm{I}, 1 \le x \le 4\}$
$W = \{x^2 \mid x \in \mathrm{I}, -2 \le x \le 2\}$

What elements may be found in the intersection of V and W? How many are there? What elements may be found in the union of V and W? How many are there?

Do you remember how to use Venn Diagrams? Based on the diagram below, (1) determine the elements that belong to both A and B; (2) determine the elements that belong to A or B or both. How many are there in each set?



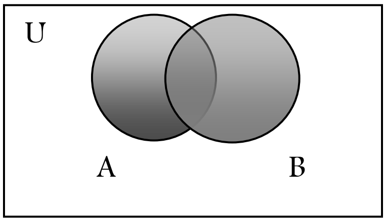
Important Terms/Symbols to Remember
The following are terms that you must remember from this point on.

1. Let A and B be sets. The *union* of the sets A and B, denoted by $A \cup B$, is the set that contains those elements that belong to A, B, or to both.

An element x belongs to the union of the sets A and B if and only if x belongs to A or x belongs to B or to both. This tells us that

$A \cup B = \{ x \mid x \in A \text{ or } x \in B \}$

Using the Venn diagram, all of the set of A and of B are shaded to show $A \cup B$.

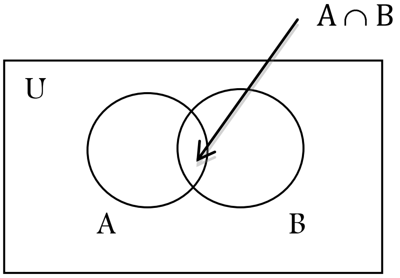


2. Let A and B be sets. The *intersection* of the sets A and B, denoted by $A \cap B$, is the set containing those elements that belong to both A and B.

An element x belongs to the intersection of the sets A and B if and only if x belongs to A and x belongs to B. This tells us that

$A \cap B = \{x \mid x \in A \text{ and } \in B\}$

Using the Venn diagram, the set $A \cap B$ consists of the shared regions of A and B.



Sets whose intersection is an empty set are called *disjoint sets*.

3. The cardinality of the union of two sets is given by the following equation:

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$.

II. Questions to Ponder (Post-Activity Discussion)

Let us answer the questions posed in the opening activity.

1. Which of the following shows the union of set A and set B? **Set 2. This is because it contains all the elements that belong to A or B or both. There are 8 elements.**
2. Which of the following shows the intersection of set A and set B? **Set 3. This is because it contains all elements that are in both A and B. There are 3 elements.**

In the second activity:

V = { 2, 4, 6, 8 }
W = { 0, 1, 4}

Therefore, $V \cap W$ = { 4 } has 1 element and $V \cup W$ = { 0, 1, 2, 4, 6, 8 } has 6 elements. Note that the element { 4 } is counted only once.

On the Venn Diagram: (1) The set that contains elements that belong to both A and B consists of two elements {1, 12 }; (2) The set that contains elements that belong to A or B or both consists of 6 elements {1, 10, 12, 20, 25, 36 }.

III. Exercises

1. Given sets A and B,

Set A Students who play the guitar	Set B Students who play the piano
Ethan Molina	Mayumi Torres
Chris Clemente	Janis Reyes
Angela Dominguez	Chris Clemente
Mayumi Torres	Ethan Molina
Joanna Cruz	Nathan Santos

determine which of the following shows (a) $A \cup B$; and (b) $A \cap B$?

Set 1	Set 2	Set 3	Set 4
Ethan Molina Chris Clemente Angela Dominguez Mayumi Torres Joanna Cruz	Mayumi Torres Ethan Molina Chris Clemente	Mayumi Torres Janis Reyes Chris Clemente Ethan Molina Nathan Santos	Ethan Molina Chris Clemente Angela Dominguez Mayumi Torres Joanna Cruz Janis Reyes Nathan Santos

2. Do the following exercises. Write your answers on the spaces provided:

   A = {0, 1, 2, 3, 4} B = {0, 2, 4, 6, 8} C = {1, 3, 5, 7, 9}

   Given the sets above, determine the elements and cardinality of:

   a. $A \cup B$ = ______________________
   b. $A \cup C$ = ______________________
   c. $A \cup B \cup C$ = ______________________
   d. $A \cap B$ = ______________________
   e. $B \cap C$ = ______________________
   f. $A \cap B \cap C$ = ______________________
   g. $(A \cap B) \cup C$ = ______________________

3. Let $W = \{ x \mid 0 < x < 3 \}$, $Y = \{ x \mid x > 2\}$, and $Z = \{x \mid 0 \leq x \leq 4 \}$. Determine (a) $(W \cup Y) \cap Z$; (b) $W \cap Y \cap Z$.

Summary

In this lesson, you learned the definition of union and intersection of sets. You also learned how use Venn diagram to represent the union and the intersection of sets. You also learned how to determine the elements that belong to the union and intersection of sets.

Lesson 2.2: Complement of a Set **Time: 1.5 hours**

Prerequisite Concepts: sets, universal set, empty set, union and intersection of sets, cardinality of sets, Venn diagrams

About the Lesson:

The complement of a set is an important concept. There will be times when one needs to consider the elements not found in a particular set A. You must know that this is when you need the complement of a set.

Objectives:

In this lesson, you are expected to:
1. Describe and define the complement of a set;
2. Find the complement of a given set;
3. Use Venn diagrams to represent the complement of a set.

Lesson Proper:

I. Problem

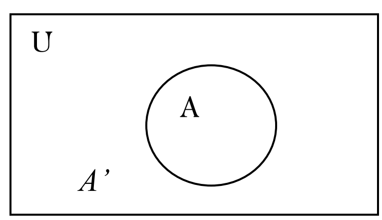
In a population of 8000 students, 2100 are Freshmen, 2000 are Sophomores, 2050 are Juniors and the remaining 1850 are either in their fourth or fifth year in university. A student is selected from the 8000 students and it is not a Sophomore, how many possible choices are there?

Discussion

Definition: The complement of a set A, written as A', is the set of all elements found in the universal set, U, that are not found in set A. The cardinality n (A') is given by

$$n(A') = n(U) - n(A) .$$

Venn diagram:



Examples:

1. Let U = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}, and A = {0, 2, 4, 6, 8}.
   Then the elements of A' are the elements from U that are not found in A.
   Therefore, A' = {1, 3, 5, 7, 9} and n (A') = 5

2. Let U = {1, 2, 3, 4, 5}, A = {2, 4} and B = {1, 5}. Then
   A' = {1, 3, 5}
   B' = {2, 3, 4}
   $A' \cup B'$ = {1, 2, 3, 4, 5} = U

3. Let U = {1, 2, 3, 4, 5, 6, 7, 8}, A = {1, 2, 3, 4} and B = {3, 4, 7, 8}.
   Then

$A' = \{5, 6, 7, 8\}$
$B' = \{1, 2, 5, 6\}$
$A' \cap B' = \{5, 6\}$

4. Let U = {1, 3, 5, 7, 9}, A = {5, 7, 9} and B = {1, 5, 7, 9}. Then
   $A \cap B = \{5, 7, 9\}$
   $(A \cap B)' = \{1, 3\}$

5. Let U be the set of whole numbers. If A = {x | x is a whole number and x > 10}, then $A' = \{x \mid x \text{ is a whole number and } 0 \leq x \leq 10\}$.

The opening problem asks for how many possible choices there are for a student that was selected and known to be a non-Sophomore. Let U be the set of all students and n (U) = 8000. Let A be the set of all Sophomores then n (A) = 2000. The set *A'* consists of all students in U that are not Sophomores and n (*A'*) = n (U) – n (A) = 6000. Therefore, there are 6000 possible choices for that selected student.

II. Activity

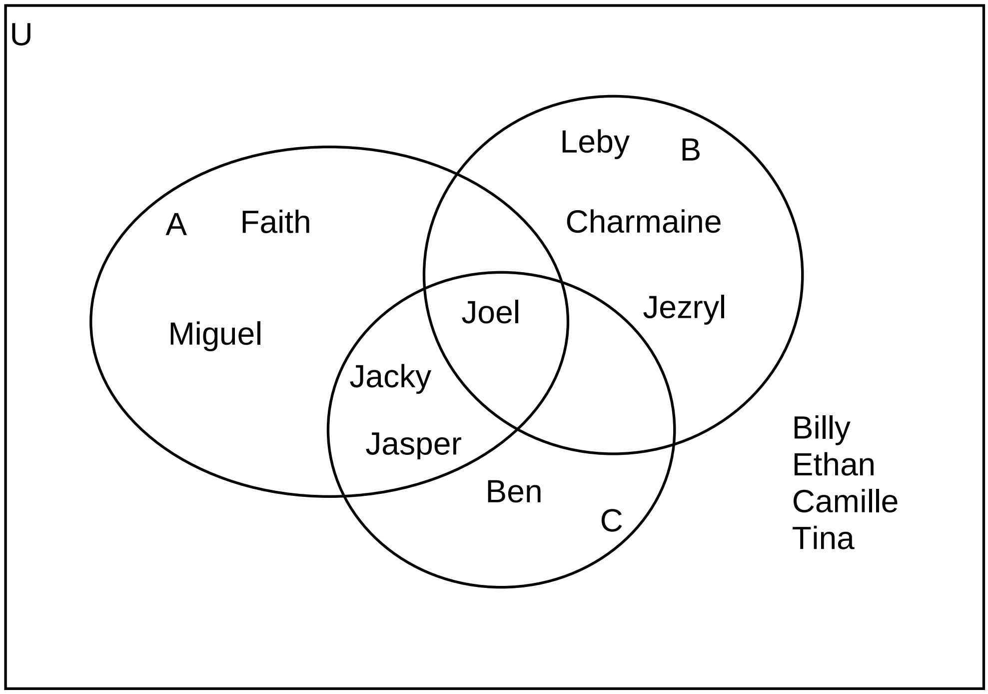
Shown in the table are names of students of a high school class by sets according to the definition of each set.

A **Likes Singing**	**B** **Likes Dancing**	**C** **Likes Acting**	**D** **Don't Like Any**
Jasper Faith Jacky Miguel Joel	Charmaine Leby Joel Jezryl	Jacky Jasper Ben Joel	Billy Ethan Camille Tina

After the survey has been completed, find the following sets.

a. U = ____
b. $A \cup B'$ = ______________________________________
c. $A' \cup C$ = ______________________________________
d. $(B \cup D)'$ = ____________________________________
e. $A' \cap B$ = ______________________________________
f. $A' \cap D'$ = _____________________________________
g. $(B \cap C)'$ = ____________________________________

The easier way to find the elements of the indicated sets is to use a Venn diagram showing the relationships of U, sets A, B, C, and D. Set D does not share any members with A, B, and C. However, these three sets share some members. The Venn diagram below is the correct picture:



Now, it is easier to identify the elements of the required sets.

a. U = {Ben, Billy, Camille, Charmaine, Ethan, Faith, Jacky, Jasper, Jezryl, Joel, Leby, Miguel, Tina}
b. A ∪ *B'* = {Faith, Miguel, Joel, Jacky, Jasper, Ben, Billy, Ethan, Camille, Tina}
c. *A'* ∪ C = {Jasper, Jacky, Joel, Ben, Leby, Charmaine, Jezryl, Billy, Ethan, Camille, Tina}
d. *(B ∪ D)'* = {Faith, Miguel, Jacky, Jasper, Ben}
e. *A'* ∩ B = {Leby, Charmaine, Jezryl}
f. *A'* ∩ *D'* = {Leby, Charmaine, Jezryl, Ben}
g. *(B ∩ C)'* = {Ben, Billy, Camille, Charmaine, Ethan, Faith, Jacky, Jasper, Jezryl, Leby, Miguel, Tina}

III. Exercises

1. True or False. If your answer is false, give the correct answer.

Let U = the set of the months of the year
X = {March, May, June, July, October}
Y = {January, June, July}
Z = {September, October, November, December}

a. *Z'* = {January, February, March, April, May, June, July, August}

____

b. *X'* ∩ *Y'* = {June, July}

____

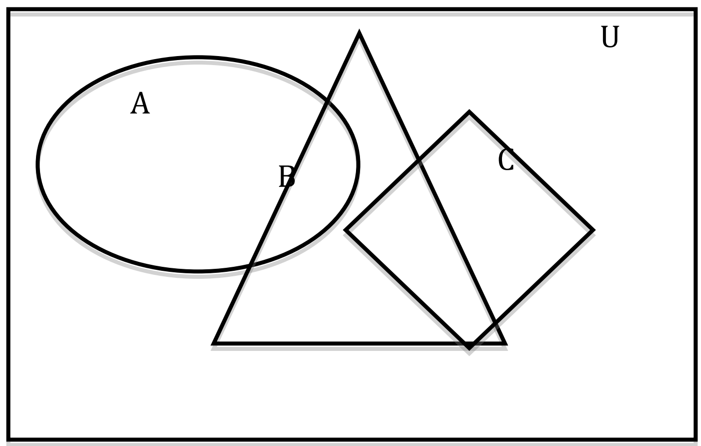
c. *X' ∪ Z'* = {January, February, March, April, May, June, July, August, September, November, December}

____

d. *(Y ∪ Z)'* = {February, March, April, May}

____

2. Place the elements in their respective sets in the diagram below based on the following elements assigned to each set:



U = {a, b, c, d, e, f, g, h, i, j}
A' = {a, c, d, e, g, j}
B' = {a, b, d, e, h, i}
C' = {a, b, c, f, h, i, j}

3. Draw a Venn diagram to show the relationships between sets U, X, Y, and Z, given the following information.

- U, the universal set contains set X, set Y, and set Z.
- $X \cup Y \cup Z = U$
- Z is the complement of X.
- Y' includes some elements of X and the set Z

Summary

In this lesson, you learned about the complement of a given set. You learned how to describe and define the complement of a set, and how it relates to the universal set, U and the given set.

Lesson 3: **Problems Involving Sets** **Time: 1 hour**

Prerequisite Concepts: Operations on Sets and Venn Diagrams

About the Lesson:

This is an application of your past lessons about sets. You will appreciate more the concepts and the use of Venn diagrams as you work through the different word problems.

Objectives:

In this lesson, you are expected to:

1. Solve word problems involving sets with the use of Venn diagrams
2. Apply set operations to solve a variety of word problems.

Lesson Proper:

I. Activity

Try solving the following problem:

In a class of 40 students, 17 have ridden an airplane, 28 have ridden a boat. 10 have ridden a train, 12 have ridden both an airplane and a boat. 3 have ridden a train only and 4 have ridden an airplane only. Some students in the class have not ridden any of the three modes of transportation and an equal number have taken all three.

a. How many students have used all three modes of transportation?
b. How many students have taken only the boat?

II. Questions/Points to Ponder (Post-Activity Discussion)

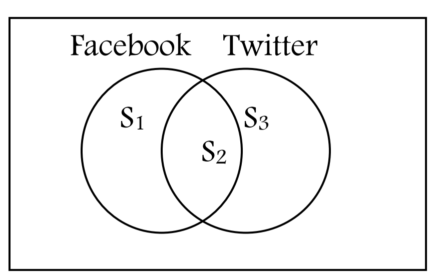
Venn diagrams can be used to solve word problems involving union and intersection of sets. Here are some worked out examples:

1. A group of 25 high school students were asked whether they use either Facebook or Twitter or both. Fifteen of these students use Facebook and twelve use Twitter.
    a. How many use Facebook only?
    b. How many use Twitter only?
    c. How many use both social networking sites?

<u>Solution:</u>

Let S_1 = set of students who use Facebook only
S_2 = set of students who use both social networking sites
S_3 = set of students who use Twitter only

The Venn diagram is shown below



Finding the elements in each region:

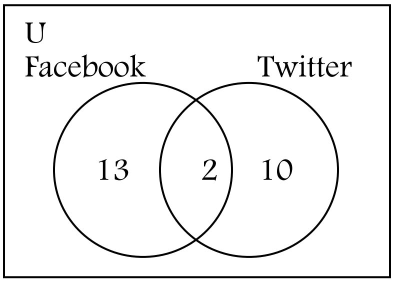
$$\begin{array}{l} n(S_1) + n(S_2) + n(S_3) = 25 \\ n(S_1) + n(S_2) \qquad\qquad = 15 \\ \hline \qquad\qquad\qquad n(S_3) = 10 \end{array}$$

But $n(S_2) + n(S_3) = 12$

$$\begin{array}{l} \hline n(S_2) \qquad = 2 \end{array}$$

$$\begin{array}{l} n(S_1) + n(S_2) + n(S_3) = 25 \\ \qquad\quad n(S_2) + n(S_3) = 12 \\ \hline n(S_1) \qquad\qquad\qquad = 13 \end{array}$$

The number of elements in each region is shown below



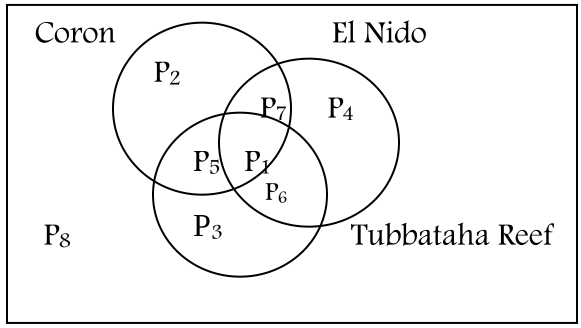
2. A group of 50 students went in a tour in Palawan province. Out of the 50 students, 24 joined the trip to Coron; 18 went to Tubbataha Reef; 20 visited El Nido; 12 made a trip to Coron and Tubbataha Reef; 15 saw Tubbataha Reef and El Nido; 11 made a trip to Coron and El Nido and 10 saw the three tourist spots.
    a. How many of the students went to Coron only?
    b. How many of the students went to Tubbataha Reef only?
    c. How many joined the El Nido trip only?
    d. How many did not go to any of the tourist spots?

<u>Solution:</u>

To solve this problem, let

P_1 = students who saw the three tourist spots
P_2 = those who visited Coron only
P_3 = those who saw Tubbataha Reef only
P_4 = those who joined the El Nido trip only
P_5 = those who visited Coron and Tubbataha Reef only
P_6 = those who joined the Tubbataha Reef and El Nido trip only
P_7 = those who saw Coron and El Nido only
P_8 = those who did not see any of the three tourist spots

Draw the Venn diagram as shown below and identify the region where the students went.



Determine the elements in each region starting from P_1.

P_1 consists of students who went to all 3 tourist spots. Thus, $n(P_1) = 10$.

$P_1 \cup P_5$ consists of students who visited Coron and Tubbataha Reef but this set includes those who also went to El Nido. Therefore, $n(P_5) = 12 – 10 = 2$ students visited Coron and Tubbatha Reef only.

$P_1 \cup P_6$ consists of students who went to El Nido and Tubbataha Reef but this set includes those who also went to Coron. Therefore, $n(P_6) = 15 – 10 = 5$ students visited El Nido and Tubbataha Reef only.

$P_1 \cup P_7$ consists of students who went to Coron and El Nido but this set includes those who also went to Tubbataha Reef. Therefore, $n(P_7) = 11 – 10 = 1$ student visited Coron and El Nido only.

From here, it follows that

$n(P_2) = 24 – n(P_1) – n(P_5) – n(P_7) = 24 – 10 – 2 – 1 =$ *11 students visited Coron only.*

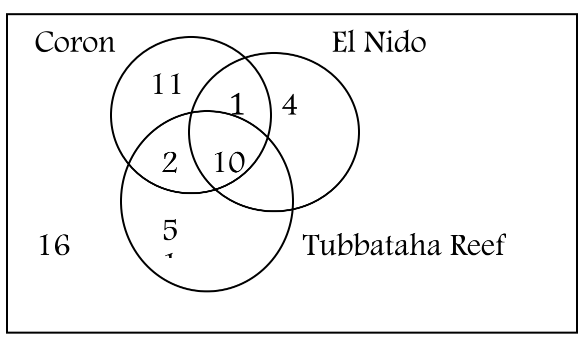
$n(P_3) = 18 – n(P_1) – n(P_5) – n(P_6) = 18 – 10 – 2 – 5 =$ *1 student visited Tubbataha Reef only*

$n(P_4) = 20 – n(P_1) – n(P_6) – n(P_7) = 20 – 10 – 5 – 1 =$ *4 students visited Coron and El Nido only.*

Therefore

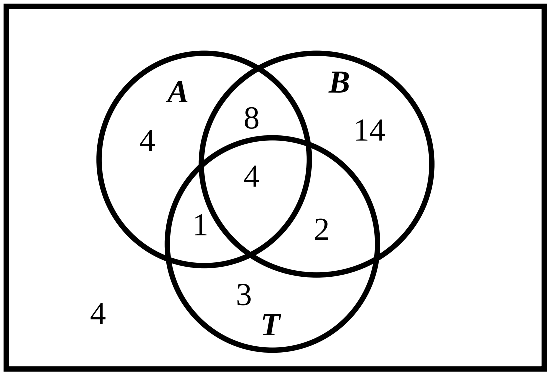
$n(P_8) = 50 – n(P_1) – n(P_2) – n(P_3) – n(P_4) – n(P_5) – n(P_6) – n(P_7) =$ *16 students did not visit any of the three spots.*

The number of elements is shown below.



Now, what about the opening problem?

Solution to the Opening Problem (Activity):

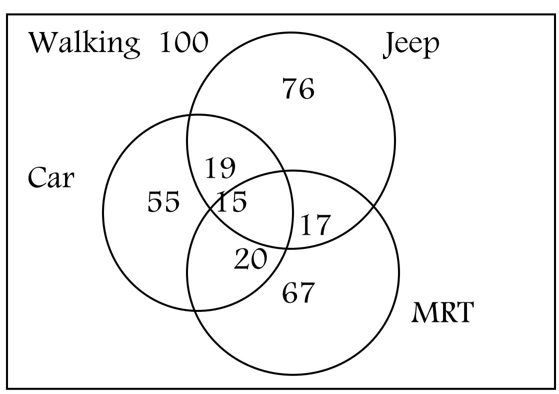


Can you explain the numbers?

III. Exercises

Do the following exercises. Represent the sets and draw a Venn diagram when needed.

1. If **A** is a set, give two subsets of **A**.
2. **(a)** If A and B are finite sets and $A \subset B$, what can you say about the cardinalities of the two sets?
   (b) If the cardinality of A is less than the cardinality of B, does it follow that $A \subset B$?
3. If **A** and **B** have the same cardinality, does it follow that **A = B?** Explain.
4. If $A \subset B$ and $B \subset C$. Does it follow that $A \subset C$? Illustrate your reasoning using a Venn diagram.
5. Among the 70 kids in Barangay Magana, 53 like eating in Jollibee while 42 like eating in McDonalds. How many like eating both in Jollibee and in McDonalds? in Jollibee only? in McDonalds only?
6. The following diagram shows how all the First Year students of Maningning High School go to school.



   a. How many students ride in a car, jeep and the MRT going to their school? ________
   b. How many students ride in both a car and a jeep? ________
   c. How many students ride in both a car and the MRT? ________

d. How many students ride in both a jeep and the MRT? ________

e. How many students go to school
   in a car only ______ a jeep only ________
   in the MRT only __________ walking ________

f. How many students First Year students of Maningning High School are there? _________

7. The blood-typing system is based on the presence of proteins called antigens in the blood. A person with antigen A has blood type A. A person with antigen B has blood type B, and a person with both antigens A and B has blood type AB. If no antigen is present, the blood type is O. Draw a Venn diagram representing the ABO System of blood typing.

   A protein that coats the red blood cells of some persons was discovered in 1940. A person with the protein is classified as Rh positive (Rh+), and a person whose blood cells lack the protein is Rh negative (Rh–). Draw a Venn diagram illustrating all the blood types in the ABO System with the corresponding Rh classifications.

Summary

In this lesson, you were able to apply what you have learned about sets, the use of a Venn diagram and set operations in solving word problems.

Lesson 4.1: Fundamental Operations on Integers: Addition of Integers
Time: 1 hour

Pre-requisite Concepts: Whole numbers, Exponents, Concept of Integers

About the Lesson: This lesson focuses on addition of integers using different approaches. It is a review of what the students learned in Grade 6.

Objectives:

In this lesson, you are expected to:

1. Add integers using different approaches;
2. Solve word problems involving addition of integers.

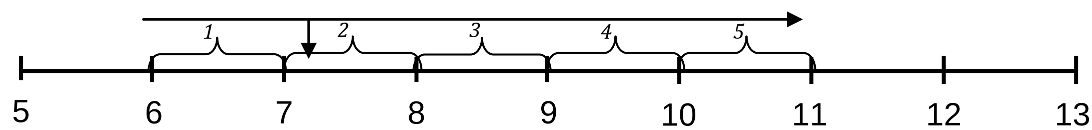
Lesson Proper:

I. Activity

Study the following examples:

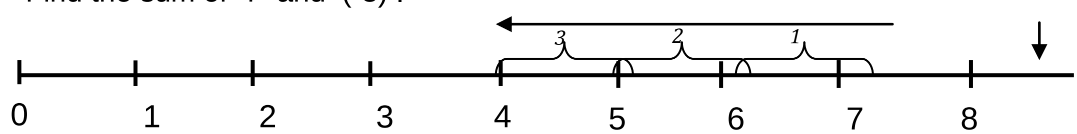
A. Addition Using Number Line

1. Use the number line to find the sum of 6 & 5.



On the number line, start with point 6 and count 5 units to the right. At what point on the number line does it stop ?
It stops at point 11; hence, 6 + 5 = 11.

2. Find the sum of 7 and (-3) .



On the number line, start from 7 and count 3 units going to the left since the sign of 3 is negative.
At which point does it stop?
It stops at point 4; hence, (-3) + (7) = 4.

After the 2 examples, can you now try the next two problems?

a. (-5) + (-4) b. (-8) + (5)

We now have the following generalization:
Adding a positive integer n to m means moving along the real line a distance of n units to the right from m. Adding a negative integer $-n$ to m means moving along the real line a distance of n units to the left from m.

B. Addition Using Signed Tiles

This is another device that can be used to represent integers.The tile $\boxed{+}$ represents integer 1, the tile $\boxed{-}$ represents -1, and the flexible $\boxed{+\quad -}$ represents 0.

Recall that a number and its negative cancel each other under the operation of addition. This means

$4 + \quad -4 \quad = 0$

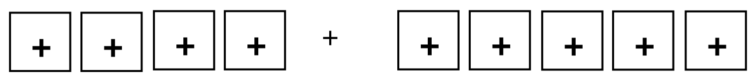
$15 + \quad -15 \quad = 0$

$-29 \quad + 29 = 0$

In general, $n + \quad -n \quad = \quad -n \quad + n = 0$.

Examples:

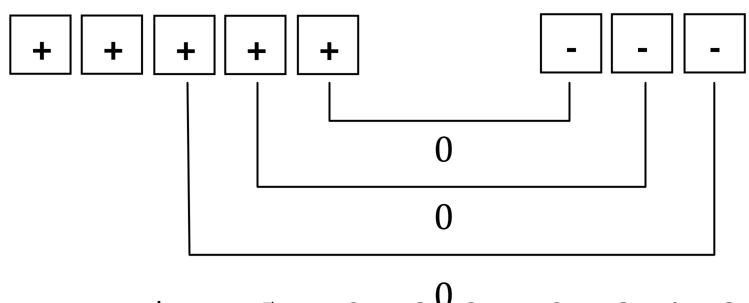
1. 4 + 5 ------→



four (+1) + five (+1)

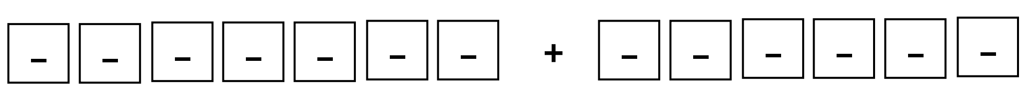
hence, 4 + 5 = 9

2. 5 + (-3) -----→



hence, $5 + \quad -3 \quad = 2 + 3 + \quad -3 \quad = 2 + 0 = 2$

3. $-7 \quad + \quad -6$



hence $-7 \quad + \quad -6 \quad = -13$

Now, try these:

1. (-5) + (-11)
2. (6) + (-9)

II. Questions/ Points to Ponder

Using the above model, we summarize the procedure for adding integers as follows:

1. If the integers have the same sign, just add the positive equivalents of the integers and attach the common sign to the result.

   a. 27 + 30 = + (/27/ + /30/)
      = + (/57/)
      = + 57

   b. (-20) + (-15) = - (/20/ + /15/)
      = - (20 + 15)
      = - (35)
      = - 35

2. If the integers have different signs, get the difference of the positive equivalents of the integers and attach the sign of the larger number to the result.

   a. (38) + (-20)
      Get the difference between 38 and 20: 18
      Since 38 is greater than 20, the sign of the sum is positive.
      Hence $38 + \; -20 \; = 18$

   b. $-42 \; + 16$
      Get the difference between 42 and 16: 26
      Since 42 is greater than 16, the sum will have a negative sign.
      Hence $-42 \; + 16 = -26$

If there are more than two addends in the problem the first step to do is to combine addends with same signs and then get the difference of their sums.

Examples:

1. $-14 + 22 + 8 + -16 = - \; 14 + 16 + \; 22 + 8$
   $= -30 + 30 = 0$

2. $31 + 70 + 9 + \; -155 = \; 31 + 70 + 9 + \; -155$
   $= 110 + \; -155 \; = -45$

III. Exercises

A. Who was the first English mathematician who first used the modern symbol of equality in 1557?

   (To get the answer, compute the sums of the given exercises below. Write the letter of the problem corresponding to the answer found in each box at the bottom).

A 25 + 95	C. (30) + (-20)	R 65 + 75
B 38 + (-15)	D. (110) + (-75)	O (-120) + (-35)

O 45 + (-20)	T. (16) + (-38)	R (165) + (-85)
R (-65) + (-20)	R (-65) + (-40)	E 47 + 98
E (78) + (-15)	E (-75) + (20)	

-105	25	63	23	-85	-22

140	-55	10	-155	80	35	145

B. Addthe following:

1. (18) + (-11) + (3)
2. (-9) + (-19) + (-6)
3. (-4) + (25) + (-15)
4. (50) + (-13) + (-12)
5. (-100) + (48) + (49)

C. Solve the following problems:

1. Mrs. Reyes charged P3,752.00 worth of groceries on her credit card. Find her balance after she made a payment of P2,530.00.
2. In a game, Team Azcals lost 5 yards in one play but gained 7 yards in the next play. What was the actual yardage gain of the team?
3. A vendor gained P50.00 on the first day; lost P28.00 on the second day, and gained P49.00 on the third day. How much profit did the vendor gain in 3 days?
4. Ronnie had PhP2280 in his checking account at the beginning of the month. He wrote checks for PhP450, P1200, and PhP900. He then made a deposit of PhP1000. If at any time during the month the account is overdrawn, a PhP300 service charge is deducted. What was Ronnie's balance at the end of the month?

Summary

In this lesson, you learned how to add integers using two different methods. The number line model is practical for small integers. For larger integers, the signed tiles model provides a more useful tool.

Lesson 4.2: Fundamental Operation on Integers:Subtraction of Integers
Time: 1 hour

Prerequisite Concepts: Whole numbers, Exponents, Concept of Integers, Addition of Integers

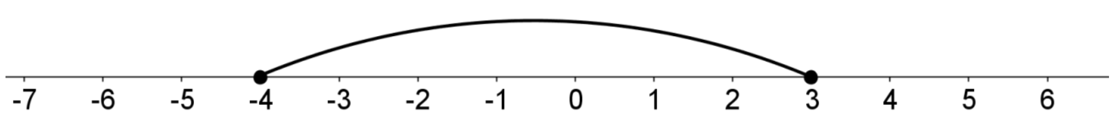
About the Lesson: This lesson focuses on the subtraction of integers using different approaches. It is a review of what the students learned in Grade 6.

Objectives:
In this lesson, you are expected to:
1. Subtract integers using
   a. Number line
   b. Signed tiles
2. Solve problems involving subtraction of integers.

Lesson Proper:

I. Activity
Study the material below.

1. *Subtraction as the reverse operation of addition.*
   Recall how subtraction is defined. We have previously defined subtraction as the reverse operation of addition. This means that when we ask "what is 5 minus 2?", we are also asking "what number do we add to 2 in order to get 5?" Using this definition of subtraction, we can deduce how subtraction is done using the number line.

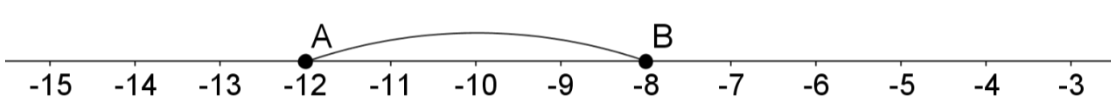


a. Suppose you want to compute $-4 \ -3$. You ask "What number must be added to 3 to get -4?

   To get from 3 to -4, you need to move 7 units to the left. This is equivalent to adding -7 to 3. Hence in order to get -4, -7 must be added to 3. Therefore,

$(-4) - 3 = -7$

b. Compute $(-8) - (-12)$
   What number must be added to -12 to get -8?



To go from -12 to -8, move 4 units to the right, or equivalently, add 4. Therefore,
$(-8) - \quad -12 \quad = 4$

2. *Subtraction as the addition of the negative*
Subtraction is also defined as the addition of the negative of the number. For example, $5 - 3 = 5 + (-3)$. Keeping in mind that n and $-n$ are negatives of each other, we can also have $5 - \quad -3 \quad = 5 + 3$. Hence the examples above can be solved as follows:

$$-4 \quad - 3 = (-4) + \quad -3 \quad = -7$$

$$-8 \quad - \quad -12 \quad = (-8) + 12 = 4$$

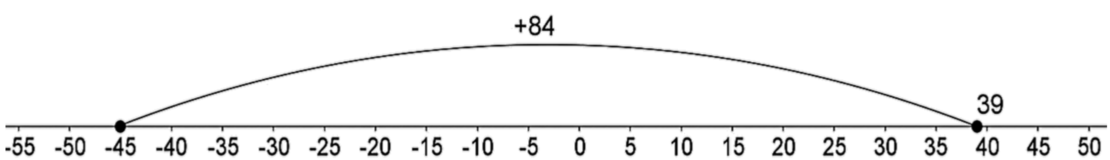
This definition of subtraction allows the conversion of a subtraction problem to an addition problem.

Problem:
Subtract (-45) from 39 using the two definitions of subtraction.

Can you draw your number line?Where do you start numbering it to make the line shorter?

Solution:

1. $39 - (-45)$
What number must be added to -45 in order to obtain 39?



$$39 - \quad -45 \quad = 84$$

2. $39 - \quad -45 \quad = 39 + 45 = 84$

II. Questions/Points to Ponder

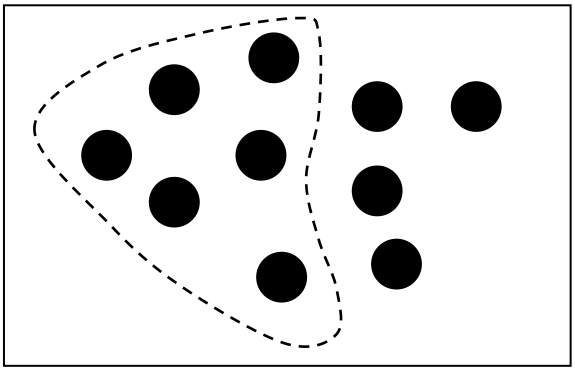
Rule in Subtracting Integers
In subtracting integers, add the negative of the subtrahend to the minuend,

$$a - b = a + \quad -b$$
$$a - \quad -b \quad = a + b$$

Using signed tiles or colored counters
Signed tiles or colored counters can also be used to model subtraction of integers. In this model, the concept of subtraction as "taking away" is utilized.

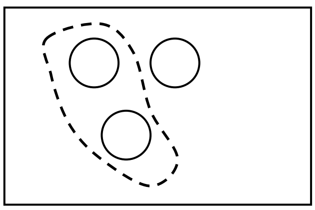
Examples:

1. $10 - 6$ means take away 6 from 10. Hence



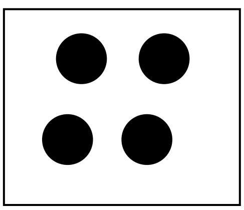
$10 - 6 = 4$

2. $-3 - (-2)$

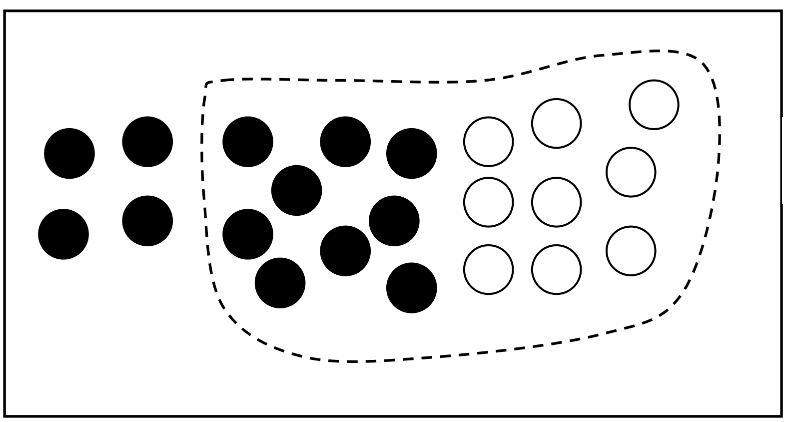


$-3 - (-2) = -1$

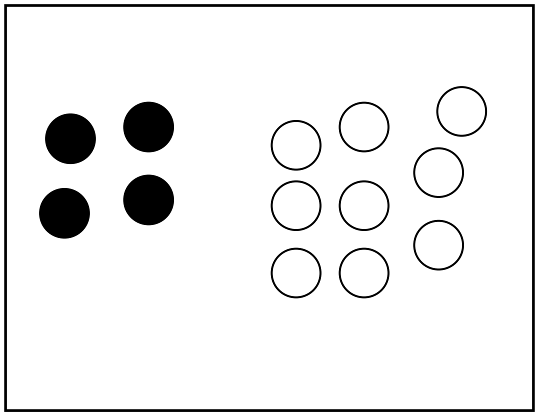
3. $4 - 9$



Since there are not enough counters from which to take away 9, we add 9 black counters and 9 white counters. Remember that these added counters are equivalent to zero.



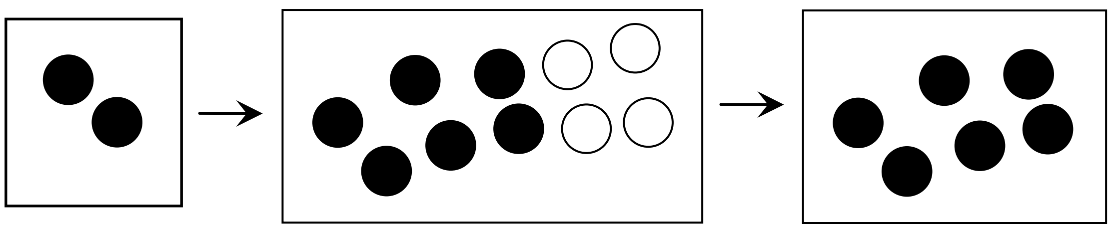
We now take away 9 black counters.



Notice that this configuration is the same configuration for $4 + (-9)$.

We proceed with the addition and obtain the answer -5

4. $2 - (-4)$



Hence $2 - \quad -4 \quad = 6$

The last two examples above illustrate the definition of subtraction as the addition of the negative.
$m - n = m - n + \ n + \ -n \ = \ m - n + n \ + \ -n \ = m + (-n)$

III. Exercices

A. What is the name of the 4th highest mountain in the world?
(Decode the answer by finding the difference of the following subtraction problems. Write the letter to the answer corresponding to the item in the box provided below:

O Subtract (-33) from 99
L Subtract (-30) from 49
H 18 less than (-77)
E Subtract (-99) from 0
T How much is 0 decreased by (-11)?
S (-42) – (-34) – (-9) - 18

79	-95	132	11	-17	99

B. Mental Math

Give the difference:

1. 53 -25
2. (-6) - 123
3. (-4) - (-9)
4. 6 - 15
5. 16 - (-20)
6. 25 - 43
7. (-30) - (-20)
8. (-19) - 2
9. 30 –(-9)
10. (-19) - (-15)

C. Solve the ff. Problems:

1. Maan deposited P53,400.00 in her account and withdrew P19,650.00 after a week. How much of her money was left in the bank?
2. Two trains start at the same station at the same time. Train A travels 92km/h, while train B travels 82km/h. If the two trains travel in opposite directions, how far apart will they be after an hour?
   If the two trains travel in the same direction, how far apart will they be in two hours?
3. During the Christmas season. The student gov't association was able to solicit 2,356 grocery items and was able to distribute 2,198 to one barangay. If this group decided to distribute 1,201 grocery items to the next barangay, how many more grocery items did they need to solicit?

Summary

In this lesson, you learned how to subtract integers by reversing the process of addition, and by converting subtraction to addition using the negative of the subtrahend.

Lesson 4.3: Fundamental Operations on Integers: Multiplication of Integers
Time: 1 hour

Prerequisite Concepts: Operations on whole numbers, addition and subtraction of integers

About the Lesson: This is the third lesson on operations on integers. The intent of the lesson is to deepen what students have learned in Grade 6, by expounding on the meaning of multiplication of integers.

Objective:
In this lesson; you are expected to:
1. Multiply integers.
2. Apply multiplication of integers in solving problems

Lesson Proper:
I. Activity
Answer the following question.

How do we define multiplication?

We learned that with whole numbers, *multiplication is repeated addition*. For example, 4×3 means three groups of 4. Or, putting it into a real context, 3 cars with 4 passengers each, how many passenger in all? Thus
$4 \times 3 = 4 + 4 + 4 = 12$.

But, if there are 4 cars with 3 passengers each, in counting the total number of passengers, the equation is $3 \times 4 = 3 + 3 + 3 + 3 = 12$. We can say then that $4 \times 3 = 3 \times 4$and

$4 \times 3 = 3 \times 4 = 3 + 3 + 3 + 3 = 12$.

We extend this definition to multiplication of a negative integer by a positive integer. Consider the situation when a boy loses P6 for 3 consecutive days.His total loss for three days is
-6×3. Hence, we could have
$-6 \times 3 = -6 + -6 + -6 = -18$.

II. Questions/Points to Ponder
The following examples illustrate further how integers are multiplied.

Example 1. Multiply : 5 ×(-2)
However,
5 × (-2) = (-2) × (5)
Therefore:
(-2) × (5)= (-2) + (-2) + (-2) + (-2) + (-2) = -10

The result shows that the product of a negative multiplier and a positive multiplicand is a negative integer.

Generalization:Multiplying unlike signs
We know that adding negative numbers means adding their positive equivalents and attaching the negative sign to the result, then

$$a \times (-b) = (-b) \times a = \underbrace{(-b) + (-b) + \cdots + (-b)}_{a addends} = -\underbrace{(b + b + \cdots + b)}_{a addends} = -ab$$

for any positive integers a and b.

We know that any whole number multiplied by 0 gives 0. Is this true for any integer as well? The answer is YES. In fact, any number multiplied by 0 gives 0. This is known as the ***Zero Property***.

What do we get when we multiply two negative integers?

Example 2. Multiply: (-8) × (-3)

We know that $(-8) \times 3 = -24$.
Therefore,
$-24 + (-8) \times (-3) = (-8) \times 3 + (-8) \times (-3)$
$= (-8) \times [3 + (-3)]$ *(Distributive Law)*
$= (-8) \times 0$ *(3 and -3 are additive inverses)*
$= 0$ *(Zero Property)*

The only number which when added to -24 gives 0 is the additive inverse of -24. Therefore, $(-8) \times (-3)$ is the additive inverse of 24, or $(-8) \times (-3) = 24$

The result shows that the product of two negative integers is a positive integer.

Generalization:Multiplying Two Negative Integers
If a and b are positive integers, then $(-a) \times (-b) = ab$.

Rules in Multiplying Integers:
In multiplying integers, find the product of their positive equivalents.
1. If the integers have the same signs, their product is positive.
2. If the integers have different signs their product is negative.

III. Exercises
A. Find the product of the following:

1. (5)(12)
2. (-8)(4)
3. (-5)(3)(2)
4. (-7)(4)(-2)
5. (3)(8)(-2)
6. (9)(-8)(-9)
7. (-9)(-4)(-6)

MATH DILEMMA

B. How can a person fairly divide 10 apples among 8 children so that each child has the same share.
To solve the dilemma, match the letter in column II with the number that corresponds to the numbers in column I.

Column I	Column II	
1. (6)(-12)	C	270
2. (-13)(-13)	P	-72
3. (19)(-17)	E	300
4. (-15)(29)	K	-323
5. (165)(0)	A	-435
6. (-18)(-15)	M	0
7. (-15)(-20)	L	16
8. (-5)(-5)(-5)	J	-125
9. (-2)(-2)(-2)(-2)	U	169
10. (4)(6)(8)	I	192

____ ____ ____ ____
5 4 3 7

____ ____ ____ ____ ____
4 1 1 9 7

____ ____ ____ ____ ____
8 2 10 6 7

C. Problem Solving
1. Jof has twenty P5 coins in her coin purse. If her niece took 5 of the coins, how much has been taken away?
2. Mark can type 45 words per minute, how many words can Mark type in 30 minutes?
3. Give an arithmetic equation which will solve the following
   a. The messenger came and delivered 6 checks worth PhP50 each. Are you richer or poorer? By how much?
   b. The messenger came and took away 3 checks worth PhP120 each. Are you richer or poorer? By how much?
   c. The messenger came and delivered 12 bills for PhP86 each. Are you richer or poorer? By how much?
   d. The messenger came and took away 15 bills for PhP72 each. Are you richer or poorer? By how much?

Summary

This lesson emphasized the meaning of multiplication to set the rules for multiplying integers. To multiply integers, first find the product of their positive equivalents. If the integers have the same signs, their product is positive. If the integers have different signs their product is negative.

Lesson 4.4: Fundamental Operations on Integers: Division of Integers
Time: 1 hour

Prerequisite Concepts: Addition and subtraction of Integers, Multiplication of Integers

About the Lesson: Like in the previous lessons, this lesson is meant to deepen students' understanding of the division operation on integers. The concept of division used here relies on its relationship to multiplication.

Objective:

In this lesson you are expected to:

1. Find the quotient of two integers.
2. Solve problems involving division of integers.

Lesson Proper:

I. Activity

Answer the following questions:

What is (-51) ÷ (-3)?
What is (-51) ÷ 3?
What is 51 ÷ (-3)?
What are the rules in dividing integers?

II. Questions/Points to Ponder

We have learned that Subtraction is the inverse operation of Addition,
In the same manner, Division is the inverse operation of Multiplication.

*Example 1.*Find the quotient of (-51) and (-3)

Solution:

Since division is the inverse of multiplication, determine whatnumber multiplied by (-3) produces (-51).

If we ignore the signs for the meantime, we know that

$$3 \times 17 = 51$$

We also know that in order to get a negative product, the factors must have different signs. Hence

$$-3 \quad \times 17 = -51$$

Therefore

(-51) ÷ (-3) = 17

Example 2. What is $-57 \quad \div 19$?

Solution: $19 \times 3 = 57$

Hence

$$19 \times \quad -3 \quad = -57$$

Therefore

$$-57 \quad \div 19 = -3$$

*Example 3.*Show why 273 ÷ (–21) = –13.

Solution: (-13) × (-21) = 57

Therefore, 273 ÷ (–21) = –13

Generalization

The quotient of two integers with the same signs is a positive integer and the quotient of two integers having unlike signs is a negative integer.However, division by zero is not possible.

When several operations have to be performed, the GEMDAS rule applies.

Example 4. Perform the indicated operations

1. $2 - 3 \times \; -4$
2. $4 \times 5 + 72 \div \; -6$
3. $9 + 6 - \; -3 \; \times 12 \div \; -9$

Solution:

1. $2 - 3 \times \; -4 \; = 2 - \; -12 \; = 14$
2. $4 \times 5 + 72 \div \; -6 \; = 20 + \; -12 \; = 8$
3. $9 + 6 - \; -3 \; \times 12 \div \; -9 \; = 9 + 6 - \; -36 \; \div \; -9 \; = 9 + 6 - 4 = 11$

III. Exercises:

A. Compute the following
   1. $\; 10 + 15 \; - 4 \times 3 + 7 \times (-2)$
   2. $22 \times 9 \div \; -6 \; - 5 \times 8$
   3. $36 \div 12 + 53 + (-30)$
   4. $\; 30 + 26 \; \div [(-2) \times 7]$
   5. $(124 - 5 \times 12) \div 8$

B. What was the original name for the butterfly?

To find the answer find the quotient of each of the following and write the letter of the problems in the box corresponding to the quotient.

R (-352) ÷

U (-120) ÷ 8

T (128) ÷ -

L (-444) ÷ (-12)

Y (144) ÷ -3

B (108) ÷ 9

E (168) ÷ 6

T (-147) ÷ 7

F (-315) ÷ (-

9	37	-15	-8	-8	28	-16	12	-48

C. Solvethe following problems:
    1. Vergara’s store earned P8750 a week, How much is her average earning in a day?
    2. Russ worked in a factory and earned P7875.00 for 15 days. How much is his earning in a day?
    3. There are 336 oranges in 12 baskets. How many oranges are there in 3 baskets?
    4. A teacher has to divide 280 pieces of graphing paper equally among his 35 students. How many pieces of graphing paper will each student recieve?
    5. A father has 976 sq. meters lot, he has to divide it among his 4 children. What is the share of each child?

D. Complete the three-by-three magic square (that is, the sums of the numbers in each row, in each column and in each of the diagonals are the same) using the numbers -10, -7, -4, -3, 0, 3, 4, 7, 10. What is the sum for each row, column and diagonal?

Summary

Division is the reverse operation of multiplication. Using this definition, it is easy to see that the quotient of two integers with the same signs is a positive integer and the quotient of two integers having unlike signs is a negative integer.

Lesson 5: Properties of the Operations on Integers **Time: 1.5 hours**

Prerequisite Concepts: Addition, Subtraction, Multiplication and Division of Integers

About the Lesson:

This lesson will strengthen the skills of students in performing the fundamental operations of integers. Knowledge of these will serve as an axiom/guide in performing said operations. In addition, this will help students solve problems including real life situations in algebra. This section also discusses how an application of the properties of real numbers in real life situations can be helpful in sustaining harmonious relationships among people.

Objectives

In this lesson, you are expected to:

1. State and illustrate the different properties of the operations on integers
   a. closure
   b. commutative
   c. associative
   d. distributive
   e. identity
   f. inverse
2. Rewrite given expressions according to the given property.

Lesson Proper:

I. A. Activity 1: Try to reflect on these . . .

1. Give at least 5 words synonymous to the word "property".

Activity 2: PICTIONARY GAME: DRAW AND TELL!

Needed Materials:
5 strips of cartolina with adhesive tape where each of the following words will be written:

- Closure
- Commutative
- Associative
- Distributive
- Identity
- Inverse

Printed Description:

- Stays the same
- Swapping /Interchange
- Bracket Together/Group Together
- Share Out /Spread Out /Disseminate
- One and the Same/Alike
- Opposite/Contrary

Rules of the Game:
The mission of each player holding a strip of cartolina is to let the classmates guess the hidden word by drawing symbols, figures or images on the board without any word.

If the hidden property is discovered, a volunteer from the class will give his/her own meaning of the identified words. Then, from the printed descriptions, he/she can choose the appropriate definition of the disclosed word and verify if his/her initial description is correct.

The game ends when all the words are revealed.

The following questions will be answered as you go along to the next activity.

- What properties of real numbers were shown in the Pictionary Game? Give one example and explain.
- How are said properties seen in real life?

Activity 3: SHOW AND TELL!
Determine what kind of property of real numbers is being illustrated in the following images:
A. Fill in the blanks with the correct numerical values of the motorbike and bicycle riders.

_______ _______

+

equals

+

If ***a*** represents the number of motorbike riders and ***b*** represents the number of bicycle riders, show the mathematical statement for the diagram below.

________ + ________ = ________ + ________

Guide Questions:

- What operation is used in illustrating the diagram?
- What happened to the terms in both sides of the equation?
- Based on the previous activity, what property is being applied?
- What if the operation is replaced by multiplication, will the same property be applicable? Give an example to prove your answer.
- Define the property.
- Give a real life situation in which the commutative property can be applied.

- Test the property on subtraction and division operations by using simple examples. What did you discover?

B. Fill in the blanks with the correct numerical values of the set of cellphones, ipods and laptops.

________ ________ ________

equals

________ ________ ________

If ***a*** represents the number of cellphones, ***b*** represents the ipods and **c** represents the laptops, show the mathematical statement for the diagram below.

(________ + ________) +________ = ________ + (________ + ________)

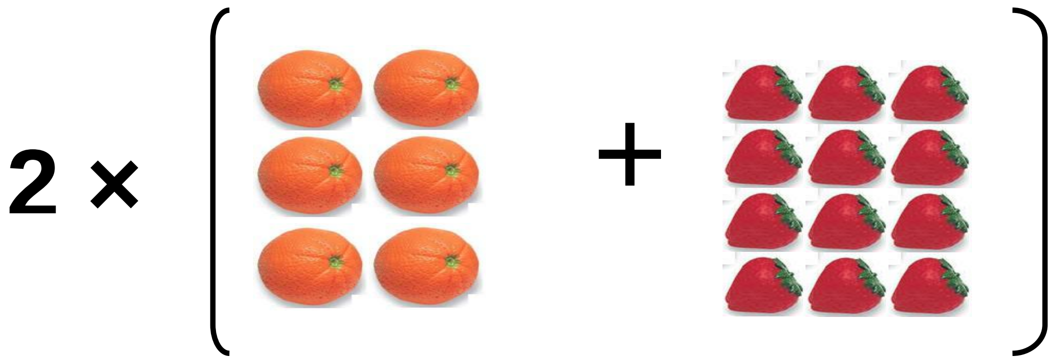
Guide Questions:

- What operation is used in illustrating the diagram?
- What happened to the groupings of the given sets that correspond to both sides of the equation?
- Based on the previous activity, what property is being applied?
- What if the operation is replaced by multiplication, will the same property be applicable? Give an example to prove your answer.
- Define the property.

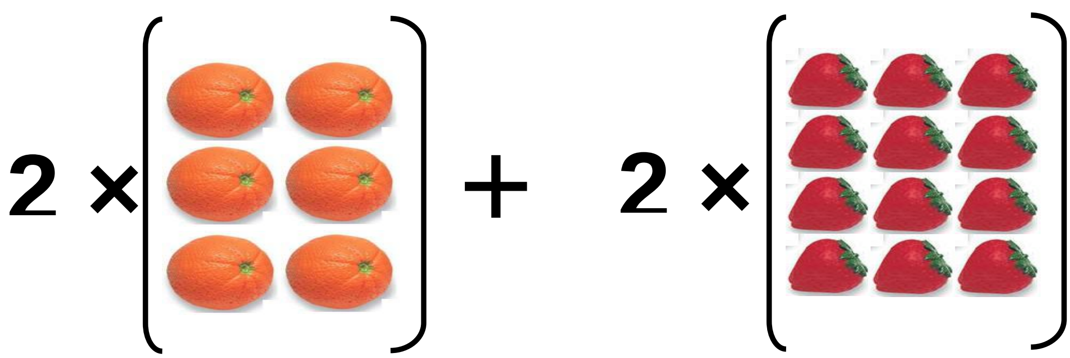
- Give a real life situation wherein associative property can be applied.
- Test the property on subtraction and division operations by using simple examples. What did you discover?

C. Fill in the blanks with the correct numerical values of the set of oranges and set of strawberries.

_______ _______



equals



_______ _______

If ***a*** represents the multiplier in front, ***b*** represents the set of oranges and **c** represents the set of strawberries, show the mathematical statement for the diagram below.

_______ (_______+_______) = ______ • _______ + _______• ______

Guide Questions:

- Based on the previous activity, what property is being applied in the images presented?
- Define the property.
- In the said property can we add/subtract the numbers inside the parentheses and then multiply or perform multiplication first and then addition/subtraction? Give an example to prove your answer.
- Give a real life situation wherein distributive property can be applied.

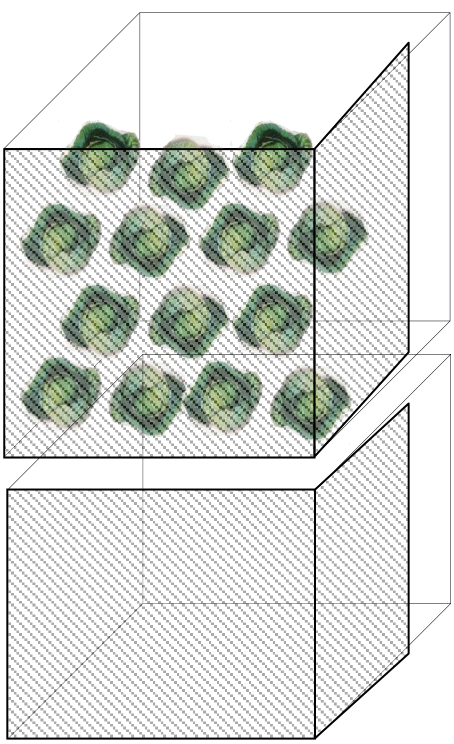
D. Fill in the blanks with the correct numerical representation of the given illustration.

_______ _______ _______

Guide Questions:

- Based on the previous activity, what property is being applied in the images presented?
- What will be the result if you add something represented by any number to nothing represented by zero?
- What do you call zero "0" in this case?
- Define the property.
- Is there a number multiplied to any number that will result to that same number? Give examples.
- What property is being illustrated? Define.
- What do you call one "1" in this case?

E. Give the correct mathematical statement of the given illustrations. To do this, refer to the guide questions below.



Guide Questions:

- How many cabbages are there in the crate?
- Using integers, represent “put in 14 cabbages” and “remove 14 cabbages”? What will be the result if you add these representations?
- Based on the previous activity, what property is being applied in the images presented?
- What will be the result if you add something to its negative?
- What do you call the opposite of a number in terms of sign? What is the opposite of a number represented by a?
- Define the property.
- What do you mean by reciprocal and what is the other term used for it?
- What if you multiply a number say 5 by its multiplicative inverse $\frac{1}{5}$, what will be the result?
- What property is being illustrated? Define.

Important Terms to Remember

The following are terms that you must remember from this point on.

1. Closure Property
   Two integers that are added and multiplied remain as integers. The set of integers is closed under addition and multiplication.
2. Commutative Property
   Changing the order of two numbers that are either being added or multiplied does not change the value.
3. Associative Property
   Changing the grouping of numbers that are either being added or multiplied does not change its value.
4. Distributive Property
   When two numbers have been added / subtracted and then multiplied by a factor, the result will be the same when each number is multiplied by the factor and the products are then added / subtracted.
5. Identity Property
   Additive Identity
   - states that the sum of any number and 0 is the given number. Zero, “0” is the additive identity.

   Multiplicative Identity
   - states that the product of any number and 1 is the given number, a • 1 = a. One, “1” is the multiplicative identity.
6. Inverse Property
   In Addition
   - states that the sum of any number and its additive inverse, is zero. The additive inverse of the number a is –a.

   In Multiplication
   - states that the product of any number and its multiplicative inverse or reciprocal, is 1.The multiplicative inverse of the number a is $\frac{1}{a}$.

Notations and Symbols

In this segment, you will learn some of the notations and symbols pertaining to properties of real number applied in the operations of integers.

Closure Property under addition and multiplication	a, b $\in$ *I*, then a+b $\in$ *I*, a•b $\in$ *I*
Commutative property of addition	a + b = b + a
Commutative property of multiplication	ab = ba
Associative property of addition	(a + b) + c = a + (b + c)
Associative property of multiplication	(ab) c = a (bc)
Distributive property	a(b + c) = ab + ac
Additive identity property	a + 0 = a
Multiplicative identity property	a • 1 = a
Multiplicative inverse property	$\frac{1}{a} \bullet a = 1$
Additive inverse property	a + (-a) = 0

III. Exercises

A. **Complete the Table:** Which property of real number justifies each statement?

Given	Property
1. 0 + (-3) = -3	
2. 2(3 - 5) = 2(3) - 2(5)	
3. (- 6) + (-7) = (-7) + (-6)	
4. 1 x (-9) = -9	
5. $-4 \times -\frac{1}{4} = 1$	
6. 2 x (3 x 7) = (2 x 3) x 7	
7. 10 + (-10) = 0	
8. 2(5) = 5(2)	
9. $1 \times (-\frac{1}{4}) = -\frac{1}{4}$	
10. (-3)(4 + 9) = (-3)(4) + (-3)(9)	

B. Rewrite the following expressions using the given property.

1. 12a – 5a → Distributive Property
2. (7a)b → Associative Property
3. 8 + 5 → Commutative Property
4. -4(1) → Identity Property
5. 25 + (-25) → Inverse Property

C. Fill in the blanks and determine what properties were used to solve the equations.

1. 5 x (_____ + 2) = 0
2. -4 + 4 = ______
3. -6 + 0 = ______
4. (-14 + 14) + 7 = ______
5. 7 x (______ + 7) = 49

Summary

The lesson on the properties or real numbers explains how numbers or values are arranged or related in an equation. It further clarifies that no matter how these numbers are arranged and what processes are used, the composition of the equation and the final answer will still be the same. Our society is much like these equations - composed of different numbers and operations, different people with varied personalities, perspectives and experiences. We can choose to look at the differences and forever highlight one's advantage or superiority over the others. Or we can focus on the commonality among people and altogether, work for the common good. A peaceful society and harmonious relationship starts with recognizing, appreciating and fully maximizing the positive traits that we, as a people, have in common.

Lesson 6: Rational Numbers in the Number Line **Time: 1 hour**

Prerequisite Concepts: Subsets of Real Numbers, Integers

About the lesson:

This lesson is a more in-depth discussion of the set of Rational Numbers and focuses on where they are found in the real number line.

Objective:

In this lesson, you, the students, are expected to

1. Define rational numbers;
2. Illustrate rational numbers on the number line;
3. Arrange rational numbers on the number line.

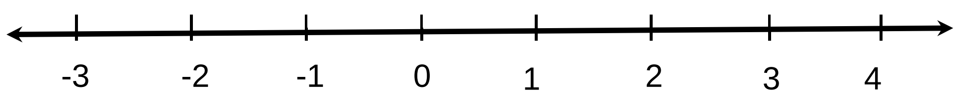
Lesson Proper

I. Activity

Determine whether the following numbers are rational numbers or not.

- 2, π, $\frac{1}{11}$, $\sqrt[3]{4}$, $\sqrt{16}$, -1.89,

Now, try to locate them on the real number line below by plotting:



II. Questions to Ponder

Consider the following examples and answer the questions that follow:

a. 7 ÷ 2 = 3 ½ ,
b. (-25) ÷ 4 = -6 ¼
c. (-6) ÷ (-12) = ½

1. Are quotients integers?
2. What kind of numbers are they?
3. Can you represent them on a number line?

Recall what rational numbers are...

3 ½, -6 ¼, ½, are rational numbers. The word *rational* is derived from the word "ratio" which means quotient. Rational numbers are numbers which can be written as a quotient of two integers, $\frac{a}{b}$ where b ≠ 0.

The following are more examples of rational numbers:

5 = $\frac{5}{1}$ 0.06 = $\frac{6}{100}$ 1.3 = $\frac{13}{10}$

From the example, we can see that an integer is also a rational number and therefore, integers are a subset of rational numbers. Why is that?

Let's check on your work earlier. Among the numbers given, - 2, π, $\frac{1}{11}$, $\sqrt[3]{4}$, $\sqrt{16}$, -1.89, the numbers π and $\sqrt[3]{4}$ are the only ones that are not rational numbers. Neither can be expressed as a quotient of two integers. However, we can express the remaining ones as a quotient of two intergers:

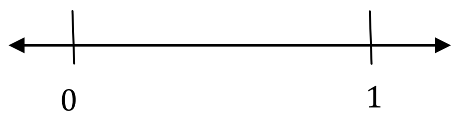
$$-2 = \tfrac{-2}{1},\ \sqrt{16} = 4 = \tfrac{4}{1},\ -1.89 = \frac{-189}{100}$$

Of course, $\frac{1}{11}$ is already a quotient by itself.

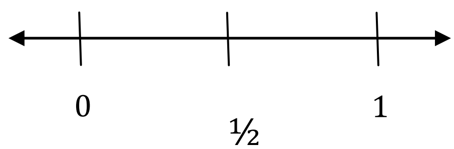
We can locate rational numbers on the real number line.

<u>Example 1</u>. Locate ½ on the number line.

a. Since 0 < ½ < 1, plot 0 and 1 on the number line.

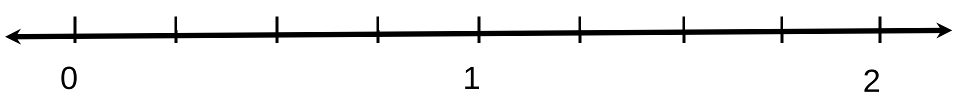


b. Get the midpoint of the segment from 0 to 1. The midpoint now corresponds to ½

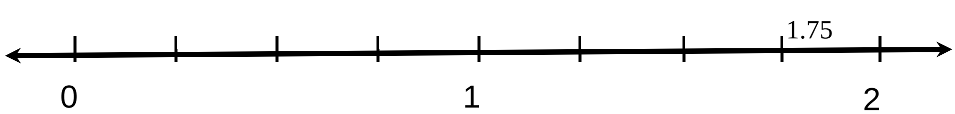


<u>Example 2</u>. Locate 1.75 on the number line.

a. The number 1.75 can be written as $\frac{7}{4}$ and, $1 < \frac{7}{4} < 2$. Divide the segment from 0 to 2 into 8 equal parts.

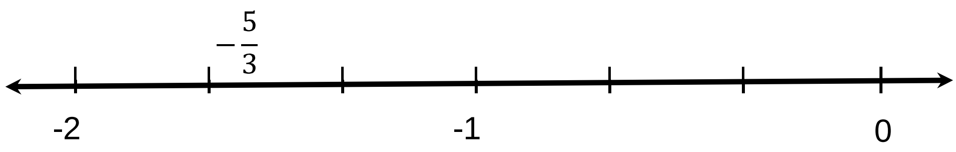


b. The 7th mark from 0 is the point 1.75.



<u>Example 3</u>. Locate the point $-\frac{5}{3}$ on the number line.

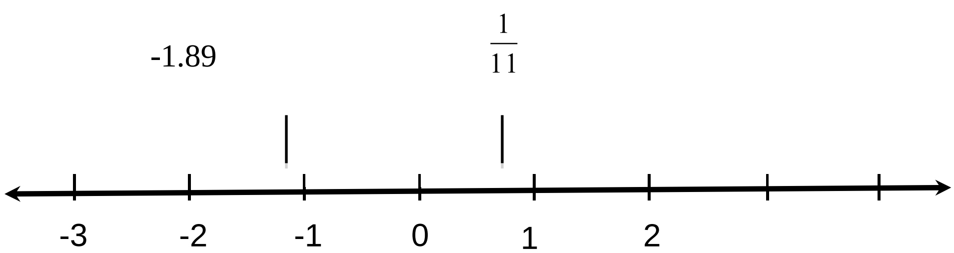
Note that $-2 < -\frac{5}{3} < -1$. Dividing the segment from -2 to 0 into 6 equal parts, it is easy to plot $-\frac{5}{3}$. The number $-\frac{5}{3}$ is the 5th mark from 0 to the left.



Go back to the opening activity. You were asked to locate the rational numbers and plot them on the real number line. Before doing that, it is useful to arrange them in order from least to greatest. To do this, express all numbers in the same form – either as similar fractions or as decimals. Because integers are easy to locate, they need not take any other form. It is easy to see that

$$-2 < -1.89 < \frac{1}{11} < \sqrt{16}$$

Can you explain why?
Therefore, plotting them by approximating their location gives



III. Exercises

1. Locate and plot the following on a number line (use only one number line).

a. $\frac{-10}{3}$	e. -0.01
b. 2.07	f. $7\frac{1}{9}$
c. $\frac{2}{5}$	g. 0
d. 12	h. $-\frac{1}{6}$

2. Name 10 rational numbers that are greater than -1 but less than 1 and arrange them from least to greatest on the real number line?

3. Name one rational number *x* that satisfies the descriptions below:
   a. $-10 \le x < -9$

   b. $\frac{1}{10} < x < \frac{1}{2}$

c. $3 < x < \pi$

d. $\frac{1}{4} < x < \frac{1}{3}$

e. $-\frac{1}{8} < x < -\frac{1}{9}$

Summary

In this lesson, you learned more about what rational numbers are and where they can be found in the real number line. By changing all rational numbers to equivalent forms, it is easy to arrange them in order, from least to greatest or vice versa.

Lesson 7: Forms of Rational Numbers and Addition and Subtraction of Rational Numbers
Time: 2 hours

Prerequisite Concepts: definition of rational numbers, subsets of real numbers, fractions, decimals

About the Lesson:

Like with any set of numbers, rational numbers can be added and subtracted. In this lesson, you will learn techniques in adding and subtracting rational numbers. Techniques include changing rational numbers into various forms convenient for the operation as well as estimation and computation techniques.

Objectives:

In this lesson, you are expected to:

1. Express rational numbers from fraction form to decimal form (terminating and repeating and non-terminating) and vice versa;
2. Add and subtract rational numbers;
3. Solve problems involving addition and subtraction of rational numbers.

Lesson Proper:

A. Forms of Rational Numbers

I. Activity

1. Change the following rational numbers in fraction form or mixed number form to decimal form:

a. $-\frac{1}{4}$ = ______ d. $\frac{5}{2}$ = ______

b. $\frac{3}{10}$ = ______ e. $-\frac{17}{10}$ = ______

c. $3\frac{5}{100}$ = _____ f. $-2\frac{1}{5}$ = ______

2. Change the following rational numbers in decimal form to fraction form.

a. 1.8 = ______ d. -0.001 = ______

b. - 3.5 = ______ e. 10.999 = ______

c. -2.2 = ______ f. $0.\overline{11}$ = ______

II. Discussion

Non-decimal Fractions

There is no doubt that most of the above exercises were easy for you. This is because all except item 2f are what we call decimal fractions. These numbers are all parts of powers of 10. For example, $-\frac{1}{4} = \frac{25}{100}$ which is easily convertible to a decimal form, 0.25. Likewise, the number -3.5 = $-3\frac{5}{10} = -\frac{35}{10}$.

What do you do when the rational number is not a decimal fraction? How do you convert from one form to the other?

Remember that a rational number is a quotient of 2 integers. To change a rational number in fraction form, you need only to divide the numerator by the denominator.

Consider the number $\frac{1}{8}$. The smallest power of 10 that is divisible by 8 is 1000. But, $\frac{1}{8}$ means you are dividing 1 whole unit into 8 equal parts. Therefore, divide 1 whole unit first into 1000 equal parts and then take $\frac{1}{8}$ of the thousandths part. That is equal to $\frac{125}{1000}$ or 0.125.

Example: Change $\frac{1}{16}$, $\frac{9}{11}$ and $-\frac{1}{3}$ to their decimal forms.

The smallest power of 10 that is divisible by 16 is 10,000. Divide 1 whole unit into 10,000 equal parts and take $\frac{1}{16}$ of the ten thousandths part. That is equal to $\frac{625}{10000}$ or 0.625. You can obtain the same value if you perform the long division $1 \div 16$.

Do the same for $\frac{9}{11}$. Perform the long division $9 \div 11$ and you should obtain $0.\overline{81}$. Therefore, $\frac{9}{11}$ = $0.\overline{81}$. Also, $-\frac{1}{3} = -0.\bar{3}$. Note that both $\frac{9}{11}$ and $-\frac{1}{3}$ are non-terminating but repeating decimals.

To change rational numbers in decimal forms, express the decimal part of the numbers as a fractional part of a power of 10. For example, -2.713 can be changed initially to $-2\frac{713}{1000}$ and then changed to $-\frac{2173}{1000}$.

What about non-terminating but repeating decimal forms? How can they be changed to fraction form? Study the following examples:

Example 1: Change $0.\bar{2}$ to its fraction form.
Solution: Let

$$r = 0.222...$$
$$10r = 2.222...$$

Since there is only 1 repeated digit, multiply the first equation by 10.

Then subtract the first equation from the second equation and obtain

$$9r = 2.0$$
$$r = \frac{2}{9}$$

Therefore, $0.\bar{2} = \frac{2}{9}$.

Example 2. Change $-1.\overline{35}$ to its fraction form.

Solution: Let

$$r = -1.353535...$$
$$100r = -135.353535...$$

Since there are 2 repeated digits, multiply the first equation by 100. In general, if there are n repeated digits, multiply the first equation by 10^n.

Then subtract the first equation from the second equation and obtain

$$99r = -134$$
$$r = -\frac{134}{99} = -1\frac{35}{99}$$

Therefore, $-1.\overline{35} = -\frac{135}{99}$.

B. Addition and Subtraction of Rational Numbers in Fraction Form

I. Activity

Recall that we added and subtracted whole numbers by using the number line or by using objects in a set.

Using linear or area models, find the sum or difference.

a. $\frac{3}{5} + \frac{1}{5} =$ ______

b. $\frac{1}{8} + \frac{5}{8} =$ ______

c. $\frac{10}{11} - \frac{3}{11} =$ ______

d. $3\frac{6}{7} - 1\frac{2}{7} =$ ______

Without using models, how would you get the sum or difference?

Consider the following examples:

1. $\frac{1}{6} + \frac{1}{2} = \frac{1}{6} + \frac{3}{6} = \frac{4}{6}$ $0r$ $\frac{2}{3}$
2. $\frac{6}{7} + -\frac{2}{3} = \frac{18}{21} + -\frac{14}{21} = \frac{4}{21}$

3. $-\frac{4}{3} + -\frac{1}{5} = -\frac{20}{15} + -\frac{3}{15} = -\frac{23}{15}\ or -1\frac{8}{15}$
4. $\frac{14}{5} - \frac{4}{7} = \frac{98}{35} - \frac{20}{35} = \frac{78}{35}\ or\ 2\frac{8}{35}$
5. $-\frac{7}{12} - -\frac{2}{3} = -\frac{7}{12} - -\frac{8}{12} = \frac{-7+8}{12} = \frac{1}{12}$
6. $-\frac{1}{6} - -\frac{11}{20} = -\frac{10}{60} - -\frac{33}{60} = \frac{-10+33}{60} = \frac{23}{60}$

Answer the following questions:

1. Is the common denominator always the same as one of the denominators of the given fractions?
2. Is the common denominator always the greater of the two denominators?
3. What is the least common denominator of the fractions in each example?
4. Is the resulting sum or difference the same when a pair of dissimilar fractions is replaced by any pair of similar fractions?

Problem: Copy and complete the fraction magic square. The sum in each row, column, and diagonal must be 2.

a ____	$\frac{1}{2}$	b ____
$\frac{7}{5}$	$\frac{1}{3}$	c ____
d ____	e ____	$\frac{2}{5}$

» What are the values of a, b, c, d and e?

Important things to remember

To Add or Subtract Fraction

- With the same denominator,
  If a, b and c denote integers, and b ≠ 0, then
  $\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b}$ and $\frac{a}{b} - \frac{c}{b} = \frac{a-c}{b}$

- With different denominators, $\frac{a}{b}\ and\ \frac{c}{d}$, where b ≠ 0 and d ≠ 0
  If the fractions to be added or subtracted are dissimilar

  » Rename the fractions to make them similar whose denominator is the least common multiple of *b* and *d*.
  » Add or subtract the numerators of the resulting fractions.
  » Write the result as a fraction whose numerator is the sum or difference of the numerators and whose denominator is the least common multiple of *b* and *d*.

Examples:

To Add:

a. $\frac{3}{7} + \frac{2}{7} = \frac{3+2}{7} = \frac{2}{7}$

To Subtract:

a. $\frac{5}{7} - \frac{2}{7} = \frac{5-2}{7} = \frac{3}{7}$

b. $\frac{2}{5} + \frac{1}{4}$

LCM/LCD of 5 and 4 is 20

$\frac{2}{5} + \frac{1}{4} = \frac{8}{20} + \frac{5}{20} = \frac{8+5}{20} = \frac{13}{20}$

b. $\frac{4}{5} - \frac{1}{4}$

$\frac{4}{5} - \frac{1}{4} = \frac{16}{20} - \frac{5}{20} = \frac{16-5}{20} = \frac{11}{20}$

II. Question to Ponder (Post –Activity Discussion)

Let us answer the questions posed in activity.
You were asked to find the sum or difference of the given fractions.

a. $\frac{3}{5} + \frac{1}{5} = \frac{4}{5}$

b. $\frac{1}{8} + \frac{5}{8} = \frac{6}{8}$ *or* $\frac{3}{4}$

c. $\frac{10}{11} - \frac{3}{11} = \frac{7}{11}$

d. $3\frac{6}{7} - 1\frac{2}{7} = 2\frac{4}{7}$

Without using the models, how would you get the sum or difference?
You would have to apply the rule for adding or subtracting similar fractions.

1. Is the common denominator always the same as one of the denominators of the given fractions?
   Not always. Consider $\frac{2}{5} + \frac{3}{4}$. *Their least common denominator is 20 not 5 or 4.*

2. Is the common denominator always the greater of the two denominators?
   Not always. The least common denominator is always greater than or equal to one of the two denominators and it may not be the greater of the two denominators.

3. What is the least common denominator of the fractions in each example?
   (1) *6* (2) *21* (3) *15* (4) *35* (5) *12* (6) *60*

4. Is the resulting sum or difference the same as when a pair of dissimilar fractions is replaced by any pair of similar fractions?
   Yes, for as long as the replacement fractions are equivalent to the original fractions.

III. Exercises

Do the following exercises.

a. Perform the indicated operations and express your answer in simplest form.

1. $\frac{2}{9} + \frac{3}{9} + \frac{1}{9}$
2. $\frac{6}{5} + \frac{3}{5} + \frac{4}{5}$
3. $\frac{2}{5} + \frac{7}{10}$
4. $\frac{16}{24} - \frac{6}{12}$
5. $2\frac{5}{12} - \frac{2}{3}$

9. $\frac{7}{9} - \frac{1}{12}$
10. $11\frac{5}{9} - 7\frac{5}{6}$
11. $\frac{1}{4} + \frac{2}{3} - \frac{1}{2}$
12. $10 - 3\frac{5}{11}$
13. $\frac{7}{20} + \frac{3}{8} + \frac{2}{5}$

6. $8\frac{1}{4} + \frac{2}{7}$
7. $3\frac{1}{4} + 6\frac{2}{3}$
8. $9\frac{5}{7} - 3\frac{2}{7}$
14. $\frac{5}{12} + \frac{4}{9} - \frac{1}{4}$
15. $2\frac{5}{8} + \frac{1}{2} + 7\frac{3}{4}$

b. Give the number asked for.
1. What is three more than three and one-fourth?
2. Subtract from $15\frac{1}{2}$ the sum of $2\frac{1}{3}$ *and* $4\frac{2}{5}$. What is the result?
3. Increase the sum of $6\frac{3}{14}$ *and* $2\frac{2}{7}$ *by* $3\frac{1}{2}$. What is the result?
4. Decrease $21\frac{3}{8}$ *by* $5\frac{1}{5}$. What is the result?
5. What is $-8\frac{4}{5}$ *minus* $3\frac{2}{7}$?

c. Solve each problem.
1. Michelle and Corazon are comparing their heights. If Michelle's height is $120\frac{3}{4}$ cm. and Corazon's height is $96\frac{1}{3}$ cm. What is the difference in their heights?
2. Angel bought $6\frac{3}{4}$ meters of silk, $3\frac{1}{2}$ meters of satin and $8\frac{2}{5}$ meters of velvet. How many meters of cloth did she buy?
3. Arah needs $10\frac{1}{4}$ kg. of meat to serve 55 guests, If she has $3\frac{1}{2}$ kg of chicken, a $2\frac{3}{4}$ kg of pork, and $4\frac{1}{4}$ kg of beef, is there enough meat for 55 guests?
4. Mr. Tan has $13\frac{2}{5}$ liters of gasoline in his car. He wants to travel far so he added $16\frac{1}{2}$ liters more. How many liters of gasoline is in the tank?
5. After boiling, the $17\frac{3}{4}$ liters of water was reduced to $9\frac{2}{3}$ liters. How much water has evaporated?

C. Addition and Subtraction of Rational Numbers in Decimal Form

There are 2 ways of adding or subtracting decimals.

1. Express the decimal numbers in fractions then add or subtract as described earlier.

Example:

Add: 2.3 + 7.21

$2\frac{3}{10} + 7\frac{21}{100}$

$2\frac{30}{100} + 7\frac{21}{100}$

$(2 + 7) + \frac{30+21}{100}$

$9 + \frac{51}{100} = 9\frac{51}{100}$ or 9.51

Subtract:: 9.6 – 3.25

$9\frac{6}{10} - 3\frac{25}{100}$

$9\frac{60}{100} - 3\frac{25}{100}$

$(9 - 3) + \frac{60-25}{100}$

$6 + \frac{35}{100} = 6\frac{35}{100}$ or 6.35

2. Arrange the decimal numbers in a column such that the decimal points are aligned, then add or subtract as with whole numbers.
   Example:

Add: 2.3 + 7.21 Subtract: 9.6- 3.25

$$\begin{array}{r} 2.3 \\ +\ \underline{\ \ 7.21} \\ 9.51 \end{array} \qquad \begin{array}{r} 9.6 \\ \underline{-\ 3.25} \\ 6.35 \end{array}$$

Exercises:

1. Perform the indicated operation.

   1) 1,902 + 21.36 + 8.7
   2) 45.08 + 9.2 + 30.545
   3) 900 + 676.34 + 78.003
   4) 0.77 + 0.9768 + 0.05301
   5) 5.44 – 4.97
   6) 700 – 678.891
   7) 7.3 – 5.182
   8) 51.005 – 21.4591
   9) (2.45 + 7.89) – 4.56
   10) (10 – 5.891) + 7.99

2. Solve the following problems:

a. Helen had P7500 for shopping money. When she got home, she had P132.75 in her pocket. How much did she spend for shopping?
b. Ken contributed P69.25, while John and Hanna gave P56.25 each for their gift to Teacher Daisy. How much were they able to gather altogether?
c. Ryan said, "I'm thinking of a number N. If I subtract 10.34 from N, the difference is 1.34." What was Ryan's number?
d. Agnes said, "I'm thinking of a number N. If I increase my number by 56.2, the sum is 14.62."What was Agnes number?
e. Kim ran the 100-meter race in 135.46 seconds. Tyron ran faster by 15.7 seconds. What was Tyron's time for the 100-meter dash?

SUMMARY

This lesson began with some activities and instruction on how to change rational numbers from one form to another and proceeded to discuss addition and subtraction of rational numbers. The exercises given were not purely computational. There were thought questions and problem solving activities that helped in deepening one's understanding of rational numbers.

Lesson 8: Multiplication and Division of Rational Numbers
Time: 2 hours

Prerequisite Concepts: addition and subtraction of rational numbers, expressing rational numbers in different forms

About the lesson:

In this lesson, you will learn how to multiply and divide rational numbers. While there are rules and algorithms to remember, this lesson also shows why those rules and algorithms work.

Objectives:

In this lesson, you are expected to:

1. Multiply rational numbers;
2. Divide rational numbers;
3. Solve problems involving multiplication and division of rational numbers.

Lesson Proper

A. Models for the Multiplication and Division

I. Activity:

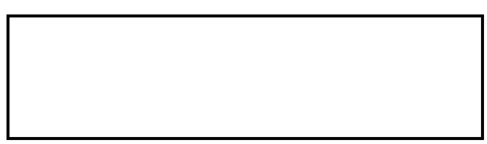
Make a model or a drawing to show the following:

1. A pizza is divided into 10 equal slices. Kim ate $\frac{3}{5}$ of $\frac{1}{2}$ of the pizza. What part of the whole pizza did Kim eat?
2. Miriam made 8 chicken sandwiches for some street children. She cut up each sandwich into 4 triangular pieces. If a child can only take a piece, how many children can she feed?

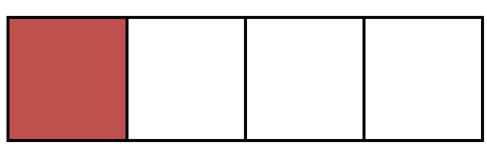
Can you make a model or a drawing to help you solve these problems?

A model that we can use to illustrate multiplication and division of rational numbers is the <u>area model</u>.

What is $\frac{1}{4} \times \frac{1}{3}$? Suppose we have one bar of chocolate represent 1 unit.

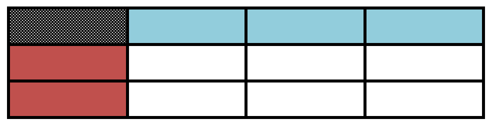


Divide the bar first into 4 equal parts vertically. One part of it is $\frac{1}{4}$



Then, divide each fourth into 3 equal parts, this time horizontally to make the divisions easy to see. One part of the horizontal division is $\frac{1}{3}$.

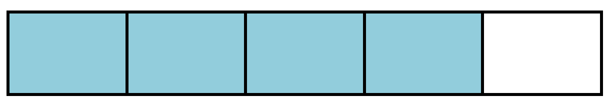
$$\frac{1}{3} \times \frac{1}{4} = \frac{1}{12}$$



There will be 12 equal-sized pieces and one piece is $\frac{1}{12}$. But, that one piece is $\frac{1}{3}$ of $\frac{1}{4}$, which we know from elementary mathematics to mean $\frac{1}{3} \times \frac{1}{4}$.

What about a model for division of rational numbers?

Take the division problem: $\frac{4}{5} \div \frac{1}{2}$. One unit is divided into 5 equal parts and 4 of them are shaded.



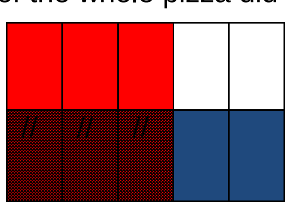
Each of the 4 parts now will be cut up in halves

Since there are 2 divisions per part (i.e. $\frac{1}{5}$) and there are 4 of them (i.e. $\frac{4}{5}$), then there will be 8 pieces out of 5 original pieces or $\frac{4}{5} \div \frac{1}{2} = \frac{8}{5}$.

II. Questions to Ponder (Post-Activity Discussion)

Let us answer the questions posed in the opening activity.

1. A pizza is divided into 10 equal slices. Kim ate $\frac{3}{5}$ of $\frac{1}{2}$ of the pizza. What part of the whole pizza did Kim eat?



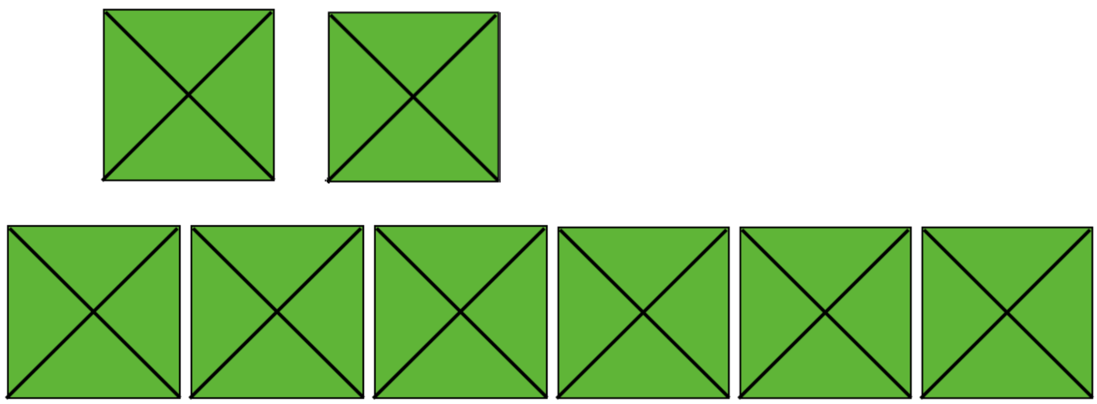
½

3/5

$$\frac{3}{5} \times \frac{1}{2} = \frac{3}{10}$$

Kim ate $\frac{3}{10}$ of the whole pizza.

2. Miriam made 8 chicken sandwiches for some street children. She cut up each sandwich into 4 triangular pieces. If a child can only take a piece, how many children can she feed?



The equation is $8 \div \frac{1}{4} = 32$. Since there are 4 fourths in one sandwich, there will be 4 x 8 = 32 triangular pieces and hence, 32 children will be fed.
How then can you multiply or divide rational numbers without using models or drawings?

Important Rules to Remember
The following are rules that you must remember. From here on, the symbols to be used for multiplication are any of the following: ●, x, $\times$, or *x*.

1. To multiply rational numbers in fraction form simply multiply the numerators and multiply the denominators.

   In symbol, $\frac{a}{b} \bullet \frac{c}{d} = \frac{ac}{bd}$ where b and d are NOT equal to zero, (b ≠ 0; d ≠ 0)

2. To divide rational numbers in fraction form, you take the reciprocal of the second fraction (called the divisor) and multiply it by the first fraction.

   In symbol, $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \bullet \frac{d}{c} = \frac{ad}{bc}$ where b, c, and d are NOT equal to zero.

<u>Example:</u>
Multiply the following and write your answer in simplest form

a. $\frac{3}{7} \bullet \frac{2}{5}$

$\frac{3}{7} \bullet \frac{2}{5} = \frac{3 \, x \, 2}{7 \, x \, 5} = \frac{6}{35}$

b. $4\frac{1}{3} \bullet 2\frac{1}{4}$

$\frac{13}{3} \bullet \frac{9}{4} = \frac{13 \bullet 3 \bullet 3}{3 \bullet 4} = \frac{13 \bullet 3}{4} = \frac{39}{4} \; or \; 9\frac{3}{4}$

The easiest way to solve for this number is to change mixed numbers to an improper fraction and then multiply it. Or use prime factors or the greatest common factor, as part of the multiplication process.

Divide: $\frac{8}{11} \div \frac{2}{3}$

$$\frac{8}{11} \div \frac{2}{3} = \frac{8}{11} \bullet \frac{3}{2}$$
$$= \frac{2 \bullet 4}{11} \bullet \frac{3}{2}$$
$$\frac{4 \bullet 3}{11} = \frac{12}{11} \text{ or } 1\frac{1}{11}$$

Take the reciprocal of $\frac{2}{3}$*, which is* $\frac{3}{2}$ *then multiply it with the first fraction. Using prime factors, it is easy to see that 2 can be factored out of the numerator then cancelled out with the denominator, leaving 4 and 3 as the remaining factors in the numerator and 11 as the remaining factors in the denominator.*

III. Exercises.

Do the following exercises. Write your answer on the spaces provided:

1. Find the products:

a. $\frac{5}{6} \bullet \frac{2}{3}$

b. $7 \bullet \frac{2}{3}$

c. $\frac{4}{20} \bullet \frac{2}{5}$

d. $10\frac{5}{6} \bullet 3\frac{1}{3}$

e. $-\frac{9}{20} \bullet \frac{25}{27}$

f. $4\frac{1}{2} \bullet 5\frac{2}{3}$

g. $\frac{2}{15} \bullet \frac{3}{4}$

h. $\frac{1}{6} \bullet \frac{2}{3} \bullet \frac{1}{4}$

i. $-\frac{5}{6} \bullet \frac{2}{3} \bullet -\frac{12}{15}$

j. $\frac{9}{16} \bullet \frac{4}{15} \bullet -2$

B. Divide:

1. $20 \div \frac{2}{3}$
2. $\frac{5}{12} \div -\frac{3}{4}$
3. $\frac{5}{50} \div \frac{20}{35}$
4. $5\frac{3}{4} \div 6\frac{2}{3}$
5. $\frac{9}{16} \div \frac{3}{4} \div \frac{1}{6}$
6. $\frac{8}{15} \div \frac{12}{25}$
7. $13\frac{1}{6} \div -2$
8. $-\frac{5}{6} \div -\frac{10}{14}$
9. $-\frac{2}{9} \div \frac{11}{15}$
10. $\frac{15}{6} \div \frac{2}{3} \div \frac{5}{8}$

C. Solve the following:

1. Julie spent $3\frac{1}{2}$ hours doing her assignment. Ken did his assignment for $1\frac{2}{3}$ times as many hours as Julie did. How many hours did Ken spend doing his assignment?
2. How many thirds are there in six-fifths?
3. Hanna donated $\frac{2}{5}$ of her monthly allowance to the Iligan survivors. If her monthly allowance is P3500, how much did she donate?
4. The enrolment for this school year is 2340. If $\frac{1}{6}$ are sophomores and $\frac{1}{4}$ are seniors, how many are freshmen and juniors?
5. At the end of the day, a store had 2/5 of a cake leftover. The four employees each took home the same amount of leftover cake. How much did each employee take home?

B. Multiplication and Division of Rational Numbers in Decimal Form

This unit will draw upon your previous knowledge of multiplication and division of whole numbers. Recall the strategies that you learned and developed when working with whole numbers.

Activity:

1. Give students several examples of multiplication sentences with the answers given. Place the decimal point in an incorrect spot and ask students to explain why the decimal place does not go there and explain where it should go and why.

   Example:

   215.2 x 3.2 = 68.864

2. Five students ordered buko pie and the total cost was P135.75. How much did each student have to pay if they shared the cost equally?

Questions and Points to Ponder:

1. In multiplying rational numbers in decimal form, note the importance of knowing where to place the decimal point in a product of two decimal numbers. Do you notice a pattern?
2. In dividing rational numbers in decimal form, how do you determine where to place the decimal point in the quotient?

Rules in Multiplying Rational Numbers in Decimal Form

1. Arrange the numbers in a vertical column.
2. Multiply the numbers, as if you are multiplying whole numbers.
3. Starting from the rightmost end of the product, move the decimal point to the left the same number of places as the sum of the decimal places in the multiplicand and the multiplier.

Rules in Dividing Rational Numbers in Decimal Form

1. If the divisor is a whole number, divide the dividend by the divisor applying the rules of a whole number. The position of the decimal point is the same as that in the dividend.
2. If the divisor is not a whole number, make the divisor a whole number by moving the decimal point in the divisor to the rightmost end, making the number seem like a whole number.
3. Move the decimal point in the dividend to the right the same number of places as the decimal point was moved to make the divisor a whole number.
4. Lastly divide the new dividend by the new divisor.

Exercises:

A. Perform the indicated operation

1. 3.5 ÷ 2
2. 78 x 0.4
3. 9.6 x 13
6. 27.3 x 2.5
7. 9.7 x 4.1
8. 3.415 ÷ 2.5

4. 3.24 ÷ 0.5
5. 1.248 ÷ 0.024

9. 53.61 x 1.02
10. 1948.324 ÷ 5.96

B. Finds the numbers that when multiplied give the products shown.

1. $\begin{array}{r} \square . \square \\ \times \quad \square \\ \hline 10.6 \end{array}$

2. $\begin{array}{r} \square . \square \\ \times \quad \square \\ \hline 16.8 \end{array}$

3. $\begin{array}{r} \square . \square \\ \times \quad \square \\ \hline 21.6 \end{array}$

4. $\begin{array}{r} \square . \square \\ \times \quad \square \\ \hline 9.5 \end{array}$

5. $\begin{array}{r} \square . \square \; \square \\ \times \quad \square \\ \hline 21.98 \end{array}$

Summary

In this lesson, you learned to use the area model to illustrate multiplication and division of rational numbers. You also learned the rules for multiplying and dividing rational numbers in both the fraction and decimal forms. You solved problems involving multiplication and division of rational numbers.

Lesson 9: Properties of the Operations on Rational Numbers
Time: 1.5 hours

Pre-requisite Concepts: Operations on rational numbers

About the Lesson: The purpose of this lesson is to use properties of operations on rational numbers when adding, subtracting, multiplying and dividing rational numbers.

Objectives:

In this lesson, you are expected to

1. Describe and illustrate the different properties of the operations on rational numbers.
2. Apply the properties in performing operations on rational numbers.

Lesson Proper:

I. Activity

Pick a Pair

$\frac{2}{14}$	$\frac{3}{5}$	0	1	$\frac{13}{40}$
	$\frac{13}{12}$	$\frac{1}{3}$	$\frac{3}{20}$	

From the box above, pick the correct rational number to be placed in the spaces provided to make the equation true.

1. $\frac{3}{14} + \_\_\_ = \frac{5}{14}$
2. $\_\_\_\_ + \frac{3}{14} = \frac{5}{14}$
3. $\frac{1}{3}\ x\ \_\_\_ = 0$
4. $1\ \text{x}\ \_\_\_\_ = \frac{3}{5}$
5. $\frac{2}{3} + \_\_\_\_ = \frac{2}{3}$
6. $\frac{1}{2} + \frac{1}{4} + \frac{1}{3} = \_\_\_\_$
7. $\frac{1}{2} + \frac{1}{4} + \_\_\_ = \frac{13}{12}$
8. $\frac{2}{5} \times \left(\_\_\_ \times \frac{3}{4}\right) = \frac{3}{20}$
9. $\frac{2}{5}\ x\ \frac{1}{2}\ x\ \frac{3}{4} = \_\_\_\_\_$
10. $\frac{1}{2}\ x\ \frac{2}{5} + \frac{1}{4} = \frac{1}{2}\ x\ \frac{2}{5} +$

$\frac{1}{2}\ x\ \frac{1}{4} = \_\_\_\_$

Answer the following questions:

1. What is the missing number in item 1?
2. How do you compare the answers in items 1 and 2
3. What about item 3? What is the missing number?
4. In item 4, what number did you multiply with 1 to get $\frac{3}{5}$?

5. What number should be added to $\frac{2}{3}$ in item 5 to get the same number?
6. What is the missing number in items 6 and 7?
7. What can you say about the grouping in items 6 and 7?
8. What do you think are the answers in items 8 and 9?
9. What operation did you apply in item 10?

Problem:

Consider the given expressions:

a. $\frac{1}{4} + \frac{1}{8} + \frac{1}{2} + \frac{2}{3} = \frac{1}{4} + \frac{1}{2} + \frac{2}{3} + \frac{1}{8}$

b. $\frac{2}{15} \bullet \frac{5}{6} = \frac{5}{6} \bullet \frac{2}{15}$

* Are the two expressions equal? If yes, state the property illustrated.

PROPERTIES OF RATIONAL NUMBERS (ADDITION & MULTIPLICATION)

1. CLOSURE PROPERTY: For any two rational numbers. $\frac{a}{b}$ and $\frac{c}{d}$, their sum $\frac{a}{b} + \frac{c}{d}$ and product $\frac{a}{b}$ x $\frac{c}{d}$ is also rational.

For example:

a. $\frac{3}{4} + \frac{2}{4} = \frac{3+2}{4} = \frac{5}{4}$

b. $\frac{3}{4} \bullet \frac{2}{4} = \frac{6}{16}$ or $\frac{3}{8}$

2. COMMUTATIVE PROPERTY: For any two rational numbers $\frac{a}{b}$ and $\frac{c}{d}$,

i. $\frac{a}{b} + \frac{c}{d} = \frac{b}{d} + \frac{a}{b}$

ii. $\frac{a}{b} \bullet \frac{c}{d} = \frac{c}{d}$ and $\frac{a}{b}$

where a, b, c and d are integers and b and d are not equal to zero.

For example:

a. $\frac{2}{7} + \frac{1}{3} = \frac{1}{3} + \frac{1}{7}$

b. $\frac{6}{7} \bullet \frac{2}{3} = \frac{3}{3} \bullet \frac{6}{7}$

3. ASSOCIATIVE PROPERTY: For any three rational numbers $\frac{a}{b}$, $\frac{c}{d}$, and $\frac{e}{f}$

i. $\frac{a}{b} + \frac{c}{d} + \frac{e}{f} = \frac{a}{b} + \frac{c}{d} + \frac{e}{f}$

ii. $\frac{a}{b} \bullet \frac{c}{d} \bullet \frac{e}{f} = \frac{a}{b} \bullet \frac{c}{d} \bullet \frac{e}{f}$

where a, b, c, d, e and f are integers and b, d and f are not equal to zero.
For example:

a. $\frac{3}{5} + \frac{2}{3} + \frac{1}{4} = \frac{3}{5} + \frac{2}{3} + \frac{1}{4}$

b. $\frac{1}{4} \bullet \frac{3}{4} \bullet \frac{2}{3} = \frac{1}{4} \bullet \frac{3}{4} \bullet \frac{2}{3}$

4. DISTRIBUTIVE PROPERTY of multiplication over addition for rational numbers.
If $\frac{a}{b}, \frac{c}{d}, and\ \frac{e}{f}$ are any rational numbers, then $\frac{a}{b} \bullet \frac{c}{d} + \frac{e}{f} = \frac{a}{b} \bullet \frac{c}{d} + \frac{a}{b} \bullet \frac{e}{f}$

For example: $\frac{3}{7} \bullet \frac{2}{3} + \frac{7}{8} = \frac{3}{7} \bullet \frac{2}{3} + \frac{3}{7} \bullet \frac{7}{8}$

5. DISTRIBUTIVE PROPERTY of multiplication over subtraction for rational numbers.
If $\frac{a}{b}, \frac{c}{d}, and\ \frac{e}{f}$ are any rational numbers, then $\frac{a}{b} \bullet \frac{c}{d} - \frac{e}{f} = \frac{a}{b} \bullet \frac{c}{d} - \frac{a}{b} \bullet \frac{e}{f}$

For example: $\frac{3}{10} \bullet \frac{2}{3} - \frac{2}{8} = \frac{3}{10} \bullet \frac{2}{3} - \frac{3}{10} \bullet \frac{2}{8}$

6. IDENTITY PROPERTY
Addition: Adding 0 to a number will not change the identity or value of that number.

$$\frac{a}{b} + 0 = \frac{a}{b}$$

For example: $\frac{1}{2} + 0 = \frac{1}{2}$

Multiplication: Multiplying a number by 1 will not change the identity or value of that number.

$$\frac{a}{b} \bullet 1 = \frac{a}{b}$$

For example: $\frac{3}{5} \bullet 1 = \frac{3}{5}$

7. ZERO PROPERTY OF MULTIPLICATION: Any number multiplied by zero equals 0, i. e. $\frac{a}{b} \bullet 0 = 0$
For example: $\frac{2}{7} \bullet 0 = 0$

II. Question to Ponder (Post-Activity Discussion)

Let us answer the questions posed in the opening activity.

1. What is the missing number in item1? » $\frac{2}{14}$
2. How do you compare the answers in items 1 and 2? » *The answer is the same, the order of the numbers is not important.*
3. What about item 3? What is the missing number? *» The missing number is 0. When you multiply a number with zero the product is zero.*
4. In item 4, what number did you multiply with 1 to get $\frac{3}{5}$? » $\frac{3}{5}$, *When you multiply a number by one the answer is the same.*
5. What number should be added to $\frac{2}{3}$ in item 5 to get the same number? » 0, *When you add zero to any number, the value of the number does not change.*
6. What do you think is the missing number in items 6 and 7?» $\frac{13}{12}$
7. What can you say about the grouping in items 6 and 7? » *The groupings are different but they do not affect the sum.*
8. What do you think are the answers in items 8 and 9? » *The answer is the same in both items,* $\frac{3}{20}$.
9. What operation did you apply in item 10? » *The Distributive Property of Multiplication over Addition*

III. Exercises:

Do the following exercises. Write your answer in the spaces provided.

A. State the property that justifies each of the following statements.

1. $\frac{2}{3} + \frac{5}{8} = \frac{5}{8} + \frac{2}{3}$ ______________________________

2. $1 \bullet \frac{9}{35} = \frac{9}{35}$ ______________________________

3. $\frac{4}{5} \bullet \frac{3}{4} + \frac{2}{3} = \frac{4}{5} \bullet \frac{3}{4} + \frac{4}{5} \bullet \frac{2}{3}$ __________________

4. $\frac{3}{5} + \frac{1}{2} + \frac{1}{4} = \frac{3}{5} + \frac{1}{2} + \frac{1}{4}$ ______________________

5. $\left(\frac{2}{7} + \frac{1}{5} + \frac{2}{3}\right) \bullet 1 = \left(\frac{2}{7} + \frac{1}{5} + \frac{2}{3}\right)$ _______________________

6. $\frac{3}{4} + 0 = \frac{3}{4}$ ______________________________

7. $\frac{1}{2} + \frac{5}{6} = \frac{4}{3}$ ______________________________

8. $\frac{3}{8} \bullet \frac{1}{4} \bullet \frac{2}{3} \bullet \frac{1}{2} = \frac{3}{8} \bullet \frac{2}{3} \bullet \frac{1}{4} \bullet \frac{1}{2}$ __________________

9. $\frac{1}{4} \bullet \frac{5}{7} - \frac{2}{3} = \frac{1}{4} \bullet \frac{5}{7} - \frac{1}{4} \bullet \frac{2}{3}$ __________________

10. $\left(\frac{2}{15} \bullet \frac{5}{7}\right) \bullet 0 = 0$ ____

B. Find the value of N in each expression

1. $N + \frac{1}{45} = \frac{1}{45}$

2. $\frac{1}{4} \bullet N \quad \bullet \frac{2}{3} = \frac{1}{4} \bullet \quad \frac{6}{7} \bullet \frac{2}{3}$

3. $\frac{2}{15} + \frac{12}{30} \quad + \frac{1}{5} = \frac{2}{15} + \quad N + \frac{1}{5}$

4. $0 + N = \frac{5}{18}$

6. $N \bullet \quad \frac{6}{14} + \frac{2}{7} \quad = \quad \frac{1}{6} \bullet \frac{6}{14} + \quad \frac{1}{6} \bullet \frac{2}{7}$

7. $\frac{8}{23} \bullet 1 = N$

8. $\frac{2}{9} + \frac{2}{3} = N$

Summary

This lesson is about the properties of operations on rational numbers. The properties are useful because they simplify computations on rational numbers. These properties are true under the operations addition and multiplication. Note that for the Distributive Property of Multiplication over Subtraction, subtraction is considered part of addition. Think of subtraction as the addition of a negative rational number.

Lesson 10: Principal Roots and Irrational Numbers **Time: 2 hours**

Prerequisite Concepts: Set of rational numbers

About the Lesson:

This is an introductory lesson on irrational numbers, which may be daunting to students at this level. The key is to introduce them by citing useful examples.

Objectives:

In this lesson, you are expected to:

1. describe and define irrational numbers;
2. describe principal roots and tell whether they are rational or irrational;
3. determine between what two integers the square root of a number is;
4. estimate the square root of a number to the nearest tenth;
5. illustrate and graph irrational numbers (square roots) on a number line with and without appropriate technology.

Lesson Proper:

I. Activities

A. Take a look at the unusual wristwatch and answer the questions below.

1. Can you tell the time?
2. What time is shown in the wristwatch?
3. What do you get when you take the $\sqrt{1}$? $\sqrt{4}$? $\sqrt{9}$? $\sqrt{16}$?
4. How will you describe the result?
5. Can you take the exact value of $\sqrt{130}$?
6. What value could you get?

Taking the square root of a number is like doing the reverse operation of squaring a number. For example, both 7 and -7 are square roots of 49 since $7^2 = 49$ and $(-7)^2 = 49$. Integers such as 1, 4, 9, 16, 25 and 36 are called perfect squares. Rational numbers such as 0.16, $\frac{4}{100}$ and 4.84 are also, perfect squares. Perfect squares are numbers that have rational numbers as square roots. The square roots of perfect squares are rational numbers while the square roots of numbers that are not perfect squares are *irrational numbers*.

Any number that cannot be expressed as a quotient of two integers is an irrational number. The numbers $\sqrt{2}$, π, and the special number *e* are all irrational numbers. Decimal numbers that are non-repeating and non-terminating are irrational numbers.

B. Activity

Use the $\sqrt[n]{\ }$ button of a scientific calculator to find the following values:

1. $\sqrt[6]{64}$ 2. $\sqrt[4]{-16}$ 3. $\sqrt[3]{90}$

4. $\sqrt[5]{-3125}$ 5. $\sqrt{24}$

II. Questions to Ponder (Post-Activity Discussions)

Let us answer the questions in the opening activity.

1. Can you tell the time? *Yes*
2. What time is it in the wristwatch? *10:07*
3. What do you get when you take the $\overline{1}$? $\overline{4}$? $\overline{9}$? $\overline{16}$? *1, 2, 3, 4*
4. How will you describe the result? *They are all positive integers*.
5. Can you take the exact value of $\overline{130}$? *No*.
6. What value could you get? *Since the number is not a perfect square you could estimate the value to be between* $\sqrt{121}$ *and* $\sqrt{144}$*, which is about 11.4.*

Let us give the values asked for in Activity B. Using a scientific calculator, you probably obtained the following:

1. $\sqrt[6]{64}$ = *2*
2. $\sqrt[4]{-16}$ *Math Error, which means not defined*
3. $\sqrt[3]{90}$ = *4.481404747, which could mean non-terminating and non-repeating since the calculator screen has a limited size*
4. $\sqrt[5]{-3125}$ = *-5*
5. $\sqrt{24}$ = *4.898979486, which could mean non-terminating and non-repeating since the calculator screen has a limited size*

<u>*On Principal n^{th} Roots*</u>

Any number, say *a*, whose n^{th} power (*n*, a positive integer), is *b* is called the n^{th} root of b. Consider the following: $(-7)^2 = 49$, $2^4 = 16$ and $(-10)^3 = -1000$. This means that -7 is a 2^{nd} or square root of 49, 2 is a 4^{th} root of 16 and -10 is a 3^{rd} or cube root of -1000.

However, we are not simply interested in any n^{th} root of a number; we are more concerned about the principal n^{th} root of a number. **The principal n^{th} root of a positive number is the positive n^{th} root. The principal n^{th} root of a negative number is the negative n^{th} root if *n* is odd. If *n* is even and the number is negative, the principal n^{th} root is not defined. The notation for the principal n^{th} root of a number *b* is $\sqrt[n]{b}$. In this expression, *n* is the index and *b* is the radicand. The n^{th} roots are also called radicals.**

Classifying Principal n^{th} Roots as Rational or Irrational Numbers
To determine whether a principal root is a rational or irrational number, determine if the radicand is a perfect n^{th} power or not. If it is, then the root is rational. Otherwise, it is irrational.

Problem 1. Tell whether the principal root of each number is rational or irrational.

(a) $\sqrt[3]{225}$ (b) $\sqrt{0.04}$ (c) $\sqrt[5]{-111}$ (d) $\sqrt{10000}$ (e) $\sqrt[4]{625}$

Answers:
a) $\sqrt[3]{225}$ *is irrational*
(b) $\sqrt{0.04}$ *= 0.2 is rational*
(c) $\sqrt[5]{-111}$ *is irrational*
(d) $\sqrt{10000}$ *= 100 is rational*
(e) $\sqrt[4]{625}$ *= 5 is rational*

If a principal root is irrational, the best you can do for now is to give an estimate of its value. Estimating is very important for all principal roots that are not roots of perfect n^{th} powers.

Problem 2. The principal roots below are between two integers. Find the two closest such integers.
(a) $\sqrt{19}$ (b) $\sqrt[3]{101}$ (c) $\sqrt{300}$

Solution:
(a) $\sqrt{19}$
16 is a perfect integer square and 4 is its principal square root. 25 is the next perfect integer square and 5 is its principal square root. Therefore, $\sqrt{19}$ *is between 4 and 5.*

(b) $\sqrt[3]{101}$
64 is a perfect integer cube and 4 is its principal cube root. 125 is the next perfect integer cube and 5 is its principal cube root. Therefore, $\sqrt[3]{101}$ *is between 4 and 5.*

(c) $\sqrt{300}$
289 is a perfect integer square and 17 is its principal square root. 324 is the next perfect integer square and 18 is its principal square root. Therefore, $\sqrt{300}$ *is between 17 and 18.*

Problem 3. Estimate each square root to the nearest *tenth*.
(a) $\sqrt{40}$ (b) $\sqrt{12}$ (c) $\sqrt{175}$

Solution:
(a) $\sqrt{40}$

The principal root $\sqrt{40}$ is between 6 and 7, principal roots of the two perfect squares 36 and 49, respectively. Now, take the square of 6.5, midway between 6 and 7. Computing, $(6.5)^2 = 42.25$. Since 42.25 > 40 then $\sqrt{40}$ is closer to 6 than to 7. Now, compute for the squares of numbers between 6 and 6.5: $(6.1)^2 = 37.21$, $(6.2)^2 = 38.44$, $(6.3)^2 = 39.69$, and $(6.4)^2 = 40.96$. Since 40 is close to 39.69 than to 40.96, $\sqrt{40}$ is approximately 6.3.

(b) $\sqrt{12}$

The principal root $\sqrt{12}$ is between 3 and 4, principal roots of the two perfect squares 9 and 16, respectively. Now take the square of 3.5, midway between 3 and 4. Computing $(3.5)^2 = 12.25$. Since 12.25 > 12 then $\sqrt{12}$ is closer to 3 than to 4. Compute for the squares of numbers between 3 and 3.5: $(3.1)^2 = 9.61$, $(3.2)^2 = 10.24$, $(3.3)^2 = 10.89$, and $(3.4)^2 = 11.56$. Since 12 is closer to 12.25 than to 11.56, $\sqrt{12}$ is approximately 3.5.

(c) $\sqrt{175}$

The principal root $\sqrt{175}$ is between 13 and 14, principal roots of the two perfect squares 169 and 196. The square of 13.5 is 182.25, which is greater than 175. Therefore, $\sqrt{175}$ is closer to 13 than to 14. Now: $(13.1)^2 = 171.61$, $(13.2)^2 = 174.24$, $(13.3)^2 = 176.89$. Since 175 is closer to 174.24 than to 176.89 then, $\sqrt{175}$ is approximately 13.2.

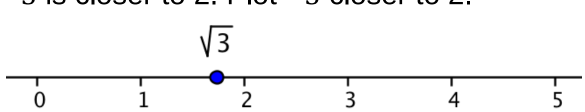
Problem 4. Locate and plot each square root on a number line.

(a) $\sqrt{3}$ (b) $\sqrt{21}$ (c) $\sqrt{87}$

Solution: You may use a program like Geogebra to plot the square roots on a number line.

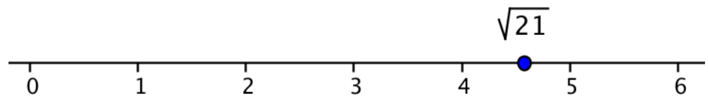
(a) $\sqrt{3}$

This number is between 1 and 2, principal roots of 1 and 4. Since 3 is closer to 4 than to 1, $\sqrt{3}$ is closer to 2. Plot $\sqrt{3}$ closer to 2.



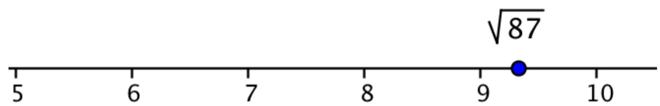
(b) $\sqrt{21}$

This number is between 4 and 5, principal roots of 16 and 25. Since 21 is closer to 25 than to 16, $\sqrt{21}$ is closer to 5 than to 4. Plot $\sqrt{21}$ closer to 5.



(c) $\sqrt{87}$

This number is between 9 and 10, principal roots of 81 and 100. Since 87 is closer to 81, then $\sqrt{87}$ is closer to 9 than to 10. Plot $\sqrt{87}$ closer to 9.



III. Exercises

A. Tell whether the principal roots of each number is rational or irrational.

1. $\sqrt{400}$	6. $\sqrt{13{,}689}$
2. $\sqrt{64}$	7. $\sqrt{1000}$
3. $\sqrt{0.01}$	8. $\sqrt{2.25}$
4. $\sqrt{26}$	9. $\sqrt{39}$
5. $\sqrt{\frac{1}{49}}$	10. $\sqrt{12.1}$

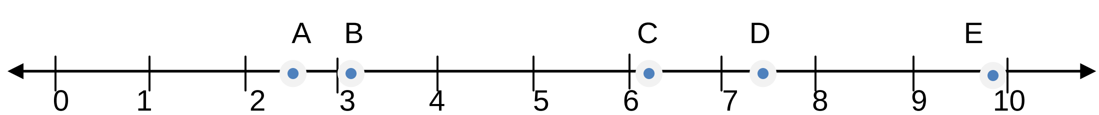
B. Between which two consecutive integers does the square root lie?

1. $\sqrt{77}$	6. $\sqrt{90}$
2. $\sqrt{700}$	7. $\sqrt{2045}$
3. $\sqrt{243}$	8. $\sqrt{903}$
4. $\sqrt{444}$	9. $\sqrt{1899}$
5. $\sqrt{48}$	10. $\sqrt{100000}$

C. Estimate each square root to the nearest *tenth* and plot on a number line.

1. $\sqrt{50}$	6. $\sqrt{250}$
2. $\sqrt{72}$	7. $\sqrt{5}$
3. $\sqrt{15}$	8. $\sqrt{85}$
4. $\sqrt{54}$	9. $\sqrt{38}$
5. $\sqrt{136}$	10. $\sqrt{101}$

D. Which point on the number line below corresponds to which square root?



1. $\overline{57}$ ______
2. $\overline{6}$ ______
3. $\overline{99}$ ______
4. $\overline{38}$ ______
5. $\overline{11}$ ______

Summary

In this lesson, you learned about irrational numbers and principal n^{th} roots, particularly square roots of numbers. You learned to find two consecutive integers between which an irrational square root lies. You also learned how to estimate the square roots of numbers to the nearest tenth and how to plot the estimated square roots on a number line.

Lesson 11: The Absolute Value of a Number **Time: 1.5 hours**

Prerequisite Concepts: Set of real numbers

About the Lesson:

This lesson explains why a distance between two points, even if represented on a number line cannot be expressed as a negative number. Intuitively, the absolute value of a number may be thought of as the non-negative value of a number. The concept of absolute value is important to designate the magnitude of a measure such as *the temperature dropped by 23* (the absolute value) *degrees*. A similar concept is applied to profit vs loss, income against expense, and so on.

Objectives:

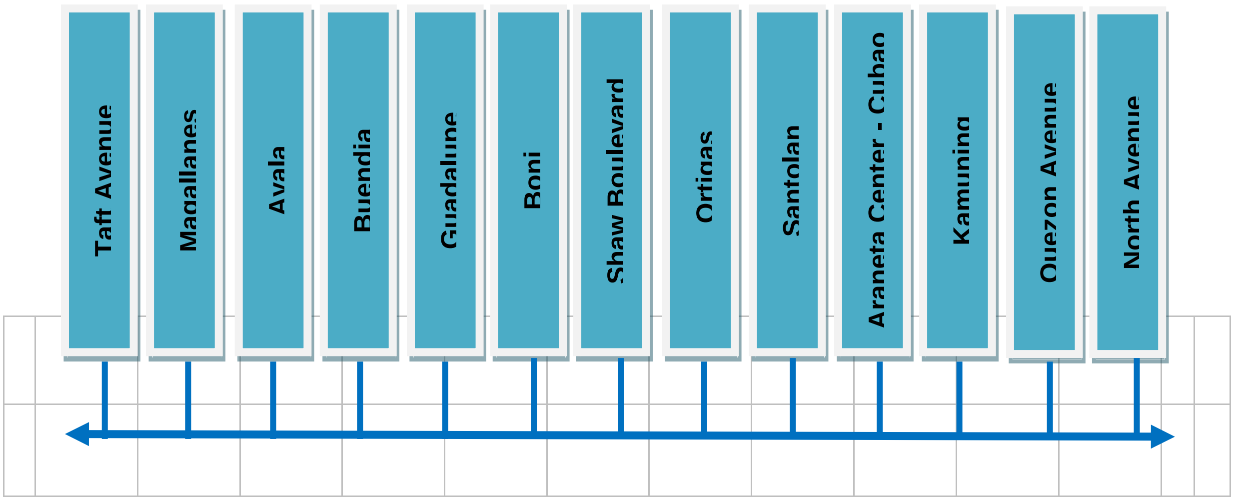
In this lesson, you are expected to describe and illustrate

a. the absolute value of a number on a number line.
b. the distance of the number from 0.

Lesson Proper:

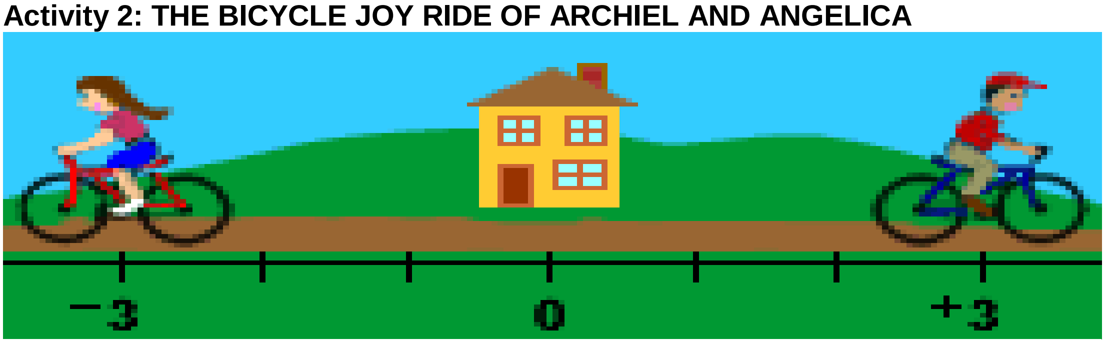
I. Activity 1: THE METRO MANILA RAIL TRANSIT (MRT) TOUR

Suppose the MRT stations from Pasay City to Quezon City were on a straight line and were 500 meters apart from each other.



1. How far would the North Avenue station be from Taft Avenue?
2. What if Elaine took the MRT from North Avenue and got off at the last station? How far would she have travelled?
3. Suppose both Archie and Angelica rode the MRT at Shaw Boulevard and the former got off in Ayala while the latter in Kamuning. How far would each have travelled from the starting point to their destinations?
4. What can you say about the directions and the distances travelled by Archie and Angelica?

Activity 2: THE BICYCLE JOY RIDE OF ARCHIEL AND ANGELICA



Problem: Archie and Angelica were at Aloys' house. Angelica rode her bicycle 3 miles west of Aloys' house, and Archie rode his bicycle 3 miles east of Aloys' house. Who travelled a greater distance from Aloys' house – Archie or Angelica?

Questions To Ponder:

1. What subsets of real numbers are used in the problem? Represent the trip of Archie and Angelica to the house of Aloys using a number line.

2. What are opposite numbers on the number line? Give examples and show on the number line.
3. What does it mean for the same distance travelled but in opposite directions? How would you interpret using the numbers -3 and +3?
4. What can you say about the absolute value of opposite numbers say -5 and +5?
5. How can we represent the absolute value of a number? What notation can we use?

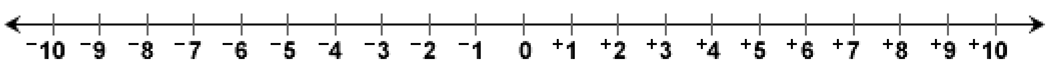
Important Terms to Remember

The following are terms that you must remember from this point on.

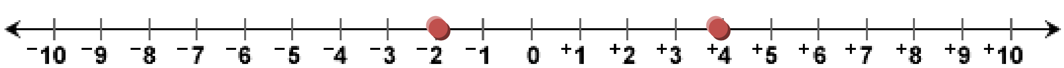
1. **Absolute Value –** of a number is the distance between that number and zero on the number line.
2. **Number Line –**is best described as a straight line which is extended in both directions as illustrated by arrowheads. A number line consists of three elements:
   a. set of positive numbers, and is located to the right of zero.
   b. set of negative numbers, and is located to the left of zero; and
   c. Zero.

Notations and Symbols

The absolute value of a number is denoted by two bars | |.
Let's look at the number line:



The absolute value of a number, denoted "| |" is the distance of the number from zero. This is why the absolute value of a number is never negative. In thinking about the absolute value of a number, one only asks "how far?" not "in which direction?" Therefore, the absolute value of 3 and of -3 is the same, which is 3 because both numbers have the same distance from zero.



Warning: The absolute-value notation is *bars*, not parentheses or brackets. Use the proper notation; the other notations do *not* mean the same thing.
It is important to note that the absolute value bars do NOT work in the same way as do parentheses. Whereas – (–3) = +3, this is NOT how it works for absolute value:

Problem: Simplify – | –3 |.

Solution: Given – | – 3 |, first find the absolute value of – 3.

– | –3 | = – (3)

Now take the negative of 3. Thus, :

– | –3 | = – (3) = **–3**

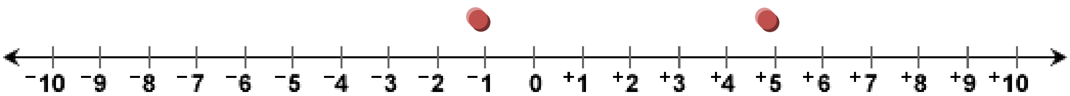
This illustrates that if you take the negative of the absolute value of a number, you will get a negative number for your answer.

II. Questons to Ponder(Post-Activity Discussion)

Let us answer the questions posed in Activity 2.

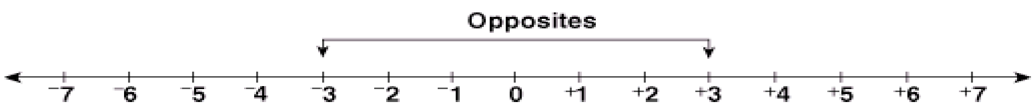
1. What subsets of real numbers are used in the problem? Represent the trip of Archie and Angelica to the house of Aloys using a number line.

The problem uses <u>integers</u>. Travelling 3 miles west can be represented by -3 (pronounced negative 3). Travelling 3 miles east can be represented by +3 (pronounced positive 3). Aloys' house can be represented by the integer 0.



2. What are opposite numbers on the number line? Give examples and show on the number line.

   Two integers that are the same distance from zero in opposite directions ***opposites****. The integers $^{+}3$ and $^{-}3$ are opposites since they are each 3 zero.*



3. What does it mean for the same distance travelled but in opposite directions? How would you interpret using the numbers -3 and +3?

   The absolute value of a number is its distance from zero on the number line. The absolute value of +3 is 3, and the absolute value of -3 is 3.

4. What can you say about the absolute value of opposite numbers say -5 and +5?

   Opposite numbers have the same absolute values.

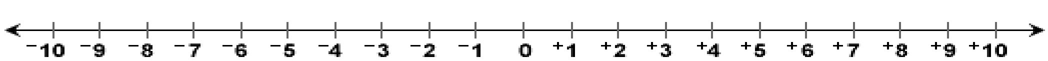
5. How can we represent the absolute value of a number? What notation can we use?

   The symbol | | *is used for the absolute value of a number.*

III. Exercises

Carry out the following tasks. Write your answers on the spaces provided for each number.

1. Find the absolute value of $^{+}3$, $^{-}3$, $^{+}7$, $^{-}5$, $^{+}9$, $^{-}8$, $^{+}4$, $^{-}4$. You may refer to the number line below. What should you remember when we talk about the absolute value of a number?



Solution: $|^{+}3| = 3$ $\quad$ $|^{+}9| = 9$

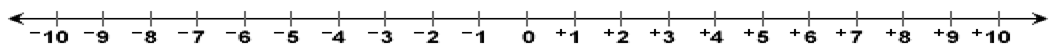
$|^{-}3| = 3$ $\quad$ $|^{-}8| = 8$

$|^{+}7| = 7$ $\quad$ $|^{+}4| = 4$

$|^{-}5| = 5$ $\quad$ $|^{-}4| = 4$

Remember that when we find the absolute value of a number, we are finding its distance from 0 on the number line. Opposite numbers have the same absolute value since they both have the same distance from 0. Also, you will notice that taking the absolute value of a number automatically means taking the positive value of that number.

2. Find the absolute value of: $^{+}11$, $^{-}9$, $^{+}14$, $^{-}10$, $^{+}17$, $^{-}19$, $^{+}20$, $^{-}20$. You may extend the number line below to help you solve this problem.



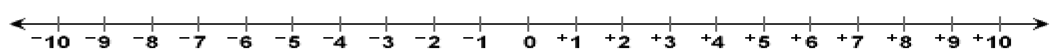
Solution: $|^{+}11| = 11$ $\quad$ $|^{+}17| = 17$

$|^{-}9| = 9$ $\quad$ $|^{-}19| = 19$

$|^{+}14| = 14$ $\quad$ $|^{+}20| = 20$

$|^{-}10| = 10$ $\quad$ $|^{-}20| = 20$

3. Use the number line below to find the value of N: $|N| = 5.1$



Solution: This problem asks us to find all numbers that are a distance of 5.1 units from zero on the number line. We let N represent all integers that satisfy this condition.

The number $^{+}5.1$ is 5.1 units from zero on the number line, and the number $^{-}5.1$ is also 5.1 units from zero on the number line. Thus both $^{+}5.1$ and $^{-}5.1$ satisfy the given condition.

4. When is the absolute value of a number equal to itself?

Solution:

When the value of the number is positive or zero.

5. Explain why the absolute value of a number is never negative. Give an example that will support your answer.

Solution: *Let $|N| = -4$. Think of a number that when you get the absolute value will give you a negative answer. There will be no solution since the distance of any number from 0 cannot be a negative quantity.*

Enrichment Exercises:

A. Simplify the following.

1. $|7.04|$
2. $|0|$
3. $|-\frac{2}{9}|$
4. $-|15 + 6|$
5. $|-2\sqrt{2}| - |-3\sqrt{2}|$

B. List at least two integers that can replace N such that.

1. $|N| = 4$
2. $|N| < 3$
3. $|N| > 5$
4. $|N| \leq 9$
5. $0 < |N| < 3$

C. Answer the following.

1. Insert the correct relation symbol(>, =, <): $|-7|$ ____ $|-4|$.
2. If $|x - 7| = 5$, what are the possible values of x?
3. If $|x| = \frac{2}{7}$, what are the possible values of x?
4. Evaluate the expression, $|x + y| - |y - x|$, if x = 4 and y = 7.
5. A submarine navigates at a depth of 50 meters below sea level while exactly above it; an aircraft flies at an altitude of 185 meters. What is the distance between the two carriers?

Summary:

In this lesson you learned about the absolute value of a number, that it is a distance from zero on the number line denoted by the notation |N|. This notation is used for the absolute value of an unknown number that satisfies a given condition. You also learned that a distance can never be a negative quantity and absolute value pertains to the magnitude rather than the direction of a number.

LESSON 12: SUBSETS OF REAL NUMBERS **Time: 1.5 hours**

Prerequisite Concepts: whole numbers and operations, set of integers, rational numbers, irrational numbers, sets and set operations, Venn diagrams

About the Lesson:

This lesson will intensify the study of mathematics since this requires a good understanding of the sets of numbers for easier communication. Classifying numbers is very helpful as it allows us to categorize what kind of numbers we are dealing with every day.

Objectives:

In this lesson, you are expected to :

2. Describe and illustrate the real number system.
3. Apply various procedures and manipulations on the different subsets of the set of real numbers.
   a. Describe, represent and compare the different subsets of real number.
   b. Find the union, intersection and complement of the set of real numbers and its subsets

Lesson Proper:

A. The Real Number System

I. Activity 1: Try to reflect on these . . .

It is difficult for us to realize that once upon a time there were no symbols or names for numbers. In the early days, primitive man showed how many animals he owned by placing an equal number of stones in a pile, or sticks in a row. Truly our number system evolved over hundreds of years.

Sharing Ideas! What do you think?

1. In what ways do you think did primitive man need to use numbers?
2. Why do you think he needed names or words to tell “how many”?
3. How did number symbols come about?
4. What led man to invent numbers, words and symbols?

Activity 2: LOOK AROUND!
Fifteen different words/partitions of numbers are hidden in this puzzle. How many can you find? Look up, down, across, backward, and diagonally. Figures are scattered around that will serve as clues to help you locate the mystery words.

π, e, $\overline{3}$

0, 1, 2, 3, ...

$\frac{2}{3}, \frac{7}{5}$

-1, 0, 1, $-\frac{1}{2}, \frac{4}{3}$,
0.25, 0.1313...

1, 2, 3, ...

-4, -5, -6, ...

0

0.25,
0.33...

...,-3, -2, -1,
0, 1, 2, 3, ...

100%,
15%, 25%

N	A	F	R	A	C	T	I	O	N	S	I
S	P	B	A	C	C	D	Z	W	N	E	L
T	E	O	F	T	O	G	E	H	E	R	A
O	R	H	S	I	U	J	R	O	G	A	M
I	C	R	K	I	N	R	O	L	A	T	I
L	E	E	L	M	T	N	A	E	T	I	C
A	N	A	O	P	I	I	Q	L	I	O	E
R	T	L	R	S	N	T	V	U	V	N	D
U	I	N	T	E	G	E	R	E	E	A	A
T	I	R	R	A	T	I	O	N	A	L	I
A	N	O	N	I	N	T	E	G	E	R	S
N	N	U	M	N	U	M	B	E	R	S	S

Answer the following questions:
1. How many words in the puzzle were familiar to you?
2. What word/s have you encountered in your early years? Define and give examples.
3. What word/s is/are still strange to you?

Activity 3: Determine what set of numbers will represent the following situations:

1. Finding out how many cows there are in a barn
2. Corresponds to no more apples inside the basket
3. Describing the temperature in the North Pole
4. Representing the amount of money each member gets when P200 prize is divided among 3 members
5. Finding the ratio of the circumference to the diameter of a circle, denoted π (read "pi")

The set of numbers called **the real number system** consists of different partitions/ **subsets** that can be represented graphically on a **number line**.

II. Questions to Ponder

Consider the activities done earlier and recall the different terms you encountered including the set of real numbers and together let us determine the various subsets. Let us go back to the first time we encountered the numbers...

Let's talk about the various subsets of real numbers.

Early Years...

1. What subset of real numbers do children learn at an early stage when they were just starting to talk? Give examples.

$$-1, 0, 1, -\frac{1}{2}, \frac{4}{3}, 0.25, 0.33..., \pi, e, \overline{3}, 10\%, 15\%, 25\%$$

One subset is the **counting (or natural) numbers**. This subset includes all the numbers we use to count starting with "1" and so on. The subset would look like this: {1, 2, 3, 4, 5...}

In School at an Early Phase...

2. What do you call the subset of real numbers that includes zero (the number that represents nothing) and is combined with the subset of real numbers learned in the early years? Give examples.

Another subset is the **whole numbers**. This subset is exactly like the subset of counting numbers, with the addition of one extra number. This extra number is "0". The subset would look like this:{0, 1, 2, 3, 4...}

In School at Middle Phase...

3. What do you call the subset of real numbers that includes negative numbers (that came from the concept of "opposites" and specifically used in describing debt or below zero temperature) and is united with the whole numbers? Give examples.

A third subset is the **integers**. This subset includes all the whole numbers and their "opposites". The subset would look like this: {... -4, -3, -2, -1, 0, 1, 2, 3, 4...}

Still in School at Middle Period...

4. What do you call the subset of real numbers that includes integers and non-integers and are useful in representing concepts like ***"half a gallon of milk"***? Give examples.

   The next subset is the **rational numbers**. This subset includes all numbers that "come to an end" or numbers that repeat and have a pattern. Examples of rational numbers are: 5.34, 0.131313..., $\frac{6}{7}$, $\frac{2}{3}$, 9

5. What do you call the subset of real numbers that is not a rational number but are physically represented like ***"the diagonal of a square"***?

   Lastly we have the set of **irrational numbers**. This subset includes numbers that cannot be exactly written as a decimal or fraction. Irrational numbers cannot be expressed as a ratio of two integers. Examples of irrational numbers are:
   $\sqrt{2}$, $\sqrt[3]{101}$, and π

Important Terms to Remember
The following are terms that you must remember from this point on.

1. **Natural/Counting Numbers** – are the numbers we use in counting things, that is {1, 2, 3, 4, . . . }. The three dots, called ellipses, indicate that the pattern continues indefinitely.

2. **Whole Numbers** – are numbers consisting of the set of natural or counting numbers and zero.

3. **Integers** – are the result of the union of the set of whole numbers and the negative of counting numbers.

4. **Rational Numbers** – are numbers that can be expressed as a quotient $\frac{a}{b}$ of two integers. The integer ***a*** is the numerator while the integer ***b***, which cannot be 0 is the denominator. This set includes fractions and some decimal numbers.

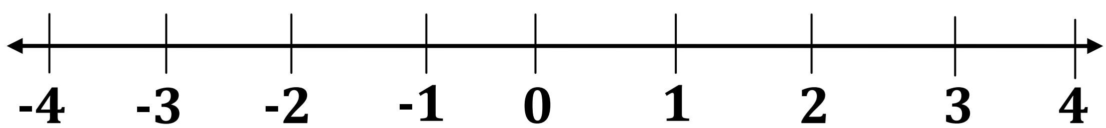
5. **Irrational Numbers** – are numbers that cannot be expressed as a quotient $\frac{a}{b}$ of two integers. Every irrational number may be represented by a decimal that neither repeats nor terminates.
6. **Real Numbers** – are any of the numbers from the preceding subsets. They can be found on the real number line. The union of rational numbers and irrational numbers is the set of real numbers.

7. **Number Line** – a straight line extended on both directions as illustrated by arrowheads and is used to represent the set of real numbers. On the real number line, there is a point for every real number and there is a real number for every point.

III. Exercises

a. Locate the following numbers on the number line by naming the correct point.

$$-2.66\ldots\ ,\ -1\frac{1}{2}\ ,\ -0.25\ ,\ \frac{3}{4}\ ,\ \sqrt{2}\ ,\ \sqrt[3]{11}$$



b. Determine the subset of real numbers to which each number belongs. Use a tick mark
(√) to answer.

Number	Whole Number	Integer	Rational	Irrational
1. -86				
2. 34.74				
3. $\frac{4}{7}$				
4. $\sqrt{64}$				
5. $\sqrt{11}$				
6. -0.125				
7. - $\sqrt{81}$				
8. e				
9. -45.37				
10. -1.252525...				

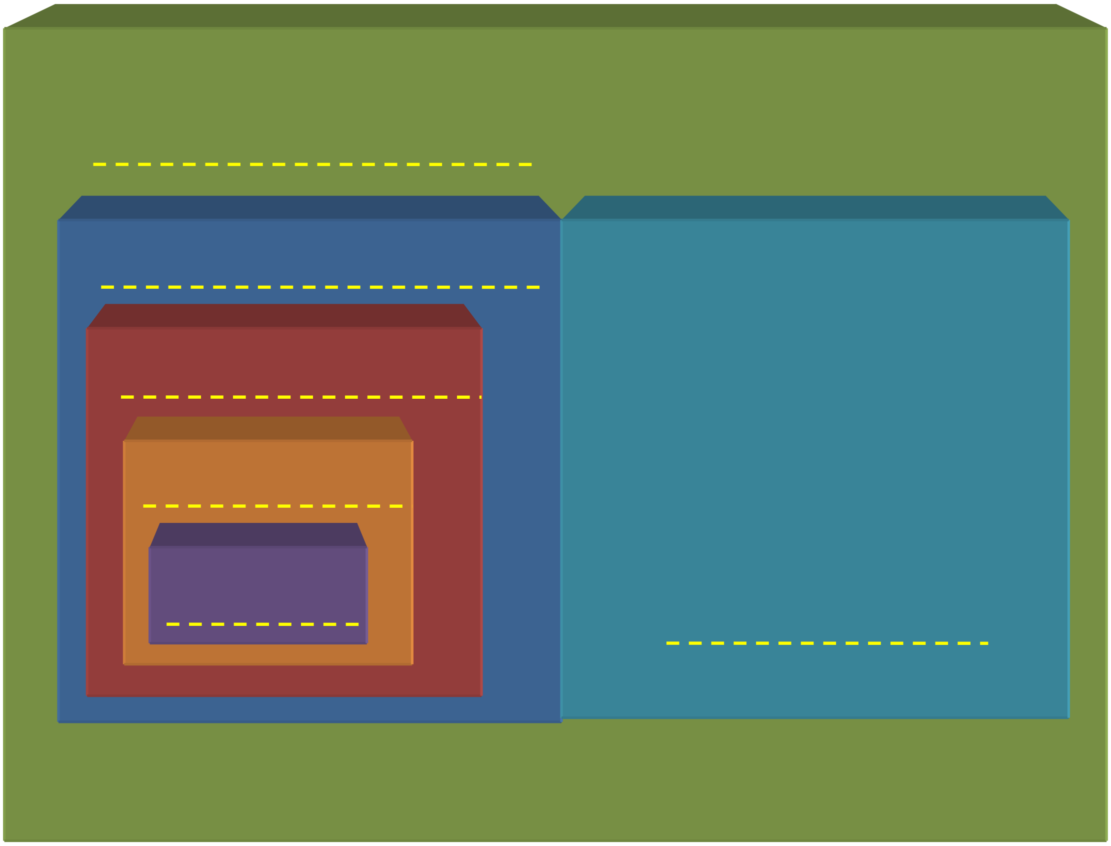
B. Points to Contemplate

It is interesting to note that the set of rational numbers and the set of irrational numbers are disjoint sets; that is their intersection is empty. In fact, they are complements of each other. The union of these two sets is the set of real numbers.

Exercises:

1. Based on the stated information, show the relationships among ***natural/counting numbers, whole numbers, integers, rational numbers, irrational numbers*** and

real numbers using the Venn diagram. Fill each broken line with its corresponding answer.



2. Answer the following questions on the space provided for each number.

a) Are all real numbers rational numbers? Prove your answer.

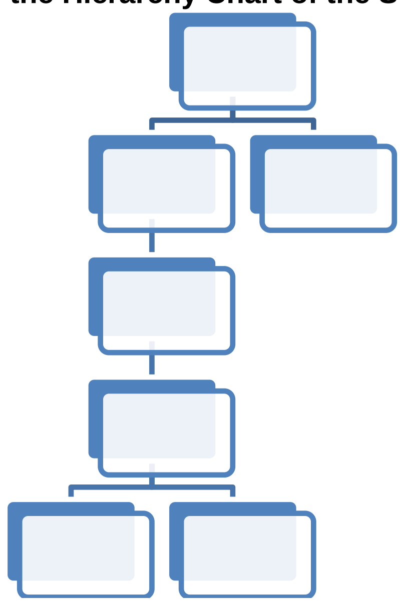
b) Are all rational numbers whole numbers? Prove your answer.

c) Are $-\frac{1}{4}$ and $-\frac{2}{5}$ negative integers? Prove your answer.

d) How is a rational number different from an irrational number?

e) How do natural numbers differ from whole numbers?

3. Complete the details in the Hierarchy Chart of the Set of Real Numbers.



THE REAL NUMBER SYSTEM

Summary

In this lesson, you learned different subsets of real numbers that enable you to name numbers in different ways. You also learned to determine the hierarchy and relationship of one subset to another that leads to the composition of the real number system using the Venn Diagram and Hierarchy Chart. You also learned that it was because of necessity that led man to invent number, words and symbols.

Lesson 13: Significant Digits and the Scientific Notation **OPTIONAL**

Prerequisite Concepts: Rational numbers and powers of 10

About the Lesson:

This is a lesson on significant digits and the scientific notation combined. The use of significant digits and the scientific notation is often in the area of measures and in the natural sciences. The scientific notation simplifies the way we write very large and very small numbers. On the other hand, numerical data become more accurate when significant digits are taken into account.

Objectives:

In this lesson, you are expected to :

1. determine the significant digits in a given situation.
2. write very large and very small numbers in scientific notation

Lesson Proper:

I. A. Activity

The following is a list of numbers. The number of significant digits in each number is written in the parenthesis after the number.

234 (3)	0.0122 (3)
745.1 (4)	0.00430 (3)
6007 (4)	0.0003668 (4)
1.3×10^{2} (2)	10000 (1)
7.50×10^{-7} (3)	1000. (4)
0.012300 (5)	2.222×10^{-3} (4)
100.0 (4)	8.004×10^{5} (4)
100 (1)	6120. (4)
7890 (3)	120.0 (4)
4970.00 (6)	530 (2)

Describe what digits are not significant. ________________________________

Important Terms to Remember

Significant digits are the digits in a number that express the precision of a measurement rather than its magnitude. The number of significant digits in a given measurement depends on the number of significant digits in the given data. In calculations involving multiplication, division, trigonometric functions, for example, the number of significant digits in the final answer is equal to the least number of significant digits in any of the factors or data involved.

Rules for Determining Significant Digits

A. All digits that are not zeros are significant.

For example: 2781 has 4 significant digits
82.973 has 5 significant digits

B. Zeros may or may not be significant. Furthermore,

1. Zeros appearing between nonzero digits are significant.
   For example: 20.1 has 3 significant digits
   79002 has 5 significant digits
2. Zeros appearing in front of nonzero digits are not significant.
   For example: 0.012 has 2 significant digits
   0.0000009 has 1 significant digit
3. Zeros at the end of a number and to the right of a decimal are significant digits. Zeros between nonzero digits and significant zeros are also significant.
   For example: 15.0 has 3 significant digits
   25000.00 has 7 significant digits
4. Zeros at the end of a number but to the left of a decimal may or may not be significant. If such a zero has been measured or is the first estimated digit, it is significant. On the other hand, if the zero has not been measured or estimated but is just a place holder it is not significant. A decimal placed after the zeros indicates that they are significant
   For example: 560000 has 2 significant digits
   560000. has 6 significant digits

Significant Figures in Calculations

1. When multiplying or dividing measured quantities, round the answer to as many significant figures in the answer as there are in the measurement with the least number of significant figures.
2. When adding or subtracting measured quantities, round the answer to the same number of decimal places as there are in the measurement with the least number of decimal places.
   For example:
   a. $3.0 \times 20.536 = 61.608$

Answer: 61 since the least number of significant digits is 2, coming from 3.0

b. 3.0 + 20.536 = 23.536

Answer: 23.5 since the addend with the least number of decimal places is 3.0

II. Questions to Ponder (Post-Activity Discussion)

Describe what digits are not significant. *The digits that are not significant are the zeros before a non-zero digit and zeros at the end of numbers without the decimal point.*

Problem 1. Four students weigh an item using different scales. These are the values they report:

a. 30.04 g
b. 30.0 g
c. 0.3004 kg
d. 30 g

How many significant digits are in each measurement?

Answer: *30.04 has 4 significant; 30.0 has 3 significant digits; 0.3004 has 4 significant digits; 30 has 1 significant digit*

Problem 2. Three students measure volumes of water with three different devices. They report the following results:

Device	Volume
Large graduated cylinder	175 mL
Small graduated cylinder	39.7 mL
Calibrated buret	18.16 mL

If the students pour all of the water into a single container, what is the total volume of water in the container? How many digits should you keep in this answer?

Answer: The total volume is 232.86 mL. Based on the measures, the final answer should be 232.9 mL.

On the Scientific Notation

The speed of light is 300 000 000 m/sec, quite a large number. It is cumbersome to write this number in full. Another way to write it is 3.0×10^8. How about a very small number like 0.000 000 089? Like with a very large number, a very small number may be written more efficiently. 0.000 000 089 may be written as 8.9×10^{-8}.

Writing a Number in Scientific Notation

1. Move the decimal point to the right or left until after the first significant digit and copy the significant digits to the right of the first digit. If the number is a whole number and has no decimal point, place a decimal point after the first significant digit and copy the significant digits to its right.

   For example, 300 000 000 has 1 significant digit, which is 3. Place a decimal point after 3.0

   The first significant digit in 0.000 000 089 is 8 and so place a decimal point after 8, (8.9).

2. Multiply the adjusted number in step 1 by a power of 10, the exponent of which is the number of digits that the decimal point moved, positive if moved to the left and negative if moved to the right.

   For example, 300 000 000 is written as 3.0×10^8 because the decimal point was moved past 8 places.

   0.0 000 089 is written as 8.9×10^{-8} because the decimal point was moved 8 places to the right past the first significant digit 8.

III. Exercises

A. Determine the number of significant digits in the following measurements. Rewrite the numbers with at least 5 digits in scientific notation.

1. 0.0000056 L
2. 4.003 kg
3. 350 m
4. 4113.000 cm
5. 700.0 mL
6. 8207 mm
7. 0.83500 kg
8. 50.800 km
9. 0.0010003 m^3
10. 8 000 L

B. a. Round off the following quantities to the specified number of significant figures.

1. 5 487 129 m to three significant figures
2. 0.013 479 265 mL to six significant figures
3. 31 947.972 cm^2 to four significant figures
4. 192.6739 m^2 to five significant figures

5. 786.9164 cm to two significant figures

b. Rewrite the answers in (a) using the scientific notation

C. Write the answers to the correct number of significant figures

1. 4.5 X 6.3 ÷ 7.22 ______________________
2. 5.567 X 3.0001 ÷ 3.45 ______________________
3. (37 X 43) ÷ (4.2 X 6.0) ______________________
4. (112 X 20) ÷ (30 X 63) ______________________
5. 47.0 ÷ 2.2 ______________________

D. Write the answers in the correct number of significant figures

1. 5.6713 + 0.31 + 8.123 ______________________
2. 3.111 + 3.11 + 3.1 ______________________
3. 1237.6 + 23 + 0.12 ______________________
4. 43.65 – 23.7 ______________________
5. 0.009 – 0.005 + 0.013 ______________________

E. Answer the following.

1. A runner runs the last 45m of a race in 6s. How many significant figures will the runner's speed have?
2. A year is 356.25 days, and a decade has exactly 10 years in it. How many significant figures should you use to express the number of days in two decades?
3. Which of the following measurements was recorded to 3 significant digits : 50 mL , 56 mL , 56.0 mL or 56.00 mL?
4. A rectangle measures 87.59 cm by 35.1 mm. Express its area with the proper number of significant figures in the specified unit: **a.** in cm^2 **b.** in mm^2
5. A 125 mL sample of liquid has a mass of 0.16 kg. What is the density of the liquid in g/mL?

Summary

In this lesson, you learned about significant digits and the scientific notation. You learned the rules in determining the number of significant digits. You also learned how to write very large and very small numbers using the scientific notation.

Lesson 14: More Problems Involving Real Numbers **Time: 1.5 hours**

Pre-requisite Concepts: Whole numbers, Integers, Rational Numbers, Real Numbers, Sets

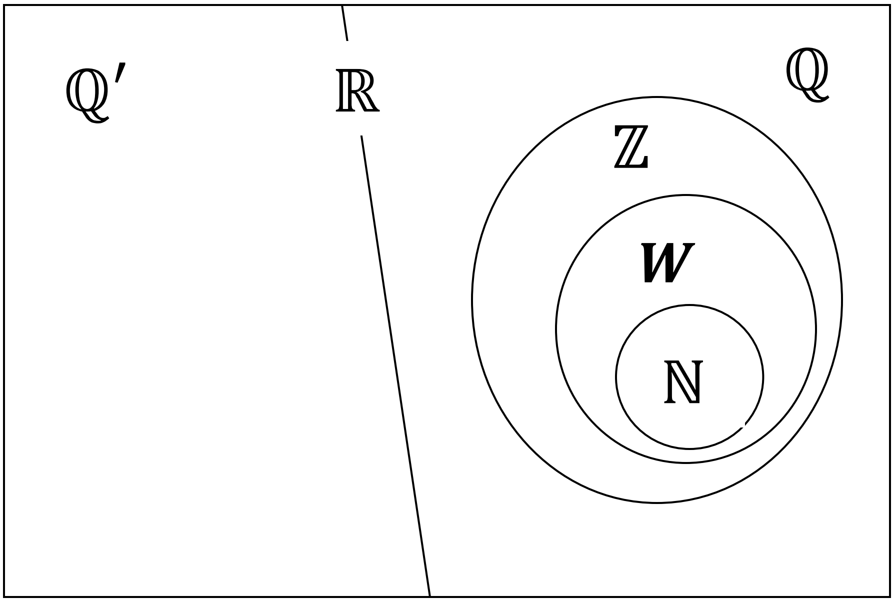
About the Lesson: This is the culminating lesson on real numbers. It combines all the concepts and skills learned in the past lessons on real numbers.

Objectives:
In this lesson, you are expected to:
1. Apply the set operations and relations to sets of real numbers
2. Describe and represent real-life situations which involve integers, rational numbers, square roots of rational numbers, and irrational numbers
3. Apply ordering and operations of real numbers in modeling and solving real-life problems

Lesson Proper:

Recall how the set of real numbers was formed and how the operations are performed. Numbers came about because people needed and learned to count. The set of counting numbers was formed. To make the task of counting easier, addition came about. Repeated addition then got simplified to multiplication. The set $\mathbb{N}$ of counting numbers is closed under both the operations of addition and multiplication. When the need to represent zero arose, the set ***W*** of whole numbers was formed. When the operation of subtraction began to be performed, the ***W*** was extended to the set $\mathbb{Z}$ or integers. $\mathbb{Z}$ is closed under the operations of addition, multiplication and subtraction. The introduction of division needed the expansion of $\mathbb{Z}$ to the set $\mathbb{Q}$ of rational numbers. $\mathbb{Q}$ is closed under all the four arithmetic operations of addition, multiplication, subtraction and division. When numbers are used to represent measures of length, the set $\mathbb{Q}$ or rational numbers no longer sufficed. Hence, the set $\mathbb{R}$ of real numbers came to be the field where properties work.

The above is a short description of the way the set of real numbers was built up to accommodate applications to counting and measurement and performance of the four arithmetic operations. We can also explore the set of real numbers by dissection – beginning from the big set, going into smaller subsets. We can say that $\mathbb{R}$ is the set of **all** decimals (positive, negative and zero). The set $\mathbb{Q}$ includes all the decimals which are repeating (we can think of terminating decimals as decimals in which all the digits after a finite number of them are zero). The set $\mathbb{Z}$ comprises all the decimals in which the digits to the right of the decimal point are all zero. This view gives us a clearer picture of the relationship among the different subsets of $\mathbb{R}$ in terms of inclusion.



We know that the *n*th root of any number which is not the *n*th power of a rational number is irrational. For instance, $\sqrt{2}$, $\sqrt{5}$, and $\sqrt[3]{9}$ are irrational.

Example 1. Explain why $3\sqrt{2}$ is irrational.

We use an argument called an indirect proof. This means that we will show why $3\sqrt{2}$ becoming rational will lead to an absurd conclusion. What happens if $3\sqrt{2}$ is rational? Because $\mathbb{Q}$ is closed under multiplication and $\frac{1}{3}$ is rational, then $3\sqrt{2}\times\frac{1}{3}$ is rational. However, $3\sqrt{2}\times\frac{1}{3}=\sqrt{2}$, which we know to be irrational. This is an absurdity. Hence we have to conclude that $3\sqrt{2}$ must be irrational.

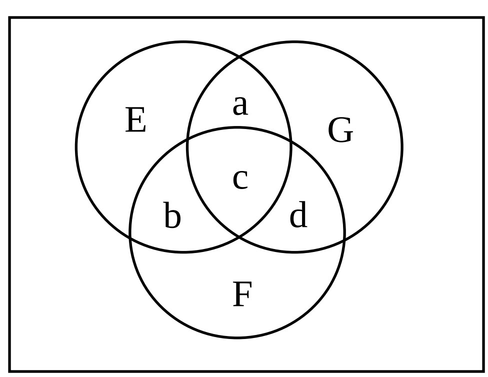
Example 2. A deep-freeze compartment is maintained at a temperature of 12°C below zero. If the room temperature is 31°C, how much warmer is the room temperature than the temperature in the deep-freeze compartment.

Get the difference between room temperature and the temperature inside the deep-freeze compartment

$31-(-12)=43$. *Hence, room temperature is 43°C warmer than the compartment.*

Example 3. Hamming Code

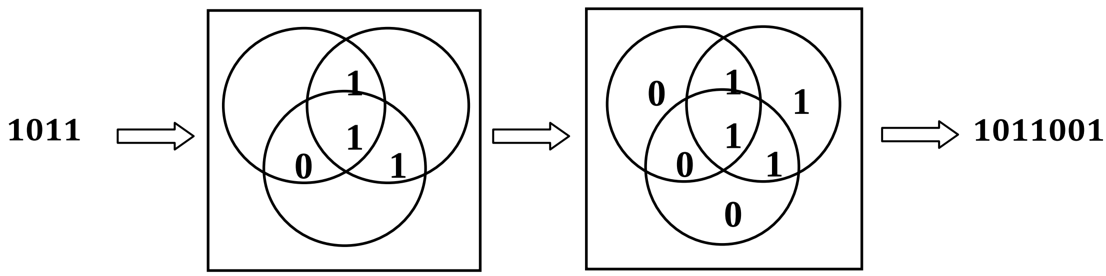
A mathematician, Richard Hamming developed an error detection code to determine if the information sent electronically is transmitted correctly. Computers store information using bits (binary digits, that is, a 0 or a 1). For example, 1011 is a four-bit code.



Hamming uses a Venn diagram with three “sets” as follows:

1. The digits of the four-bit code are placed in regions a, b, c, and d, in this order.
2. Three additional digits of 0’s and 1’s are put in the regions E, F, and G so that each “set” has an even number of 1’s.
3. The code is then extended to a 7-bit code using (in order) the digits in the regions a, b, c, d, E, F, G.

For example, the code 1011 is encoded as follows:



Example 4. Two students are vying to represent their school in the regional chess competition. Felix won 12 of the 17 games he played this year, while Rommel won 11 of the 14 games he played this year. If you were the principal of the school, which student would you choose? Explain.

The Prinicpal will likely use fractions to get the winning ratio or percentage of each player. Felix has a $\frac{12}{17}$*winning ratio, while Rommel has a* $\frac{11}{14}$*winning ratio. Since* $\frac{11}{14} > \frac{12}{17}$*, Rommel will be a logical choice.*

Example 5. A class is having an election to decide whether they will go on a fieldtrip. They will have a fieldtrip if more than 50% of the class will vote <u>Yes</u>. Assume that every member of the class will vote. If 34% of the girls and 28% of the boys will vote Yes, will the class go on a fieldtrip? Explain.

Although $38 + 28 = 64 > 50$*, less than half of the girls and less than half of the boys voted Yes. This means that less than half all students voted Yes.*

Example 6. A sale item was marked down by the same percentage for three years in a row. After two years the item was 51% off the original price. By how much was the price off the original price in the first year?

Since the price after 2 years is 51% off the original price, this means that the price is then 49% of the original. Since the percentage ratio must be multiplied to the original price twice (one per year), and $0.7 \times 0.7 = 0.49$, then the price per year is 70% of the price in the preceding year. Hence the discount is 30% off the original.

Exercises:

1. The following table shows the mean temperature in Moscow by month from 2001 to 2011

January	-6.4℃	May	13.7℃	September	12.6℃
February	-7.6℃	June	16.9℃	October	6.0℃
March	-0.8℃	July	21.0℃	November	0.5℃
April	6.8℃	August	18.4℃	December	-4.9℃

Plot each temperature point on the number line and list from lowest to highest.

2. Below are the ingredients for chocolate oatmeal raisin cookies. The recipe yields 32 cookies. Make a list of ingredients for a batch of 2 dozen cookies.

1 ½ cups all-purpose flour
1 tsp baking soda
1 tsp salt
1 cup unsalted butter
¾ cup light-brown sugar
¾ cup granulated sugar
2 large eggs
1 tsp vanilla extract
2 ½ cups rolled oats
1 ½ cups raisins
12 ounces semi-sweet chocolate chips

3. In high-rise buildings, floors are numbered in increasing sequence from the ground-level floor to second, third, etc, going up. The basement immediately below the ground floor is usually labeled B1, the floor below it is B2, and so on. How many floors does an elevator travel from the 39th floor of a hotel to the basement parking at level B6?

4. A piece of ribbon 25 m long is cut into pieces of equal length. Is it possible to get a piece with irrational length? Explain.

5. Explain why $5 + \sqrt{3}$ is irrational. (See Example 1.)

Lesson 15: Measurement and Measuring Length **Time: 2.5 hours**

Prerequisite Concepts: Real Numbers and Operations

About the Lesson:

This is a lesson on the English and Metric System of Measurement and using these systems to measure length. Since these systems are widely used in our community, a good grasp of this concept will help you be more accurate in dealing with concepts involving length such as distance, perimeter and area.

Objective

At the end of the lesson, you should be able to:

1. Describe what it means to measure;
2. Describe the development of measurement from the primitive to the present international system of unit;
3. Estimate or approximate length;
4. Use appropriate instruments to measure length;
5. Convert length measurement from one unit to another, including the English system;
6. Solve problems involving length, perimeter and area.

Lesson Proper

A.

I. Activity:

Instructions: Determine the dimension of the following using only parts of your arms. Record your results in the table below. Choose a classmate and compare your results.

	SHEET OF INTERMEDIATE PAPER		TEACHER'S TABLE		CLASSROOM	
	Length	Width	Length	Width	Length	Width
Arm part used*						
Measurement						
Comparison to: (classmate's name)						

* For the arm part, please use any of the following only: the palm, the handspan and the forearm length

Important Terms to Remember:

>palm – the width of one's hand excluding the thumb

> handspan – the distance from the tip of the thumb to the tip of the little finger of one's hand with fingers spread apart.

> forearm length – the length of one's forearm: the distance from the elbow to the tip of the middle finger.

Answer the following questions:
1. What was your reason for choosing which arm part to use? Why?
2. Did you experience any difficulty when you were doing the actual measuring?
3. Were there differences in your data and your classmate's data? Were the differences significant? What do you think caused those differences?

II. Questions to Ponder (Post-Activity Discussion)
Let us answer the questions in the opening activity:
1. What is the appropriate arm part to use in measuring the length and width of the sheet of paper? of the teacher's table? Of the classroom? What was your reason for choosing which arm part to use? Why?
   - While all of the units may be used, there are appropriate units of measurement to be used depending on the length you are trying to measure.
   - For the sheet of paper, the palm is the appropriate unit to use since the handspan and the forearm length is too long.
   - For the teacher's table, either the palm or the handspan will do but the forearm length might be too long to get an accurate measurement.
   - For the classroom, the palm and handspan may be used but you may end up with a lot of repetitions. The best unit to use would be the forearm length.
2. Did you experience any difficulty when you were doing the actual measuring?
   The difficulties you may have experienced might include having to use too many repetitions.
3. Were there differences in your data and your classmate's data? Were the differences significant? What do you think caused those differences?
   If you and your partner vary a lot in height, then chances are your forearm length, handspan and palm may also vary, leading to different measurements of the same thing.

History of Measurement

One of the earliest tools that human beings invented was the unit of measurement. In olden times, people needed measurement to determine how long or wide things are; things they needed to build their houses or make their clothes. Later, units of measurement were used in trade and commerce. In the 3rd century BC Egypt, people used their body parts to determine measurements of things; the same body parts that you used to measure the assigned things to you.

The forearm length, as described in the table below, was called a cubit. The handspan was considered a half cubit while the palm was considered 1/6 of a cubit. Go ahead, check out how many handspans your forearm length is. The Egyptians came up with these units to be more accurate in measuring different lengths.

However, using these units of measurement had a disadvantage. Not everyone had the same forearm length. Discrepancies arose when the people started comparing their measurements to one another because measurements of the same thing differed, depending on who was measuring it. Because of this, these units of measurement are called non-standard units of measurement which later on evolved into what is now the inch, foot and yard, basic units of length in the English system of measurement.

III. Exercise:

1. Can you name other body measurements which could have been used as a non-standard unit of measurement? Do some research on other non-standard units of measurement used by people other than the Egyptians.
2. Can you relate an experience in your community where a non-standard unit of measurement was used?

B.

I. Activity

Instructions: Determine the dimension of the following using the specified English units only. Record your results in the table below. Choose a classmate and compare your results.

	SHEET OF INTERMEDIATE PAPER		TEACHER'S TABLE		CLASSROOM	
	Length	Width	Length	Width	Length	Width
Arm part used*						
Measurement						
Comparison to: (classmate's name)						

For the unit used, choose which of the following SHOULD be used: inch or foot.

Answer the following questions:

1. What was your reason for choosing which unit to use? Why?
2. Did you experience any difficulty when you were doing the actual measuring?
3. Were there differences in your data and your classmate's data? Were the differences as big as the differences when you used non-standard units of measurement? What do you think caused those differences?

II. Questions to Ponder (Post-Activity Discussion)

Let us answer the questions in the activity above:

1. What was your reason for choosing which unit to use? Why?
    - For the sheet of paper, the appropriate unit to use is inches since its length and width might be shorter than a foot.
    - For the table and the classroom, a combination of both inches and feet may be used for accuracy and convenience of not having to deal with a large number.
2. What difficulty, if any, did you experience when you were doing the actual measuring?
3. Were there differences in your data and your classmate's data? Were the differences as big as the differences when you used non-standard units of measurement? What do you think caused those differences?
    - If you and your partner used the steel tape correctly, both your data should have little or no difference at all. The difference should not be as big or as significant as the difference when non-standard units of measurement were

used. The slight difference might be caused by how accurately you tried to measure each dimension or by how you read the ticks on the steel tape. In doing actual measurement, a margin of error should be considered.

History of Measurement (Continued)

As mentioned in the first activity, the inch, foot and yard are said to be based on the cubit. They are the basic units of length of the English System of Measurement, which also includes units for mass, volume, time, temperature and angle. Since the inch and foot are both units of length, each can be converted into the other. Here are the conversion factors, as you may recall from previous lessons:

1 foot = 12 inches

1 yard = 3 feet

For long distances, the mile is used:

1 mile = 1,760 yards = 5,280 feet

Converting from one unit to another might be tricky at first, so an organized way of doing it would be a good starting point. As the identity property of multiplication states, the product of any value and 1 is the value itself. Consequently, dividing a value by the same value would be equal to one. Thus, dividing a unit by its equivalent in another unit is equal to 1. For example:

1 foot / 12 inches = 1

3 feet / 1 yard = 1

These conversion factors may be used to convert from one unit to another. Just remember that you're converting from one unit to another so cancelling same units would guide you in how to use your conversion factors. For example:

1. Convert 36 inches into feet:

$$36\ \cancel{inches}\ x\ \frac{1\ foot}{12\ \cancel{inches}} = \mathbf{3feet}$$

2. Convert 2 miles into inches:

$$2\ \cancel{miles}\ x\ \frac{5280\ \cancel{feet}}{1\ \cancel{mile}}\ x\ \frac{12\ inches}{1\ \cancel{foot}} = \frac{2x5280x12}{1x1}\ inches = \mathbf{126,720inches}$$

Again, since the given measurement was multiplied by conversion factors which are equal to 1, only the unit was converted but the given length was not changed.
Try it yourself.

III. Exercise:

Convert the following lengths into the desired unit:

1. Convert 30 inches to feet
2. Convert 130 yards to inches
3. Sarah is running in a 42-mile marathon. How many more feet does Sarah need to run if she has already covered64,240 yards?

C.

I. Activity:

Answer the following questions:

1. When a Filipina girl is described as 1.7 meters tall, would she be considered tall or short? How about if the Filipina girl is described as 5 ft, 7 inches tall, would she be considered tall or short?

2. Which particular unit of height were you more familiar with? Why?

II. Questions to Ponder (Post-Activity Discussion)

Let us answer the questions in the activity above:

1. When a Filipina girl is described as 1.7 meters tall, would she be considered tall or short? How about if the Filipina girl is described as 5 ft, 7 inches tall, would she be considered tall or short?
   - Chances are, you would find it difficult to answer the first question. As for the second question, a Filipina girl with a height of 5 feet, 7 inches would be considered tall by Filipino standards.
2. Which particular unit of height were you more familiar with? Why?
   - Again, chances are you would be more familiar with feet and inches since feet and inches are still being widely used in measuring and describing height here in the Philippines.

History of Measurement (Continued)

The English System of Measurement was widely used until the 1800s and the 1900s when the Metric System of Measurement started to gain ground and became the most used system of measurement worldwide. First described by Belgian Mathematician Simon Stevin in his booklet, De Thiende (The Art of Tenths) and proposed by English philosopher, John Wilkins, the Metric System of Measurement was first adopted by France in 1799. In 1875, the General Conference on Weights and Measures (*Conférence générale des poids et mesures or CGPM)* was tasked to define the different measurements. By 1960, CGPM released the International System of Units (SI) which is now being used by majority of the countries with the biggest exception being the United States of America. Since our country used to be a colony of the United States, the Filipino people were schooled in the use of the English instead of the Metric System of Measurement. Thus, the older generation of Filipinos is more comfortable with English System rather than the Metric System although the Philippines have already adopted the Metric System as its official system of measurement.

The Metric System of Measurement is easier to use than the English System of Measurement since its conversion factors would consistently be in the decimal system, unlike the English System of Measurement where units of lengths have different conversion factors. Check out the units used in your steep tape measure, most likely they are inches and centimeters. The base unit for length is the meter and units longer or shorter than the meter would be achieved by adding prefixes to the base unit. These prefixes may also be used for the base units for mass, volume, time and other measurements. Here are the common prefixes used in the Metric System:

PREFIX	SYMBOL	FACTOR
tera	T	x 1,000,000,000,000
giga	G	x 1,000,000,000
mega	M	x 1,000,000
kilo	k	x 1,000
hecto	h	x 100
deka	da	x 10

deci	d	x 1/10
centi	c	x 1/100
milli	m	x 1/1,000
micro	µ	x 1/1,000,000
nano	n	x 1/1,000,000,000

For example:

1 kilometer = 1,000 meters

1 millimeter = 1/1,000 meter or 1,000 millimeters = 1 meter

These conversion factors may be used to convert from big to small units or vice versa. For example:

1. Convert 3 km to m:

$$3\ \cancel{km}\ x\ \frac{1{,}000\ m}{1\ \cancel{km}} = \mathbf{3{,}000}\boldsymbol{m}$$

2. Convert 10 mm to m:

$$10\ \cancel{mm}\ x\ \frac{1\ m}{1000\ \cancel{mm}} = \frac{1}{100}\ \boldsymbol{or}\mathbf{0.01}\boldsymbol{m}$$

As you can see in the examples above, any length or distance may be measured using the appropriate English or Metric units. In the question about the Filipina girl whose height was expressed in meters, her height can be converted to the more familiar feet and inches. So, in the Philippines where the official system of measurements is the Metric System yet the English System continues to be used, or as long as we have relatives and friends residing in the United States, knowing how to convert from the English System to the Metric System (or vice versa) would be useful. The following are common conversion factors for length:

1 inch = 2.54 cm

3.3 feet ≈ 1 meter

For example:

Convert 20 inches to cm:

$$20\ \cancel{in}\ x\ \frac{2.54\ cm}{1\ \cancel{in}} = \mathbf{50.8}\boldsymbol{cm}$$

III. Exercise:

1. Using the tape measure, determine the length of each of the following in cm. Convert these lengths to meters.

	PALM	HANDSPAN	FOREARM LENGTH
Centimeters			
Meters			

2. Using the data in the table above, estimate the lengths of the following without using the steel tape measure or ruler:

	BALLPEN	LENGTH OF WINDOW PANE	LENGTH OF YOUR FOOT FROM THE TIP OF YOUR HEEL TO THE TIP OF YOUR TOES	HEIGHT OF THE CHALK BOARD	LENGTH OF THE CHALK BOARD
NON-STANDARD UNIT					
METRIC UNIT					

3. Using the data from table 1, convert the dimensions of the sheet of paper, teacher's table and the classroom into Metric units. Recall past lessons on perimeter and area and fill in the appropriate columns:

	SHEET OF INTERMEDIATE PAPER				TEACHER'S TABLE				CLASSROOM			
	Length	Width	Peri-meter	Area	Length	Width	Peri-meter	Area	Length	Width	Peri-meter	Area
English units												
Metric Units												

4. Two friends, Zale and En zo, run in marathons. Zale finished a 21-km marathon in Cebu while Enzo finished a 15-mile marathon in Los Angeles. Who between the two ran a longer distance? By how many meters?
5. Georgia wants to fence her square garden, which has a side of 20 feet, with two rows of barb wire. The store sold barb wire by the meter at P12/meter. How much money will Georgia need to buy the barb wire she needs?
5. A rectangular room has a floor area of 32 square meters. How many tiles, each measuring 50 cm x 50 cm, are needed to cover the entire floor?

Summary

In this lesson, you learned: 1) that ancient Egyptians used units of measurement based on body parts such as the cubit and the half cubit. The cubit is the length of the forearm from the elbow to the tip of the middle finger; 2) that the inch and foot, the units for length of the English System of Measurement, are believed to be based on the cubit; 3) that the Metric System of Measurement became the dominant system in the 1900s and is now used by most of the countries with a few exceptions, the biggest exception being the United States of America; 4) that it is appropriate to use short base units of length for measuring short lengths and long units of lengths to measure long lengths or distances; 5) how to convert common English units of length into other English units of length using conversion factors; 6) that the Metric System of Measurement is based on the decimal system and is therefore easier to use; 7) that the Metric System of Measurement has a base unit for length (meter) and prefixes to signify long or short lengths or distances; 8) how to estimate lengths and distances using your arm parts and their equivalent Metric lengths; 9) how to convert common Metric units of length into other Metric units of length using the conversion factors based on prefixes; 10) how to convert common English units of length into Metric units of length (and vice versa) using conversion factors; 11) how to solve length, perimeter and area problems using English and Metric units.

Lesson 16: Measuring Weight/Mass and Volume **Time: 2.5 hours**

Prerequisite Concepts: Basic concepts of measurement, measurement of length

About the Lesson:

This is a lesson on measuring volume & mass/weight and converting its units from one to another. A good grasp of this concept is essential since volume & weight are commonplace and have practical applications.

Objectives:

At the end of the lesson, you should be able to:

7. estimate or approximate measures of weight/mass and volume;
8. use appropriate instruments to measure weight/mass and volume;
9. convert weight/mass and volume measurements from one unit to another, including the English system;
10. Solve problems involving weight/mass and volume/capacity.

Lesson Proper

A.

I. Activity:

Read the following narrative to help you review the concept of volume.

Volume

Volume is the amount of space an object contains or occupies. The volume of a container is considered to be the capacity of the container. This is measured by the number of cubic units or the amount of fluid it can contain and not the amount of space the container occupies. The base SI unit for volume is the cubic meter (m^3). Aside from cubic meter, another commonly used metric unit for volume of solids is the cubic centimeter (cm^3 or cc) while the commonly used metric units for volume of fluids are the liter (L) and the milliliter (mL).

Hereunder are the volume formulae of some regularly-shaped objects:

Cube: Volume = edge x edge x edge ($V = e^3$)

Rectangular prism: Volume = length x width x height (V = lwh)

Triangular prism: Volume = ½ x base of the triangular base x height of the triangular base x Height of the prism

$$(V = (\frac{1}{2}bh)H)$$

Cylinder: Volume = π x $(\text{radius})^2$ x height of the cylinder ($V = \pi r^2 h$)

Other common regularly-shaped objects are the different pyramids, the cone and the sphere. The volumes of different pyramids depend on the shape of its base. Here are their formulae:

Square-based pyramids: Volume = 1/3 x $(\text{side of base})^2$ x height of pyramid($V = 1/3\ \mathbf{s^2}h$)

Rectangle-based pyramid: Volume=1/3 x length of the base x width of the base x height of pyramid(V=1/3 lwh)

Triangle-based pyramid: Volume = 1/3 x ½ x base of the triangle x height of the triangle x Height of the pyramid

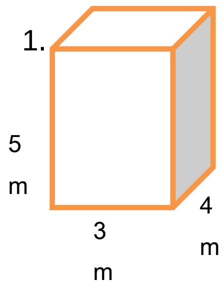
$(V = \frac{1}{3}(\frac{1}{2}bh)H)$

Cone: Volume = 1/3 x π x (radius)^2 x height

Sphere: Volume = 4/3 x πx (radius)^3 (V = 4/3 πr^3)

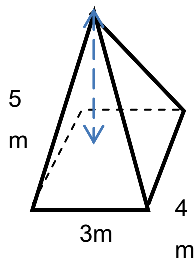
Here are some examples:

1.



V = lwh = 3 m x 4 m x 5 m
= (3 x 4 x 5) x (m x m x m) = 60 m^3

2.



V = 1/3 lwh = 1/3 x 3 m x 4 m x 5 m
= (1/3 x 3 x 4 x 5) x (m x m x m) = 20 m^3

Answer the following questions:

1. Cite a practical application of volume.
2. What do you notice about the parts of the formulas that have been underlined? Come up with a general formula for the volume of all the given prisms and for the cylinder.
3. What do you notice about the parts of the formulas that have been shaded? Come up with a general formula for the volume of all the given pyramids and for the cone.

II. Questions to Ponder (Post-Activity Discussion)

Let us answer the questions in the opening activity:

1. Cite a practical application of volume.
   Volume is widely used from baking to construction. Baking requires a degree of precision in the measurement of the ingredients to be used thus measuring spoons and cups are used. In construction, volume is used to measure the size of a room, the amount of concrete needed to create a specific column or beam or the amount of water a water tank could hold.
2. What do you notice about the parts of the formulas that have been underlined? Come up with a general formula for the volume of all the given prisms and for the cylinder.
   The formulas that have been underlined are formulas for area. The general formula for the volume of the given prisms and cylinder is just the area of the base of the prisms or cylinder times the height of the prism or cylinder (V = $A_{base}h$*).*

3. What do you notice about the parts of the formulas that have been shaded? Come up with a general formula for the volume of all the given pyramids and for the cone.
   The formulas that have been shaded are formulas for the volume of prisms or cylinders. The volume of the given pyramids is just 1/3 of the volume of a prism whose base and height are equal to that of the pyramid while the formula for the cone is just 1/3 of the volume of a cylinder with the same base and height as the cone ($V = 1/3\ V_{\text{prism or cylinder}}$).

III. Exercise:

Instructions: Answer the following items. Show your solution.

1. How big is a Toblerone box (triangular prism) if its triangular side has a base of 3 cm and a height of 4.5 cm and the box's height is 25 cm?
2. How much water is in a cylindrical tin can with a radius of 7 cm and a height of 20 cm if it is only a quarter full?
3. Which of the following occupies more space, a ball with a radius of 4 cm or a cube with an edge of 60 mm?

B.

I. Activity

Materials Needed:

Ruler / Steel tape measure

Different regularly-shaped objects (brick, cylindrical drinking glass, balikbayan box)

Instructions: Determine the dimension of the following using the specified metric units only. Record your results in the table below and compute for each object's volume using the unit used to measure the object's dimensions. Complete the table by expressing/converting the volume using the specified units.

		BRICK			DRINKING GLASS		BALIKBAYAN BOX			CLASSROOM		
		Length	Width	Height	Radius	Height	Length	Width	Height	Length	Width	Height
Unit used*												
Measurement												
Volume	cm^3											
	m^3											
	in^3											
	ft^3											

For the unit used, choose ONLY one: centimeter or meter.

Answer the following questions:

1. What was your reason for choosing which unit to use? Why?
2. How did you convert the volume from cc to m^3 or vice versa?
3. How did you convert the volume from cc to the English units for volume?

Volume (continued)

The English System of Measurement also has its own units for measuring volume or capacity. The commonly used English units for volume are cubic feet (ft^3) or cubic inches (in^3) while the commonly used English units for fluid volume are the pint, quart or gallon. Recall from the lesson on length and area that while the Philippine government has mandated the use of the Metric system, English units are still very much in use in our society so it is an advantage if we know how to convert from the English to the Metric system and vice versa. Recall as well from the previous lesson on measuring length that a unit can be converted into another unit using conversion factors. Hereunder are some of the conversion factors which would help you convert given volume units into the desired volume units:

1 m^3 = 1 million cm^3	1 gal = 3.79 L
1 ft^3 = 1,728 in^3	1 gal = 4 quarts
1 in^3 = 16.4 cm^3	1 quart = 2 pints
1 m^3 = 35.3 ft^3	1 pint = 2 cups
	1 cup = 16 tablespoons
	1 tablespoon = 3 teaspoons

Since the formula for volume only requires length measurements, another alternative to converting volume from one unit to another is to convert the object's dimensions into the desired unit before solving for the volume.

For example:

1. How much water, in cubic centimeters, can a cubical water tank hold if it has an edge of 3 meters?

Solution 1 (using a conversion factor):

i. Volume = $e^3 = (3\text{ m})^3 = 27\text{ m}^3$

ii. $27\text{ m}^3 \times {}^{1\text{ million cm3}}/_{1\text{ m3}}$ = **27 million cm^3**

Solution 2 (converting dimensions first):

i. $3\text{ m} \times {}^{100\text{ cm}}/_{1\text{ m}} = 300\text{ cm}$

ii. Volume = $e^3 = (300\text{ cm})^3$ = **27 million cm^3**

II. Questions to Ponder (Post-Activity Discussion)

Let us answer the questions in the activity above:

1. What was your reason for choosing which unit to use?

   Any unit on the measuring instrument may be used but the decision on what unit to use would depend on how big the object is. In measuring the brick, the glass and the balikbayan box, the appropriate unit to use would be centimeter. In measuring the dimensions of the classroom, the appropriate unit to use would be meter.

2. How did you convert the volume from cc to m^3 or vice versa?

   Possible answer would be converting the dimensions to the desired units first before solving for the volume.

3. How did you convert the volume from cc or m^3 to the English units for volume?

   Possible answer would be by converting the dimensions into English units first before solving for the volume.

III. Exercises:

Answer the following items. Show your solutions.

1. Convert 10 m^3 to ft^3
2. Convert 12 cups to mL
3. A cylindrical water tank has a diameter of 4 feet and a height of 7 feet while a water tank shaped like a rectangular prism has a length of 1 m, a width of 2 meters and a height of 2 meters. Which of the two tanks can hold more water? By how many cubic meters?

C.

I. Activity:

Problem: The rectangular water tank of a fire truckmeasures 3 m by 4 m by 5 m. How many liters of water can the fire truck hold?

Volume (Continued)

While capacities of containers are obtained by measuring its dimensions, fluid volume may also be expressed using Metric or English units for fluid volume such as liters or gallons. It is then essential to know how to convert commonly used units for volume into commonly used units for measuring fluid volume.

While the cubic meter is the SI unit for volume, the liter is also widely accepted as a SI-derived unit for capacity. In 1964, after several revisions of its definition, the General Conference on Weights and Measures (CGPM) finally defined a liter as equal to one cubic decimeter. Later, the letter L was also accepted as the symbol for liter.

This conversion factor may also be interpreted in other ways. Check out the conversion factors below:

1 L = 1 dm^3
1 mL = 1 cc
1,000 L = 1 m^3

II. Questions to Ponder (Post-Activity Discussion)

Let us answer the problem above:

Step 1: V = lwh
= 3m x 4m x 5m
= 60 m^3

Step 2: 60 m^3 x $^{1,000\ L}/_{1\ m3}$ = ***60,000 L***

III. Exercise:

Instructions: Answer the following items. Show your solution.

1. A spherical fish bowl has a radius of 21 cm. How many mL of water is needed to fill half the bowl?
2. A rectangular container van needs to be filled with identical cubical balikbayan boxes. If the container van's length, width and height are 16 ft, 4 ft and 6ft, respectively, while each balikbayan box has an edge of 2 ft, what is the maximum number of balikbayan boxes that can be placed inside the van?
3. A drinking glass has a height of 4 in, a length of 2 in and a width of 2 in while a baking pan has a width of 4 in, a length of 8 in and a depth of 2 in. If the baking pan is to be filled with water up to half its depth using the drinking glass, how many glasses full of water would be needed?

D.

Activity:

Instructions: Fill the table below according to the column headings. Choose which of the available instruments is the most appropriate in measuring the given object's weight. For the weight, choose only one of the given units.

	INSTRUMENT*	WEIGHT		
		Gram	Kilogram	Pound
¢25-coin				
₱5-coin				
Small toy marble				
Piece of brick				
Yourself				

*Available instruments: triple-beam balance, nutrition (kitchen) scale, bathroom scale

Answer the following questions:

1. What was your reason for choosing which instrument to use?
2. What was your reason for choosing which unit to use?
3. What other kinds of instruments for measuring weight do you know?
4. What other units of weight do you know?

Mass/ Weight

In common language, mass and weight are used interchangeably although weight is the more popular term. Oftentimes in daily life, it is the mass of the given object which is called its weight. However, in the scientific community, mass and weight are two different measurements. Mass refers to the amount of matter an object has while weight is the gravitational force acting on an object.

Weight is often used in daily life, from commerce to food production. The base SI unit for weight is the kilogram (kg) which is almost exactly equal to the mass of one liter of water. For the English System of Measurement, the base unit for weight is the pound (lb). Since both these units are used in Philippine society, knowing how to convert from pound to kilogram or vice versa is important. Some of the more common Metric units are the gram (g) and the milligram (mg) while another commonly used English unit for weight is ounces (oz). Here are some of the conversion factors for these units:

1 kg = 2.2 lb 1 g = 1000 mg 1 metric ton = 1000 kg

1 kg = 1000 g 1 lb = 16 oz

Use these conversion factors to convert common weight units to the desired unit. For example:

Convert 190 lb to kg: $19\cancel{0}\, lb\ x\ \frac{1\, kg}{2\cancel{.2}\, lb} = 86.18\ kg$

II. Questions to Ponder (Post-Activity Discussion)

1. What was your reason for choosing which instrument to use?
   Possible reasons would include how heavy the object to be weighed to the capacity of the weighing instrument.
2. What was your reason for choosing which unit to use?
   The decision on which unit to use would depend on the unit used by the weighing instrument. This decision will also be influenced by how heavy the object is.

3. What other kinds of instruments for measuring weight do you know?
   Other weighing instruments include the two-pan balance, the spring scale, the digital scales.
4. What other common units of weight do you know?
   Possible answers include ounce, carat and ton.

III. Exercise:

Answer the following items. Show your solution.

1. Complete the table above by converting the measured weight into the specified units.
2. When Sebastian weighed his balikbayan box, its weight was 34 kg. When he got to the airport, he found out that the airline charged $5 for each lb in excess of the free baggage allowance of 50 lb. How much will Sebastian pay for the excess weight?
3. A forwarding company charges P1,100 for the first 20 kg and P60 for each succeeding 2 kg for freight sent to Europe. How much do you need to pay for a box weighing 88 lb?

Summary

In this lesson, you learned: 1) how to determine the volume of selected regularly-shaped solids; 2) that the base SI unit for volume is the cubic meter; 3) how to convert Metric and English units of volume from one to another; 4) how to solve problems involving volume or capacity; 5) that mass and weight are two different measurements and that what is commonly referred to as weight in daily life is actually the mass; 6) how to use weighing intruments to measure the mass/weight of objects and people; 7) how to convert common Metric and English units of weight from one to another; 8) how to solve problems involving mass / weight.

Lesson 17: Measuring Angles, Time and Temperature

Prerequisite Concepts: Basic concepts of measurement, ratios
About the Lesson:

This lesson should reinforce your prior knowledge and skills on measuring angle, time and temperature as well as meter reading. A good understanding of this concept would not only be useful in your daily lives but would also help you in geometry and physical sciences.

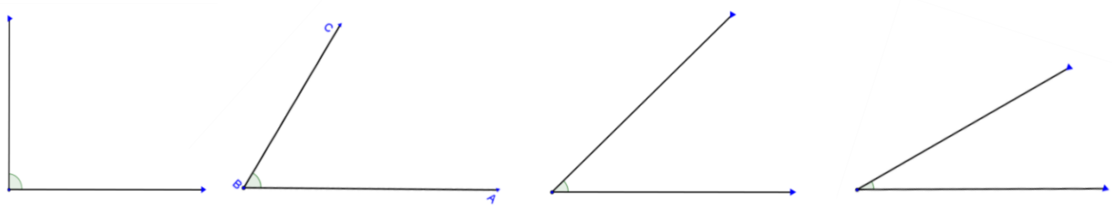
Objectives:
At the end of the lesson, you should be able to:

11. estimate or approximate measures of angle, time and temperature;
12. use appropriate instruments to measure angles, time and temperature;
13. solve problems involving time, speed, temperature and utilities usage (meter reading).

Lesson Proper
A.
I. Activity:
Material needed:

Protractor

Instruction: Use your protractor to measure the angles given below. Write your answer on the line provided.



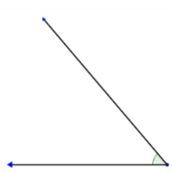
1.__________ 2.______________ 3._____________ 4._____________

Angles

Derived from the Latin word *angulus*, which means corner, an angle is defined as a figure formed when two rays share a common endpoint called the vertex. Angles are measured either in degree or radian measures. A protractor is used to determine the measure of an angle in degrees. In using the protractor, make sure that the cross bar in the middle of the protractor is aligned with the vertex and one of the legs of the angle is aligned with one side of the line passing through the cross bar. The measurement of the angle is determined by its other leg.

Answer the following items:

1. Estimate the measurement of the angle below. Use your protractor to check your estimate.



Estimate________________
Measurement using the protractor_______

2. What difficulties did you meet in using your protractor to measure the angles?

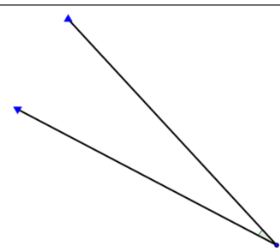
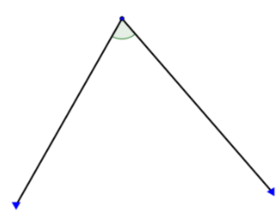
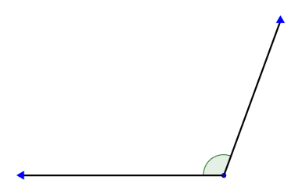
3. What can be done to improve your skill in estimating angle measurements?

II. Questions to Ponder (Post-activity discussion):

1. Estimate the measurement of the angles below. Use your protractor to check your estimates.
   Measurement = 50°
2. What difficulties did you meet in using your protractor to measure the angles?
   One of the difficulties you may encounter would be on the use of the protractor and the angle orientation. Aligning the cross bar and base line of the protractor with the vertex and an angle leg, respectively, might prove to be confusing at first, especially if the angle opens in the clockwise orientation. Another difficulty arises if the length of the leg is too short such that it won't reach the tick marks on the protractor. This can be remedied by extending the leg.
3. What can be done to improve your skill in estimating angle measurements?
   You may familiarize yourself with the measurements of the common angles like the angles in the first activity and use these angles in estimating the measurement of other angles.

III. Exercise:

Instructions: Estimate the measurement of the given angles, then check your estimates by measuring the same angles using your protractor.

ANGLE	 A	 B	 C
ESTIMATE			
MEASURE MENT			

B.

I. Activity

Problem: An airplane bound for Beijing took off from the Ninoy Aquino International Airport at 11:15 a.m. Its estimated time of arrival in Beijing is at1550 hrs. The distance from Manila to Beijing is 2839 km.

1. What time (in standard time) is the plane supposed to arrive in Beijing?
2. How long is the flight?
3. What is the plane's average speed?

Time and Speed

The concept of time is very basic and is integral in the discussion of other concepts such as speed. Currently, there are two types of notation in stating time, the 12-hr notation (standard time) or the 24-hr notation (military or astronomical time). Standard time makes use of a.m. and p.m. to distinguish between the time from 12midnight to 12 noon (a.m. or *ante meridiem*) and from 12 noon to 12 midnight (p.m. or *post meridiem*). This sometimes leads to ambiguity when the suffix of a.m. and p.m. are left out. Military time prevents this ambiguity by using the 24-hour notation where the counting of the time continues all the way to 24. In this notation, 1:00 p.m. is expressed as 1300 hours or 5:30 p.m. is expressed as 1730 hours.

Speed is the rate of an object's change in position along a line. Average speed is determined by dividing the distance travelled by the time spent to cover the distance (Speed = $^{\text{distance}}/_{\text{time}}$ or S = $^{d}/_{t}$, read as "distance per time"). The base SI unit for speed is meters per second ($^{m}/_{s}$). The commonly used unit for speed is $^{\text{Kilometers}}/_{\text{hour}}$ (kph or $^{km}/_{h}$) for the Metric system and miles/hour (mph or $^{mi}/_{hr}$) for the English system.

II. Questions to Ponder (Post-Activity Discussion)

Let us answer the questions in the activity above:

1. What time (in standard time) is the plane supposed to arrive in Beijing?
   3:50 p.m.
2. How long is the flight?
   1555 hrs – 1115 hrs = 4 hrs, 40 minutes or 4.67 hours
3. What is the plane's average speed?
   S = d/t
   = 2839 km / 4.67 hrs
   = 607.92 kph

III. Exercise:

Answer the following items. Show your solutions.

1. A car left the house and travelled at an average speed of 60 kph. How many minutes will it take for the car to reach the school which is 8 km away from the house?
2. Sebastian stood at the edge of the cliff and shouted facing down. He heard the echo of his voice 4 seconds after he shouted. Given that the speed of sound in air is 340 m / s, how deep is the cliff?
3. Maria ran in a 42-km marathon. She covered the first half of the marathon from 0600 hrs to 0715 hours and stopped to rest. She resumed running and was able to cover the remaining distance from 0720 hrs to 0935 hrs. What was Maria's average speed for the entire marathon?

C.

I. Activity:

Problem: Zale, a Cebu resident, was packing his suitcase for his trip to New York City the next day for a 2-week vacation. He googled New York weather and found out the average temperature there is 59°F. Should he bring a sweater? What data should Zale consider before making a decision?

Temperature

Temperature is the measurement of the degree of hotness or coldness of an object or substance. While the commonly used units are Celsius (°C) for the Metric system and Fahrenheit (°F) for the English system, the base SI unit for temperature is the Kelvin (K). Unlike the Celsius and Fahrenheit which are considered degrees, the Kelvin is considered as an absolute unit of measure and therefore can be worked on algebraically.

Hereunder are some conversion factors:

$°C = (^5/_9)(°F – 32)$

$°F = (^9/_5)(°C) + 32$

$K = °C + 273.15$

For example:

Convert 100°C to °F: *$°F = (^9/_5)(100\ °C) + 32$*

$= 180 + 32$

$= 212\ °F$

II. Questions to Ponder (Post-Activity Discussion)

Let us answer the problem above:

1. What data should Zale consider before making a decision?

   In order to determine whether he should bring a sweater or not, Zale needs to compare the average temperature in NYC to the temperature he is used to which is the average temperature in Cebu. He should also express both the average temperature in NYC and in Cebu in the same units for comparison.

2. Should Zale bring a sweater?

   The average temperature in Cebu is between 24 – 32 °C. Since the average temperature in NYC is 59°F which is equivalent to 15°C, Zale should probably bring a sweater since the NYC temperature is way below the temperature he is used to. Better yet, he should bring a jacket just to be safe.

III. Exercise:

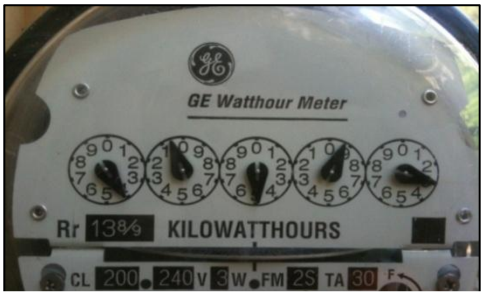
Instructions: Answer the following items. Show your solution.

1. Convert 14°F to K.
2. Maria was preparing the oven to bake brownies. The recipe's direction was to pre-heat the oven to 350°F but her oven thermometer was in °C. What should be the thermometer reading before Maria puts the baking pan full of the brownie mix in the oven?

D.

Activity:

Instructions: Use the pictures below to answer the questions that follow.



Initial electric meter reading at 0812 hrs on 14 Feb 2012

Final electric meter reading at 0812 hrs on 15 Feb 2012

1. What was the initial meter reading? Final meter reading?
2. How much electricity was consumed during the given period?

3. How much will the electric bill be for the given time period if the electricity charge is P9.50 / kiloWatthour?

Reading Your Electric Meter

Nowadays, reading the electric meter would be easier considering that the newly-installed meters are digital but most of the installed meters are still dial-based. Here are the steps in reading the electric meter:

a. To read your dial-based electric meter, read the dials from left to right.
b. If the dial hand is between numbers,the smaller of the two numbers should be used. If the dial hand is on the number, check out the dial to the right. If the dial hand has passed zero, use the number at which the dial hand is pointing. If the dial hand has not passed zero, use the smaller number than the number at which the dial hand is pointing.
c. To determine the electric consumption for a given period, subtract the initial reading from the final reading.

II. Questions to Ponder (Post-Activity Discussion)

Let us answer the questions above:

1. What was the initial meter reading? final meter reading?
   The initial reading is 40493 kWh. For the first dial from the left, the dial hand is on the number 4 so you look at the dial immediately to the right which is the second dial. Since the dial hand of the second dial is past zero already, then the reading for the first dial is 4. For the second dial, since the dial hand is between 0 and 1 then the reading for the second dial is 0. For the third dial from the left, the dial hand is on the number 5 so you look at the dial immediately to the right which is the fourth dial. Since the dial hand of the fourth dial has not yet passed zero, then the reading for the third dial is 4. The final reading is 40515 kWh.
2. How much electricity was consumed during the given period?
   Final reading – initial reading = 40515 kWh – 40493 kWh = 22 kWh
3. How much will the electric bill be for the given time period if the electricity charge is ₱ 9.50 / kiloWatthour?
   Electric bill = total consumption x electricity charge
   = 22 kWh x P9.50 / kWh
   = P209

III. Exercise:

Answer the following items. Show your solution.

1. The pictures below show the water meter reading of Sebastian's house.

Initial meter reading at 0726 hrs
on 20 February 2012

Final meter reading at 0725 hrs
on 21 February 2012

If the water company charges P14 / cubic meter of water used, how much must Sebastian pay the water company for the given period?

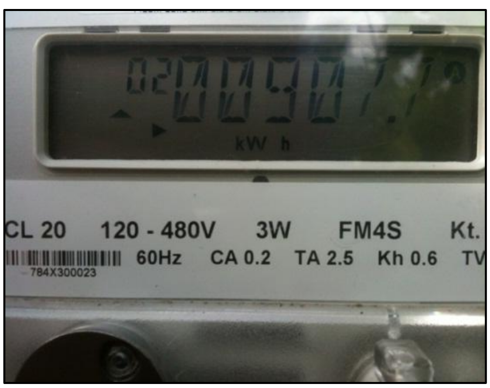
2. The pictures below show the electric meter reading of Maria's canteen.

Initial meter reading at 1600 hrs on 20 Feb 2012

Final meter reading @ 1100 hrs on 22 Feb 2012

If the electric charge is P9.50 / kWh, how much will Maria pay the electric company for the given period?

3. The pictures below show the electric meter reading of a school.



Initial meter reading @1700 hrs on 15 July 2012

Final meter reading @ 1200 hrs on 16 July 2012

Assuming that the school's average consumption remains the same until 1700 hrs of 15 August 2012 and the electricity charge is P9.50 / kWh, how much will the school be paying the electric company?

Summary

In this lesson, you learned:

1. how to measure angles using a protractor;
2. how to estimate angle measurement;
3. express time in 12-hr or 24-hr notation;
4. how to measure the average speed as the quotient of distance over time;
5. convert units of temperature from one to the other;
6. solve problems involving time, speed and temperature;
7. read utilities usage.

Lesson 18: Constants, Variables and Algebraic Expressions

Prerequisite Concepts: Real Number Properties and Operations

About the Lesson:

This lesson is an introduction to the concept of constants, unknowns and variables and algebraic expressions. Familiarity with this concept is necessary in laying a good foundation for Algebra and in understanding and translating mathematical phrases and sentences, solving equations and algebraic word problems as well as in grasping the concept of functions.

Objectives:

At the end of the lesson, you should be able to:

1. Differentiate between constants and variables in a given algebraic expression
2. Evaluate algebraic expressions for given values of the variables

Lesson Proper

I. Activity

A. Instructions: Complete the table below according to the pattern you see.

TABLE A		
ROW	1^{ST} TERM	2^{ND} TERM
a.	1	5
b.	2	6
c.	3	7
d.	4	
e.	5	
f.	6	
g.	59	
h.	Any number *n*	

B. Using Table A as your basis, answer the following questions:

1. What did you do to determine the 2^{nd} term for rows d to f?
2. What did you do to determine the 2^{nd} term for row g?
3. How did you come up with your answer in row h?
4. What is the relation between the 1^{st} and 2^{nd} terms?
5. Express the relation of the 1^{st} and 2^{nd} terms in a mathematical sentence.

II. Questions to Ponder (Post-Activity Discussion)

A. The 2^{nd} terms for rows d to f are 8, 9 and 10, respectively. The 2^{nd} term in row g is 63. The 2^{nd} term in row h is the sum of a given number *n* and 4.

B.

1. One way of determining the 2^{nd} terms for rows d to f is to add 1 to the 2^{nd} term of the preceding row (e.g 7 + 1 = 8). Another way to determine the 2^{nd} term would be to add 4 to its corresponding 1^{st} term (e.g. 4 + 4 = 8).
2. Since from row f, the first term is 6, and from 6 you add 53 to get 59, to get the 2^{nd} term of row g, 10 + 53 = 63. Of course, you could have simply added 4 to 59.

3. The answer in row h is determined by adding 4 to *n*, which represents any number.
4. The 2nd term is the sum of the 1st term and 4.
5. To answer this item better, we need to be introduced to Algebra first.

Algebra

We need to learn a new language to answer item 5. The name of this language is Algebra. You must have heard about it. However, Algebra is not entirely a new language to you. In fact, you have been using its applications and some of the terms used for a long time already. You just need to see it from a different perspective.

Algebra comes from the Arabic word, *al-jabr (*which means restoration), which in turn was part of the title of a mathematical book written around the 820 AD by Arab mathematician, *Muhammad ibn Musa al-Khwarizmi*. While this book is widely considered to have laid the foundation of modern Algebra, history shows that ancient Babylonian, Greek, Chinese and Indian mathematicians were discussing and using algebra a long time before this book was published.

Once you've learned this new language, you'll begin to appreciate how powerful it is and how its applications have drastically improved our way of life.

III. Activity

Instructions: How do you understand the following symbols and expressions?

SYMBOLS / EXPRESSIONS	MEANING
1. *x*	
2. 2 + 3	
3. =	

IV. Questions to Ponder (Post-Activity Discussion)

Let us answer the questions in the previous activity:

1. You might have thought of *x* as the multiplication sign. From here on, *x* will be considered a symbol that stands for any value or number.
2. You probably thought of 2 + 3 as equal to 5 and must have written the number 5. Another way to think of 2 + 3 is to read it as the sum of 2 and 3.
3. You must have thought, “Alright, what am I supposed to compute?” The sign “=” may be called the equal sign by most people but may be interpreted as a command to carry out an operation or operations. However, the equal sign is also a symbol for the relation between the expressions on its left and right sides, much like the less than “$<$” and greater than “$>$” signs.

The Language Of Algebra

The following are important terms to remember.

a. **constant** – a constant is a number on its own. For example, 1 or 127;
b. **variable** – a variable is a symbol, usually letters, which represent a value or a number. For example, *a* or *x*. In truth, you have been dealing with variables since pre-school in the form of squares (□), blank lines (___) or other symbols used to represent the unknowns in some mathematical sentences or phrases;

c. **term** – a term is a constant or a variable or constants and variables multiplied together. For example, 4, xy or $8yz$. The term's number part is called the **numerical coefficient** while the variable or variables is/are called the **literal coefficient/s**. For the term $8yz$, the **numerical coefficient** is 8 and the **literal coefficients** are yz;

d. **expression** – an Algebraic expression is a group of terms separated by the plus or minus sign. For example, $x - 2$ or $4x + \frac{1}{2}y - 45$

Problem: Which of the following is/are equal to 5?

a. 2 + 3 b. 6 – 1 c. $^{10}/_2$ d. 1+4 e. all of these

Discussion: The answer is e since 2 + 3, 6 – 1, $^{10}/_2$ and 1 + 4 are all equal to 5.

Notation

Since the letter x is now used as a variable in Algebra, it would not only be funny but confusing as well to still use x as a multiplication symbol. Imagine writing the product of 4 and a value x as *4xx*! Thus, Algebra simplifies multiplication of constants and variables by just writing them down beside each other or by separating them using only parentheses or the symbol " •" . For example, the product of 4 and the value x (often read as four x) may be expressed as $4x$, $4(x)$ or $4 \bullet x$. Furthermore, division is more often expressed in fraction form. The division sign ÷ is now seldom used.

Problem: Which of the following equations is true?

a. 12 + 5 = 17
b. 8 + 9 = 12 + 5
c. 6 + 11 = 3(4 + 1) + 2

Discussion: All of the equations are true. In each of the equations, both sides of the equal sign give the same number though expressed in different forms. In a) 17 is the same as the sum of 12 and 5. In b) the sum of 8 and 9 is 17 thus it is equal to the sum of 12 and 5. In c) the sum of 6 and 11 is equal to the sum of 2 and the product of 3 and the sum of 4 and 1.

On Letters and Variables

Problem: Let x be any real number. Find the value of the expression $3x$ (the product of 3 and x, remember?) if

a) $x = 5$ b) $x = {}^{1}/_{2}$ c) $x = -0.25$

Discussion: The expression $3x$ means multiply 3 by any real number x. Therefore,

a) If $x = 5$, then $3x = 3(5) = 15$.
b) If $x = {}^{1}/_{2}$, then $3x = 3({}^{1}/_{2}) = {}^{3}/_{2}$
c) If $x = -0.25$, then $3x = 3(-0.25) = -0.75$

The letters such as x, y, n, etc. do not always have specific values assigned to them. When that is the case, simply think of each of them as any number. Thus, they can be added ($x + y$), subtracted ($x - y$), multiplied (xy), and divided (${}^{x}/_{y}$) like any real number.

Problem: Recall the formula for finding the perimeter of a rectangle, P = 2L + 2W. This means you take the sum of twice the length and twice the width of the rectangle to get the perimeter. Suppose the length of a rectangle is 6.2 cm and the width is $^{1}/_{8}$ cm. What is the perimeter?

Discussion: Let L = 6.2 cm and W = $^{1}/_{8}$ cm. Then,

$$P = 2(6.2) + 2(^{1}/_{8}) = 12.4 + ¼ = 12.65 \text{ cm}$$

V. Exercises:

1. Which of the following is considered a constant?

a. *f* b. □ c. 500 d. 42*x*

2. Which of the following is a term?

a. 23*m* + 5 b. (2)(6*x*) c. *x* – *y* + 2 d. ½ *x* – *y*

3. Which of the following is equal to the product of 27 and 2?

a. 29 b. 49 + 6 c. 60 – 6 d. 11(5)

4. Which of the following makes the sentence 69 – 3 = ___ + 2 true?

a. 33 b. 64 c. 66 d. 68

5. Let *y* = 2*x* + 9. What is *y* when *x* = 5?

a. 118 b. 34 c. 28 d. 19

Let us now answer item B.5. of the initial problem using Algebra:

1. The relation of the 1st and 2nd terms of Table A is “the 2nd term is the sum of the 1st term and 4”. To express this using an algebraic expression, we use the letters *n* and *y* as the variables to represent the 1st and 2nd terms, respectively. Thus, if *n* represents the 1st term and *y* represents the 2nd term, then

$$y = n + 4.$$

FINAL PROBLEM:

A. Fill the table below:

TABLE B		
ROW	1ST TERM	2ND TERM
a.	10	23
b.	11	25
c.	12	27
d.	13	
e.	15	
f.	18	
g.	37	
h.	*n*	

B. Using Table B as your basis, answer the following questions:

1. What did you do to determine the 2^{nd} term for rows d to f?
2. What did you do to determine the 2^{nd} term for row g?
3. How did you come up with your answer in row h?
4. What is the relation between the 1^{st} and 2^{nd} terms?
5. Express the relation of the 1^{st} and 2^{nd} terms using an algebraic expression.

Summary

In this lesson, you learned about constants, letters and variables, and algebraic expressions. You learned that the equal sign means more than getting an answer to an operation; it just means that expressions on either side have equal values. You also learned how to evaluate algebraic expressions when values are assigned to letters.

Lesson 19: Verbal Phrases and Mathematical Phrases
Time: 2 hours

Prerequisite Concepts: Real Numbers and Operations on Real Numbers

About the Lesson:
This lesson is about verbal phrases and sentences and their equivalent expressions in mathematics. This lesson will show that mathematical or algebraic expressions are also meaningful.

Objectives
In this lesson, you will be able to translate verbal phrases to mathematical phrases and vice versa.

Lesson Proper
I. Activity 1

Directions: Match each verbal phrase under Column A to its mathematical phrase under Column B. Each number corresponds to a letter which will reveal a quotation if answered correctly. A letter may be used more than once.

		Column A		Column B
______	1.	The sum of a number and three	A.	$x + 3$
______	2.	Four times a certain number decreased by one	B.	$3 + 4x$
______	3.	One subtracted from four times a number	E.	$4 + x$
______	4.	A certain number decreased by two	I.	$x + 4$
______	5.	Four increased by a certain number	L.	$4x - 1$
______	6.	A certain number decreased by three	M.	$x - 2$
______	7.	Three more than a number	N.	$x - 3$
______	8.	Twice a number decreased by three	P.	$3 - x$
______	9.	A number added to four	Q.	$2 - x$
______	10.	The sum of four and a number	R.	$2x - 3$
______	11.	The difference of two and a number	U.	$4x + 3$
______	12.	The sum of four times a number and three		
______	13.	A number increased by three		
______	14.	The difference of four times a number and one		

II. Question to Ponder (Post-Activity Discussion)

Which phrase was easy to translate? ___________________________________

Translate the mathematical expression *2(x-3)* in at least two ways.
____
____

Did you get the quote, “ALL MEN ARE EQUAL”? If not, what was your mistake?

____

III. Activity 2

Directions: Choose the words or expressions inside the boxes and write it under its respective symbol.

plus	more than	times	divided by	is less than
increased by	subtracted from	multiplied by	ratio of	is greater than or equal to
is greater than	the quotient of	of	is at most	is less than or equal to
the sum of	the difference of	diminished by	less than	added to
is at least	the product of	decreased by	is not equal to	minus

+	–	x	÷	<	>	≤	≥	≠
increased by	**decreased by**	**multiplied by**	**ratio of**	**is less than**	**is greater than**	**is less than or equal to**	**is greater than or equal to**	**is not equal to**
added to	**subtracted from**	**of**	**the quotient of**			**is at most**	**is at least**	
the sum of	**the difference of**	**the product of**						
more than	**less than**							
	diminished by							

IV. Question to Ponder (Post-Activity Discussion)

1. Addition would indicate an increase, a putting together, or combining. Thus, phrases like *increased by* and *added to* are addition phrases.
2. Subtraction would indicate a lessening, diminishing action. Thus, phrases like *decreased by*, *less*, *diminished by* are subtraction phrases.
3. Multiplication would indicate a multiplying action. Phrases like *multiplied by* or *n times* are multiplication phrases.
4. Division would indicate partitioning, a quotient, and a ratio. Phrases such as *divided by*, *ratio of*, and *quotient of* are common for division.
5. The inequalities are indicated by phrases such as *less than*, *greater than*, *at least*, and *at most*.
6. Equalities are indicated by phrases like *the same as* and *equal to*.

V. THE TRANSLATION OF THE "=" SIGN

Directions: The table below shows two columns, A and B. Column A contains mathematical sentences while Column B contains their verbal translations. Observe the items under each column and compare. Answer the proceeding questions.

Column A Mathematical Sentence	**Column B Verbal Sentence**
x + 5 = 4	The sum of a number and 5 is 4.
2x – 1 = 1	Twice a number decreased by 1 is equal to 1.
7 + x = 2x + 3	Seven added by a number x is equal to twice the same number increased by 3.
3x = 15	Thrice a number x yields 15.
x – 2 = 3	Two less than a number x results to 3.

VI. Question to Ponder (Post-Activity Discussion)

1) Based on the table, what do you observe are the common verbal translations of the "=" sign? "is", "is equal to"
2) Can you think of other verbal translations for the "=" sign? "results in", "becomes"
3) Use the phrase "is equal to" on your own sentence.
4) Write your own pair mathematical sentence and its verbal translation on the last row of the table.

VII. Exercises:

A. Directions: Write your responses on the space provided.

1. Write the verbal translation of the formula for converting temperature from Celsius (C) to Fahrenheit (F) which is $F=\frac{9}{5}C+32$.

2. Write the verbal translation of the formula for converting temperature from Fahrenheit (F) to Celsius (C) which is $C=\frac{5}{9}\ F-32$.

3. Write the verbal translation of the formula for simple interest: *I* = *PRT*, where *I* is simple interest, *P* is Principal Amount, *R* is Rate and *T* is time in years.

4. The perimeter (P) of a rectangle is twice the sum of the length (L) and width (W). Express the formula of the perimeter of a rectangle in algebraic expressions using the indicated variables.

5. The area (A) of a rectangle is the product of length (L) and width (W).

___

6. The perimeter (P) of a square is four times its side (S).

___

7. Write the verbal translation of the formula for Area of a Square (A): $A = s^2$, where s is the length of a side of a square.

___

8. The circumference (C) of a circle is twice the product of π and radius (r).

___

9. Write the verbal translation of the formula for Area of a Circle (A): $A = \pi r^2$, where r is the radius.

___

10. The midline (k) of a trapezoid is half the sum of the bases (a and b) or the sum of the bases (a and b) divided by 2.

___

11. The area (A) of a trapezoid is half the product of the sum of the bases (a and b) and height (h).

___

12. The area (A) of a triangle is half the product of the base (b) and height (h).

___

13. The sum of the angles of a triangle (A, B and C) is 180^0.

___

14. Write the verbal translation of the formula for Area of a Rhombus (A): $A = \frac{1}{2} d_1 d_2$, where d_1 *and* d_2 are the lengths of diagonals.

___

15. Write the verbal translation of the formula for the Volume of a rectangular parallelepiped (V): $A = lwh$, where l is the length, w is the width and h is the height.

___

16. Write the verbal translation of the formula for the Volume of a sphere (V): $V = \frac{4}{3}\pi r^3$, where r is the radius.

17. Write the verbal translation of the formula for the Volume of a cylinder (V): $V = \pi r^2 h$, where r is the radius and h is the height.

18. The volume of the cube (V) is the cube of the length of its edge (a). Or the volume of the cube (V) is the length of its edge (a) raised to 3. Write its formula.

B. Directions: Write as many verbal translations as you can for this mathematical sentence.

3x – 2 = – 4

C. REBUS PUZZLE
Try to answer this puzzle!
What number must replace the letter x?

x + ("  " – "b") = "  – "kit"

SUMMARY

In this lesson, you learned that verbal phrases can be written in both words and in mathematical expressions. You learned common phrases associated with addition, subtraction, multiplication, division, the inequalities and the equality. With this lesson, you must realize by now that mathematical expressions are also meaningful.

GRADE 7 MATH LEARNING GUIDE

Lesson 20: Polynomials **Time: 1.5 hours**

Pre-requisite Concepts: Constants, Variables, Algebraic expressions

About the Lesson: This lesson introduces to students the terms associated with polynomials. It discusses what polynomials are.

Objectives:

In this lesson, the students must be able to:

1) Give examples of polynomials, monomials, binomials, and trinomials;
2) Identify the base, coefficient, terms and exponent sin a given polynomial.

Lesson Proper:

I. A. Activity 1: Word Hunt

Find the following words inside the box.

BASE
COEFFICIENT
DEGREE
EXPONENT
TERM
CONSTANT
BINOMIAL
MONOMIAL
POLYNOMIAL
TRINOMIAL
CUBIC
LINEAR
QUADRATIC
QUINTIC
QUARTIC

P	C	I	T	N	I	U	Q	Y	N	E	T
P	M	E	X	P	O	N	E	N	T	S	C
C	O	E	F	F	I	C	I	E	N	T	O
Q	N	L	I	N	E	A	R	B	D	R	N
U	O	C	Y	A	P	M	R	A	E	I	S
A	M	R	I	N	L	M	T	S	G	N	T
D	I	U	N	B	O	Q	U	N	R	O	A
R	A	E	Q	P	U	M	V	T	E	M	N
A	L	S	O	B	D	C	I	R	E	I	T
T	A	A	C	U	B	I	N	A	S	A	A
I	U	B	I	N	O	M	I	A	L	L	C
C	I	T	R	A	U	Q	R	T	I	C	B

Definition of Terms

In the algebraic expression $3x^2 - x + 5$, $3x^2$, $-x$ and 5 are called the terms.

Term is a constant, a variable or a product of constant and variable.

In the term $3x^2$, 3 is called the numerical coefficient and x^2 is called the **literal coefficient**.

In the term $-x$ has a **numerical coefficient** which is -1 and a literal coefficient which is x.

The term 5 is called the ***constant,*** which is usually referred to as the term without a variable.

Numerical coefficient is the constant/number.

Literal coefficient is the variable including its exponent.

The word ***Coefficient*** alone is referred to as the numerical coefficient.

In the literal coefficient x^2, x is called the ***base*** and 2 is called the ***exponent***.

Degree is the highest exponent or the highest sum of exponents of the variables in a term.

In $3x^2 - x + 5$, the degree is 2.

In $3x^2y^3 - x^4y^3$ the degree is 7.

Similar Terms are terms having the same literal coefficients.

$3x^2$ and $-5x^2$ are similar because their literal coefficients are the same.

5x and $5x^2$ are NOT similar because their literal coefficients are NOT the same.

$2x^3y^2$ and $-4x^2y^3$ are NOT similar because their literal coefficients are NOT the same.

A ***polynomial*** is a kind of algebraic expression where each term is a constant, a variable or a product of a constant and variable in which the variable has a whole number (non-negative number) exponent. A polynomial can be a monomial, binomial, trinomial or a multinomial.

An algebraic expression is **NOT** a polynomial if

1) the exponent of the variable is NOT a whole number {0, 1, 2, 3..}.
2) the variable is inside the radical sign.
3) the variable is in the denominator.

Kinds of Polynomial according to the number of terms

1) Monomial – is a polynomial with only one term
2) Binomial – is polynomial with two terms
3) Trinomial – is a polynomial with three terms
4) Polynomial – is a polynomial with four or more terms

B. Activity 2

Tell whether the given expression is a polynomial or not. If it is a polynomial, determine its degree and tell its kind according to the number of terms. If it is NOT, explain why.

1)	$3x^2$	6)	$x^{½} - 3x + 4$
2)	$x^2 - 5xy$	7)	$\sqrt{2}\,x^4 - x^7 + 3$
3)	10	8)	$3x^2\sqrt{2x} - 1$
4)	$3x^2 - 5xy + x^3 + 5$	9)	$\frac{1}{3}x - \frac{3x^3}{4} + 6$

5) $x^3 - 5x^{-2} + 3$ 10) $\frac{3}{x^2} - x^2 - 1$

Kinds of Polynomial according to its degree

1) Constant – a polynomial of degree zero
2) Linear – a polynomial of degree one
3) Quadratic – a polynomial of degree two
4) Cubic – a polynomial of degree three
5) Quartic – a polynomial of degree four
6) Quintic – a polynomial of degree five

* The next degrees have no universal name yet so they are just called "polynomial of degree ____."

A polynomial is in ***Standard Form*** if its terms are arranged from the term with the highest degree, up to the term with the lowest degree.

If the polynomial is in standard form the first term is called the ***Leading Term***, the numerical coefficient of the leading term is called the ***Leading Coefficient*** and the exponent or the sum of the exponents of the variable in the leading term the ***Degree*** of the polynomial.

The standard form of $2x^2 - 5x^5 - 2x^3 + 3x - 10$ is ***$-5x^5 - 2x^3 + 2x^2 + 3x - 10$***.
The terms $-5x^5$ is the leading term, -5 is its leading coefficient and 5 is its degree.
It is a quintic polynomial because its degree is 5.

C. Activity 3

Complete the table.

Given	Leading Term	Leading Coefficient	Degree	Kind of Polynomial according to the no. of terms	Kind of Polynomial According to the degree	Standard Form
1) $2x + 7$						
2) $3 - 4x + 7x^2$						
3) 10						
4) $x^4 - 5x^3 + 2x - x^2 - 1$						
5) $5x^5 + 3x^3 - x$						
6) $3 - 8x$						
7) $x^2 - 9$						
8) $13 - 2x + x^5$						
9) $100x^3$						

10) $2x^3 - 4x^2 + x^4 - 6$						

Summary

In this lesson, you learned about the terminologies in polynomials: term, coefficient, degree, similar terms, polynomial, standard form, leading term, leading coefficient.

Lesson 21: Laws of Exponents **Time: 1.5 hours**

Pre-requisite Concepts: Multiplication of real numbers

About the Lesson: This lesson is all about the laws of exponents.

Objectives:
In this lesson, the students must be able to:
1) define and interpret the meaning of a^n where n is a positive integer;

2) derive inductively the Laws of Exponents (restricted to positive integers)
3) illustrate the Laws of Exponents.

Lesson Proper
I. Activity 1
Give the product of each of the following as fast as you can.
1) 3 x 3 = ________
2) 4 x 4 x 4 = ________
3) 5 x 5 x 5 = ________
4) 2 x 2 x 2 = ________
5) 2 x 2 x 2 x 2 = ________
6) 2 x 2 x 2 x 2 x 2 = ________

II. Development of the Lesson
Discovering the Laws of Exponents

A) a^n = a x a x a x a ….. (n times)
In ***a^n*, *a*** is called the base and ***n*** is called the exponent

Exercises
1) Which of the following is/are correct?
a) $4^2 = 4 \times 4 = 16$
b) $2^4 = 2 \times 2 \times 2 \times 2 = 8$
c) $2^5 = 2 \times 5 = 10$
d) $3^3 = 3 \times 3 \times 3 = 27$
2) Give the value of each of the following as fast as you can.

a) 2^3 b) 2^5 c) 3^4
d) 10^6

Activity 2
Evaluate the following. Investigate the result. Make a simple conjecture on it. The first two are done for you.
1) $(2^3)^2 = 2^3 \bullet 2^3 = 2 \bullet 2 \bullet 2 \bullet 2 \bullet 2 \bullet 2 = 64$
2) $(x^4)^3 = x^4 \bullet x^4 \bullet x^4 = x \bullet x \bullet x \bullet x \bullet x \bullet x \bullet x \bullet x \bullet x \bullet x \bullet x \bullet x = x^{12}$
3) $(3^2)^2 =$
4) $(2^2)^3 =$

5) $(a^2)^5$=

Did you notice something?

What can you conclude about $(\boldsymbol{a}^{n})^{m}$? What will you do with ***a***, ***n*** and ***m***?

What about these?

1) $(x^{100})^3$

2) $(y^{12})^5$

Activity 3

Evaluate the following. Notice that the bases are the same. The first example is done for you.

1) $(2^3)(2^2) = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^5 = 32$
2) $(x^5)(x^4)$ =
3) $(3^2)(3^4)$ =
4) $(2^4)(2^5)$ =
5) $(x^3)(x^4)$ =

Did you notice something?

What can you conclude about $\boldsymbol{a}^{n} \cdot \boldsymbol{a}^{m}$? What will you do with ***a***, ***n*** and ***m***?

What about these?

1) $(x^{32})(x^{25})$
2) $(y^{59})(y^{51})$

Activity 4

Evaluate each of the following. Notice that the bases are the same. The first example is done for you.

1) $\frac{2^7}{2^3} = \frac{128}{8} = 16$ ---→ remember that 16 is the same as 2^4

2) $\frac{3^5}{3^3}$ =

3) $\frac{4^3}{4^2}$ =

4) $\frac{2^8}{2^6}$ =

Did you notice something?

What can you conclude about $\frac{a^n}{a^m}$? What will you do with ***a, n*** and ***m***?

What about these?

1) $\frac{x^{20}}{x^{13}}$

2) $\frac{y^{105}}{y^{87}}$

Summary:
Laws of exponent

1) $a^n = a \bullet a \bullet a \bullet a \bullet a$….. (n times)

2) $(a^n)^m = a^{nm}$ *power of powers*

3) $a^n \bullet a^m = a^{m+n}$ *product of a power*

4) $\frac{a^n}{a^m} = a^{n-m}$ *quotient of a power*

5) $a^0 = 1$ where $a \neq 0$ ***law for zero exponent***

What about these?

a) $(7{,}654{,}321)^0$

b) $3^0 + x^0 + (3y)^0$

Exercise**:**

Choose a Law of Exponent to apply. Complete the table and observe. Make a conjecture.

No.	Result	Applying a law of Exponent	***GIVEN (Start here)***	ANSWER	REASON
1)	$\leftarrow$		$\leftarrow \frac{5}{5}$	$\rightarrow$	
2)	$\leftarrow$		$\leftarrow \frac{100}{100}$	$\rightarrow$	
3)	$\leftarrow$		$\leftarrow \frac{x}{x}$	$\rightarrow$	
4)	$\leftarrow$		$\leftarrow \frac{a^5}{a^5}$	$\rightarrow$	

6) a^{-n} and $\frac{1}{a^{-n}}$ ***law for negative exponent***

Can you rewrite the fractions below using exponents and simplify them?

a) $\frac{2}{4}$

b) $\frac{4}{32}$

c) $\frac{27}{81}$

What did you notice?

What about these?

d) x^{-2}

e) 3^{-3}

f) $(5-3)^{-2}$

III. Exercises

A. Evaluate each of the following.

1) 2^8
2) 8^2
3) 5^{-1}
4) 3^{-2}
5) 18^0
6) $(2^3)^3$
7) $(2^4)(2^3)$
8) $(3^2)(2^3)$
9) $x^0 + 3^{-1} – 2^2$
10 $[2^2 – 3^3 + 4^4]^0$

B. Simplify each of the following.

1) $(x^{10})(x^{12})$

2) $(y^{-3})(y^8)$

3) $(m^{15})^3$

4) $(d^{-3})^2$

5) $(a^{-4})^{-4}$

6) $\frac{z^{23}}{z^{15}}$

7) $\frac{b^8}{b^{12}}$

8) $\frac{c^3}{c^{-2}}$

9) $\frac{x^7 y^{10}}{x^3 y^5}$

10) $\frac{a^8 b^2 c^0}{a^5 b^5}$

11) $\frac{a^8 a^3 b^{-2}}{a^{-1} b^{-5}}$

Summary

In these lessons, you learned some laws of exponents.

Lesson 22: Addition and Subtraction of Polynomials **Time: 2 hours**

Pre-requisite Concepts: Similar Terms, Addition and Subtraction of Integers

About the Lesson: This lesson will teach students how to add and subtract polynomials using tiles at first and then by paper and pencil after.

Objectives:

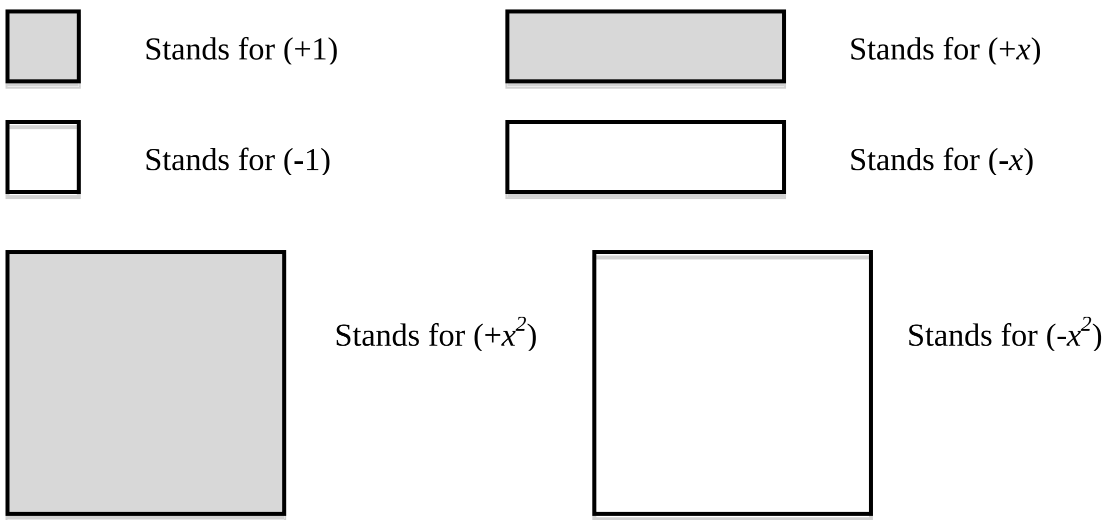
In this lesson, the students are expected to:

1) add and subtract polynomials;
2) solve problems involving polynomials.

Lesson Proper:

I. Activity 1

Familiarize yourself with the Tiles below:



Can you represent the following quantities using the above tiles?

1. $x - 2$
2. $4x + 1$

Activity 2.

Use the tiles to find the sum of the following polynomials;

1. $5x + 3x$
2. $(3x - 4) - 6x$
3. $(2x^2 - 5x + 2) + (3x^2 + 2x)$

Can you come up with the rules for adding polynomials?

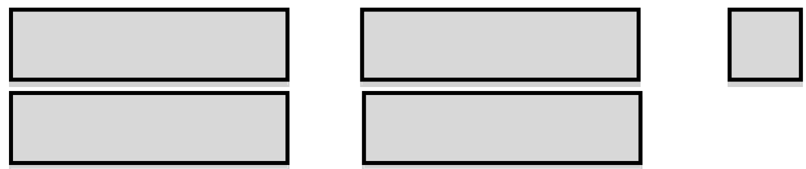
II. Questions/Points to Ponder (Post-Activity Discussion)

The tiles can make operations on polynomials easy to understand and do.

Let us discuss the first activity.

1. To represent $x - 2$, we get one ($+x$) tile and two (-1) tiles.

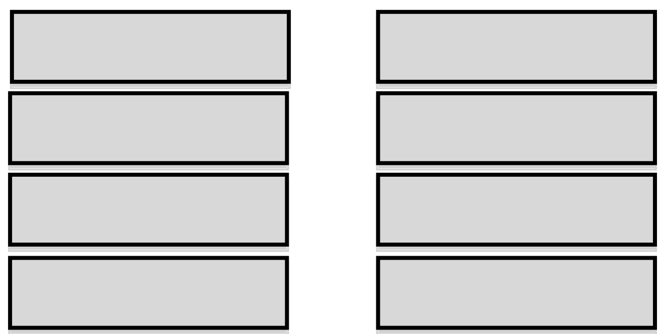
2. To represent $4x + 1$, we get four ($+x$) tiles and one ($+1$) tile.



What about the second activity? Did you pick out the correct tiles?

1. $5x + 3x$

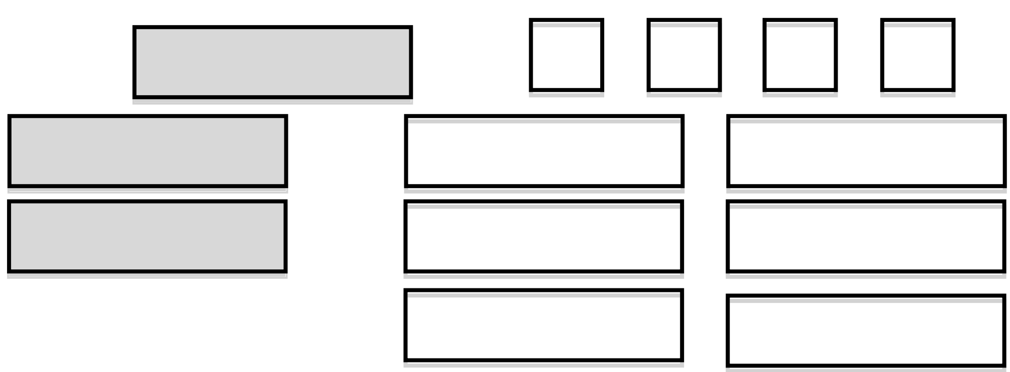
Get five (+x tiles) and three more (+x) tiles. How many do you have in all?



There are eight (+x) altogether. Therefore, $5x + 3x = 8x$.

2. $(3x - 4) - 6x$

Get three ($+x$) tiles and four (-1) tiles to represent ($3x - 4$). Add six ($-x$) tiles.
[Recall that subtraction also means adding the negative of the quantity.]



Now, recall further that a pair of one (+*x*) and one (-*x*) is zero. What tiles do you have left?

That's right, if you have with you three (-*x*) and four (-1), then you are correct. That means the sum is ($-3x - 4$).

3. $(2x^2 – 5x + 2) + (3x^2 + 2x)$

What tiles would you put together? You should have two ($+x^2$), five (-x) and two (+1) tiles then add three ($+x^2$) and two (+*x*) tiles. Matching the pairs that make zero, you have in the end five ($+x^2$), three (-x), and two (+1) tiles. The sum is $5x^2 – 3x + 2$.

Or, using your pen and paper,

$$(2x^2 – 5x + 2) + (3x^2 + 2x) = (2x^2+3x^2) + (– 5x + 2x) + 2 = 5x^2 – 3x + 2$$

Rules for Adding Polynomials

To add polynomials, simply combine similar terms. To combine similar terms, get the sum of the numerical coefficients and annex the same literal coefficients. If there is more than one term, for convenience, write similar terms in the same column.

Do you think you can add polynomials now without the tiles?

Perform the operation.

1) Add 4a – 3b + 2c, 5a + 8b – 10c and -12a + c.

$$\begin{array}{rrr} 4a & -\ 3b & +\ 2c \\ 5a & +\ 8b & -\ 10c \\ \underline{+\ -12a} & \underline{} & \underline{+\ c} \end{array}$$

2) Add $13x^4 – 20x^3 + 5x – 10$ and $-10x^2 – 8x^4 – 15x + 10$.

$$\begin{array}{rrrrr} 13x^4 & -\ 20x^3 & & +\ 5x & -\ 10 \\ \underline{+\ -8x^4} & \underline{} & \underline{-\ 10x^2} & \underline{-\ 15x} & \underline{+\ 10} \end{array}$$

Rules for Subtracting Polynomials

To subtract polynomials, change the sign of the subtrahend then proceed to the addition rule. Also, remember what subtraction means. It is adding the negative of the quantity.

Perform the operation.

1) $5x – 13x = 5x + (-5x) + (-8x) = -8x$

2) $$\begin{array}{rrr} 2x^2 & -\ 15x & +\ 25 \\ \underline{-\ \ 3x^2} & \underline{+\ 12x} & \underline{-\ 18} \end{array} \qquad \begin{array}{rrr} 2x^2 & -\ 15x & +\ 25 \\ \underline{+\ -3x^2} & \underline{-\ 12x} & \underline{+\ 18} \end{array}$$

3) $(30x^3 – 50x^2 + 20x – 80) – (17x^3 + 26x + 19)$

$$\begin{array}{rrrr} 30x^3 & -\ 50x^2 & +\ 20x & -\ 80 \\ \underline{+\ -17x^3} & \underline{} & \underline{-\ 26x} & \underline{-\ 19} \end{array}$$

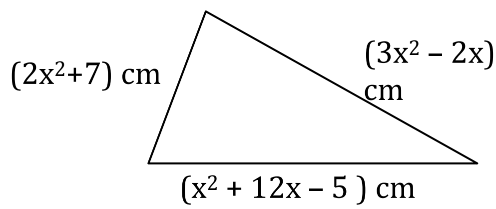
III. Exercises

A. Perform the indicated operation, first using the tiles when applicable, then using paper and pen.

1) $3x + 10x$
2) $12y – 18y$
3) $14x^3 + (-16x^3)$
4) $-5x^3 -4x^3$
5) $2x – 3y$
6) $10xy – 8xy$
7) $20x^2y^2 + 30x^2y^2$
8) $-9x^2y + 9x^2y$
9) $10x^2y^3 – 10x^3y^2$
10) $5x – 3x – 8x + 6x$

B. Answer the following questions. Show your solution.

1) What is the sum of $3x^2 – 11x + 12$ and $18x^2 + 20x – 100$?
2) What is $12x^3 – 5x^2 + 3x + 4$ less than $15x^3 + 10x + 4x^2 – 10$?
3) What is the perimeter of the triangle shown at the right?



4) If you have $(100x^3 – 5x + 3)$ pesos in your wallet and you spent $(80x^3 – 2x^2 + 9)$ pesos in buying foods, how much money is left in your pocket?
5) What must be added to $3x + 10$ to get a result of $5x – 3$?

Summary

In this lesson, you learned about tiles and how to use them to represent algebraic expressions. You learned how to add and subtract terms and polynomials using these tiles. You were also able to come up with the rules in adding and subtracting polynomials. To add polynomials, simply combine similar terms. To combine similar terms, get the sum of the numerical coefficients and annex the same literal coefficients. If there is more than one term, for convenience, write similar terms in the same column. To subtract polynomials, change the sign of the subtrahend then proceed to the addition rule.

Lesson 23: Multiplying Polynomials **Time: 3 hours**

Pre-requisite Concepts: Laws of exponents, Adding and Subtracting Polynomials, Distributive Property of Real Numbers

About the Lesson: In this lesson, we use the context of area to show how to multiply polynomials. Tiles will be used to illustrate the action of multiplying terms of a polynomial. Other ways of multiplying polynomials will also be taught.

Objectives:

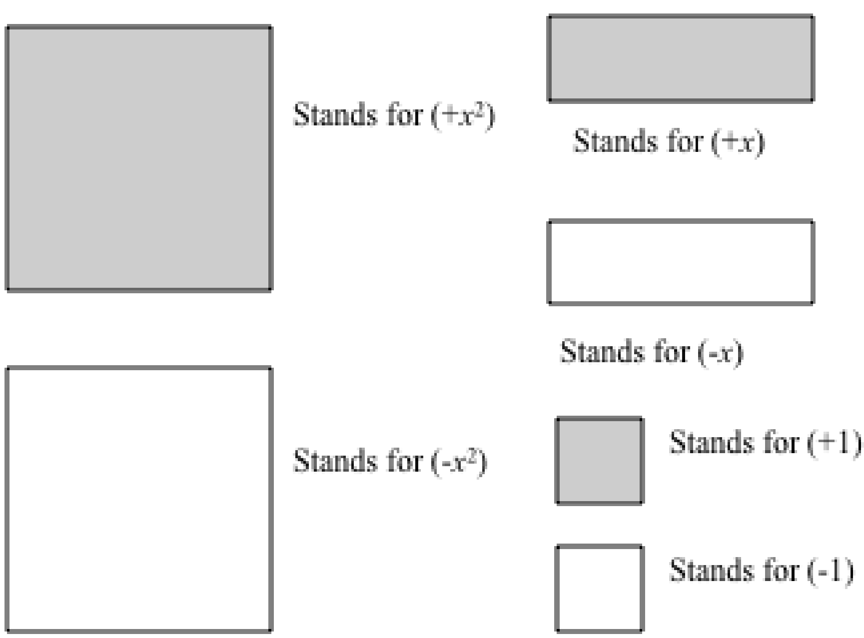
In this lesson, you should be able to:

1) multiply polynomials such as;
   a) monomial by monomial,
   b) monomial by polynomial with more than one term,
   c) binomial by binomial,
   d) polynomial with more than one term to polynomial with three or more terms.
2) solve problems involving multiplying polynomials.

Lesson Proper

I. Activity

Familiarize yourself with the following tiles:



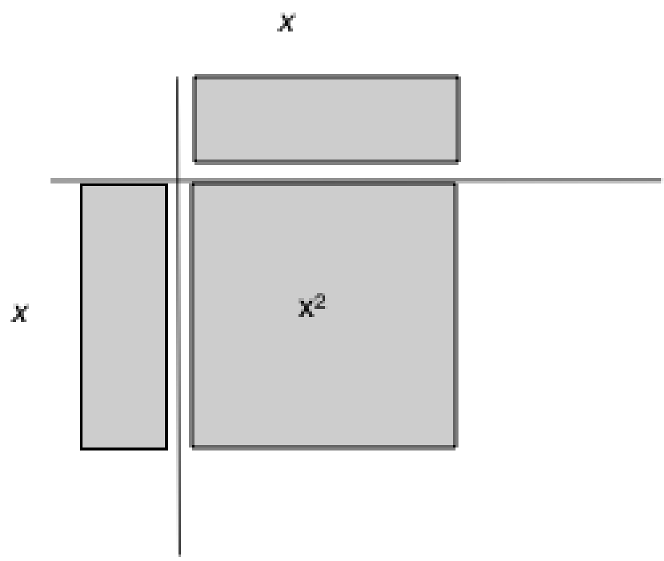
Now, find the following products and use the tiles whenever applicable:

1) $(3x)\ (x)$ 2) $(-x)(1+x)$ 3) $(3 - x)(x + 2)$

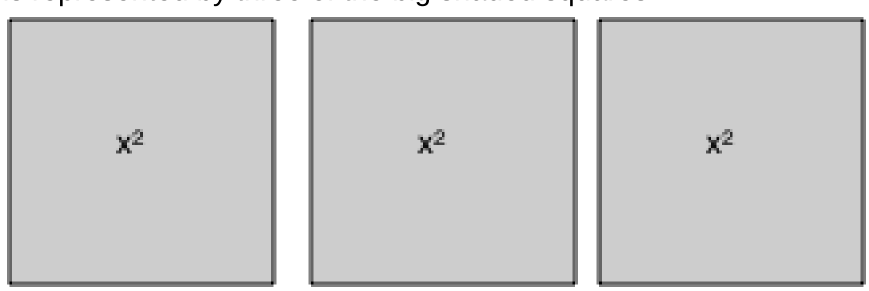
Can you tell what the algorithms are in multiplying polynomials?

II. Questions/Points to Ponder (Post-Activity Discussion)

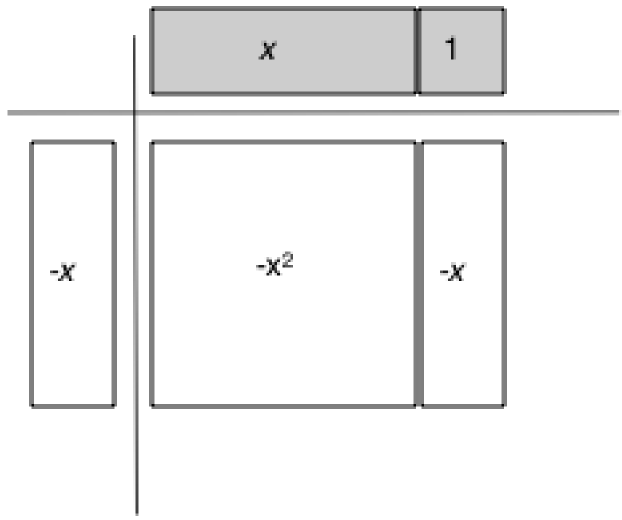
Recall the Laws of Exponents. The answer to item (1) should not be a surprise. By the Laws of Exponents, (3x) (x) = 3x^2. Can you use the tiles to show this product?



So, 3x^2 is represented by three of the big shaded squares.

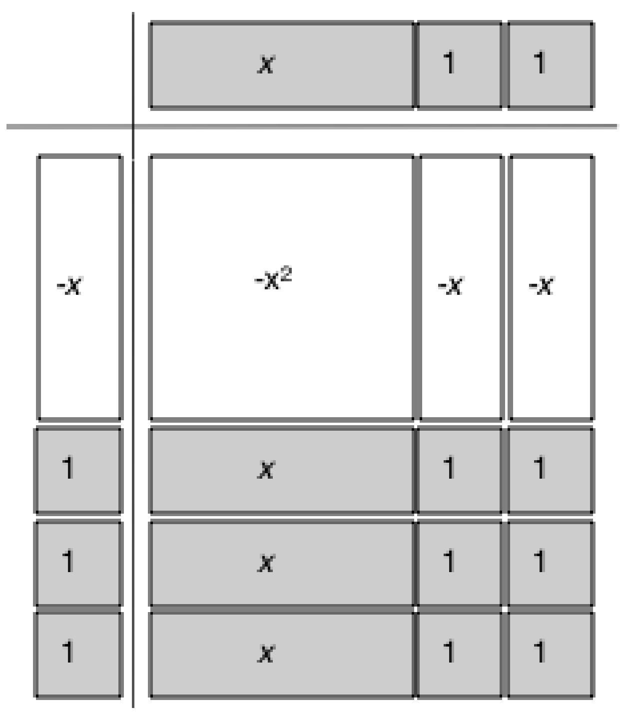


What about item (2)? The product (-x)(1+ x) can be represented by the following.



The picture shows that the product is $\left(-x^2\right)+\left(-x\right)$. Can you explain what happened? Recall the sign rules for multiplying.

The third item is (3 - *x*)(*x* + 2). How can you use the Tiles to show the product?



$(-x^2) + (-2x) + 3x + 6 = (-x^2) + x + 6$

Rules in Multiplying Polynomials

A. To multiply a monomial by another monomial, simply multiply the numerical coefficients then multiply the literal coefficients by applying the basic laws of exponent.

Examples:

1) $(x^3)(x^5) = x^8$
2) $(3x^2)(-5x^{10}) = -15x^{12}$
3) $(-8x^2y^3)(-9xy^8) = 72x^3y^{11}$

B. To multiply monomial by a polynomial, simply apply the distributive property and follow the rule in multiplying monomial by a monomial.

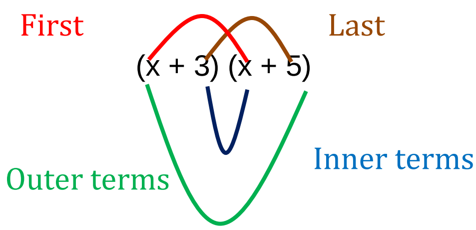
Examples:

1) $3x(x^2 - 5x + 7) = 3x^3 - 15x^2 + 21x$
2) $-5x^2y^3(2x^2y - 3x + 4y^5) = -10x^4y^4 + 15x^3y^3 - 20x^2y^8$

C. To multiply binomial by another binomial, simply distribute the first term of the first binomial to each term of the other binomial then distribute the second term to each term of the other binomial and simplify the results by combining similar terms. This procedure is also known as the F-O-I-L method or Smile method. Another way is the vertical way of multiplying which is the conventional one.

Examples

1) $(x + 3)(x + 5) = \mathbf{x^2 + 8x + 15}$



F –> (x)(x) = x^2

O –> (x)(5) = 5x

I –> (3)(x) = 3x

L –> (3)(5)= 15

Since 5x and 3x are similar terms we can combine them. 5x + 3x = 8x. The final answer is

$x^2 + 8x + 15$

2) $(x - 5)(x + 5) = x^2 + 5x - 5x - 25 =$ ***$x^2 - 25$***
3) $(x + 6)^2 = (x + 6)(x + 6) = x^2 + 6x + 6x + 36 =$ ***$x^2 + 12x + 36$***
4) $(2x + 3y)(3x - 2y) = 6x^2 - 4xy + 9xy - 6y^2 =$ ***$6x^2 + 5xy - 6y^2$***
5) $(3a - 5b)(4a + 7) =$ ***$12a^2 + 21a - 20ab - 35b$***

There are no similar terms so it is already in simplest form.

Guide questions to check whether the students understand the process or not

If you multiply (2x + 3) and (x – 7) by F-O-I-L method,

a) the product of the first terms is __________.
b) the product of the outer terms is __________.
c) the product of the inner terms is __________.
d) the product of the last terms is __________.
e) Do you see any similar terms? What are they?
f) What is the result when you combine those similar terms?
g) The final answer is ____________________.

Another Way of Multiplying Polynomials

1) Consider this example.

$$\begin{array}{r} 7\,8 \\ \underline{\times \quad 5\,9} \\ 7\,0\,2 \\ \underline{3\,9\,0} \\ \boldsymbol{4\,6\,0\,2} \end{array}$$

This procedure also applies the distributive property.

2) Now, consider this.

$$\begin{array}{r} 2x + 3 \\ \underline{x - 7} \\ 14x + 21 \\ \underline{2x^2 + 3x \quad\quad} \\ \boldsymbol{2x^2 + 17x + 21} \end{array}$$

This one looks the same as the first one.

Consider the example below.

$$\begin{array}{r} 3a - 5b \\ \underline{4a + 7} \\ 21a - 35b \\ \underline{12a^2 - 20ab \quad\quad\quad\quad} \\ \boldsymbol{12a^2 - 20ab + 21a - 35b} \end{array}$$

In this case, although 21a and -20ab are aligned, you cannot combine them because they are not similar.

D. To multiply a polynomial with more than one term by a polynomial with three or more terms, simply distribute the first term of the first polynomial to each term of the other polynomial. Repeat the procedure up to the last term and simplify the results by combining similar terms.

Examples:

1) $(x + 3)(x^2 - 2x + 3) = x(x^2 - 2x + 3) - 3(x^2 - 2x + 3)$
$= x^3 - 2x^2 + 3x - 3x^2 + 6x - 9$
$= \mathbf{x^3 - 5x^2 + 9x - 9}$

2) $(x^2 + 3x - 4)(4x^3 + 5x - 1) = x^2(4x^3 + 5x - 1) + 3x(4x^3 + 5x - 1) - 4(4x^3 + 5x - 1)$
$= 4x^5 + 5x^3 - x^2 + 12x^4 + 15x^2 - 3x - 16x^3 - 20x + 4$
$= \mathbf{4x^5 + 12x^4 - 11x^3 + 14x^2 - 23x + 4}$

3) $(2x - 3)(3x + 2)(x^2 - 2x - 1) = (6x^2 - 5x - 6)(x^2 - 2x - 1)$
$= 6x^4 - 17x^3 - 22x^2 + 17x + 6$

**Do the distribution one by one.*

III. Exercises

A. Simplify each of the following by combining like terms.

1) $6x + 7x$
2) $3x - 8x$
3) $3x - 4x - 6x + 2x$
4) $x^2 + 3x - 8x + 3x^2$
5) $x^2 - 5x + 3x - 15$

B. Call a student or ask for volunteers to recite the basic laws of exponent but focus more on the “product of a power” or ”multiplying with the same base”. Give follow up exercises through flashcards.

1) $x^{12} \div x^5$
2) $a^{-10} \bullet a^{12}$
3) $x^2 \bullet x^3$
4) $2^2 \bullet 2^3$
5) $x^{100} \bullet x$

C. Answer the following.

1) Give the product of each of the following.
   a) $(12x^2y^3z)(-13ax^3z^4)$
   b) $2x^2(3x^2 – 5x – 6)$

   c) $(x – 2)(x^2 – x + 5)$

2) What is the area of the square whose side measures (2x – 5) cm? (*Hint: Area of the square = s^2*)

3) Find the volume of the rectangular prism whose length, width and height are (x + 3) meter, (x – 3) meter and (2x + 5) meter. *(Hint: Volume of rectangular prism = l x w x h)*

4) If I bought (3x + 5) pencils which cost (5x – 1) pesos each, how much will I pay for it?

Summary

In this lesson, you learned about multiplying polynomials using different approaches: using the Tiles, using the FOIL, and using the vertical way of multiplying numbers.

Lesson 24: Dividing Polynomials **Time: 3 hours**

Pre-requisite Concepts: Addition, Subtraction, and Multiplication of Polynomials

About the Lesson: In this lesson, students will continue to work with Tiles to help reinforce the association of terms of a polynomial with some concrete objects, hence helping them remember the rules for dividing polynomials.

Objectives:

In this lesson, the students must be able to:

1) divide polynomials such as:
   a) polynomial by a monomial and
   b) polynomial by a polynomial with more than one term.
2) solve problems involving division of polynomials.

Lesson Proper

I. Activity 1:

Decoding

" I am the father of Archimedes." Do you know my name? Find it out by decoding the hidden message below.

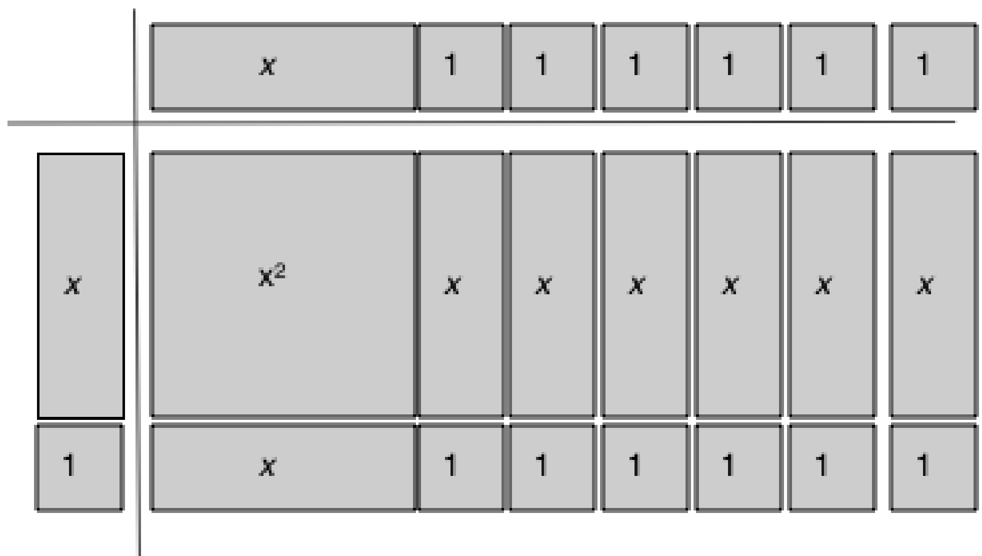
Match Column A with its answer in Column B to know the name of Archimedes' father. Put the letter of the correct answer in the space provided below.

Column A (Perform the indicated operation)			Column B
1)	$(3x^2 - 6x - 12) + (x^2 + x + 3)$	S	$4x^2 + 12x + 9$
2)	$(2x - 3)(2x + 3)$	H	$4x^2 - 9$
3)	$(3x^2 + 2x - 5) - (2x^2 - x + 5)$	I	$x^2 + 3x - 10$
4)	$(3x^2 + 4) + (2x - 9)$	P	$4x^2 - 5x - 9$
5)	$(x + 5)(x - 2)$	A	$2x^2 - 3x + 6$
6)	$3x^2 - 5x + 2x - x^2 + 6$	E	$4x^2 - 6x - 9$
7)	$(2x + 3)(2x + 3)$	D	$3x^2 + 2x - 5$
		V	$5x^3 - 5$

____ ____ ____ ____ ____ ____ ____
1 2 3 4 5 6 7

Activity 2.

Recall the Tiles. We can use these tiles to divide polynomials of a certain type. Recall also that division is the reverse operation of multiplication. Let's see if you can work out this problem using Tiles: $\left(x^2+7x+6\right)\div\left(x+1\right)$



The answer is $x + 6$.

II. Questions/Points to Ponder (Post-Activity Discussion)

The answer to Activity 1 is PHIDIAS. Di you get it? If not, what went wrong?

In Activity 2, note that the dividend is under the horizontal bar similar to the long division process on whole numbers.

Rules in Dividing Polynomials

To divide polynomial by a monomial, simply divide each term of the polynomial by the given divisor.

Examples:

1) Divide $12x^4 - 16x^3 + 8x^2$ by $4x^2$

a) $\dfrac{12x^4-16x^3+8x^2}{4x^2}$

$=\dfrac{12x^4}{4x^2}-\dfrac{16x^3}{4x^2}+\dfrac{8x^2}{4x^2}$

$= 3x^2 - 4x + 2$

b.
$$\begin{array}{r|l} & 3x^2 - 4x + 2 \\ \hline 4x^2 & 12x^4 - 16x^3 + 8x^2 \\ & \underline{12x^4} \\ & \qquad -16x^3 \\ & \qquad \underline{-16x^3} \\ & \qquad\qquad 8x^2 \\ & \qquad\qquad \underline{8x^2} \\ & \qquad\qquad 0 \end{array}$$

2) Divide $15x^4y^3 + 25x^3y^3 - 20x^2y^4$ by $-5x^2y^3$

$$= \frac{15x^4y^3}{-5x^2y^3} + \frac{25x^3y^3}{-5x^2y^3} - \frac{20x^2y^4}{-5x^2y^3}$$

$$= -3x^2 - 5x + 4y$$

To divide polynomial by a polynomial with more than one term (by long division), simply follow the procedure in dividing numbers by long division.

These are some suggested steps to follow:

1) Check the dividend and the divisor if it is in standard form.

2) Set-up the long division by writing the division symbol where the divisor is outside the division symbol and the dividend inside it.

3) You may now start the Division, Multiplication, Subtraction and Bring Down cycle.

4) You can stop the cycle when:

a) the quotient (answer) has reached the constant term.
b) the exponent of the divisor is greater than the exponent of the dividend

Examples:

1) Divide 2485 by 12.

$$\begin{array}{r} 207 \\ 12\overline{)2485} \\ \underline{24} \\ 8 \\ \underline{0} \\ 85 \\ \underline{84} \\ 1 \end{array}$$

r. 1 or $207\frac{1}{12}$

2) Divide $x^2 - 3x - 10$ by $x + 2$

$$\begin{array}{r} x - 5 \\ x+2\overline{)x^2 - 3x - 10} \\ \underline{x^2 + 2x} \\ -5x - 10 \\ \underline{-5x - 10} \\ 0 \end{array}$$

1) divide x^2 by x and put the result on top
2) multiply that result to x + 2
3) subtract the product to the dividend
4) bring down the remaining term/s
5) repeat the procedure from 1.

3) Divide $x^3 + 6x^2 + 11x + 6$ by $x - 3$

$$
\begin{array}{r}
x^2 - 3x + 2 \\
x-3\overline{\smash{)}\,x^3 - 6x^2 + 11x - 6} \\
\underline{x^3 - 3x^2} \qquad\qquad \\
-3x + 11x \quad \\
\underline{-3x + 9x} \quad \\
2x - 6 \\
\underline{2x - 6} \\
0
\end{array}
$$

4) Divide $2x^3 - 3x^2 - 10x - 4$ by $2x - 1$

$$
\begin{array}{l}
\qquad\qquad x^2 - 2x - 4 - \dfrac{2}{2x+1} \\
2x+1\overline{\smash{)}\,2x^3 - 3x^2 - 10x - 6} \\
\qquad\quad \underline{2x^3 + x^2} \\
\qquad\qquad -4x^2 - 10x \\
\qquad\qquad \underline{-4x^2 - 2x} \\
\qquad\qquad\quad -8x - 6 \\
\qquad\qquad\quad \underline{-8x - 4} \\
\qquad\qquad\qquad\quad -2
\end{array}
$$

5) Divide $x^4 - 3x^2 + 2$ by $x^2 - 2x + 3$

$$
\begin{array}{l}
\qquad\qquad\qquad x^2 + 2x - 2 + \dfrac{-10x + 18}{x^2 - 2x + 3} \\
x^2 - 2x + 3\overline{\smash{)}\,x^4 + 0x^3 - 3x^2 + 0x + 12} \\
\qquad\qquad\quad \underline{x^4 - 2x^3 + 3x^2} \\
\qquad\qquad\qquad 2x^3 - 6x^2 + 0x \\
\qquad\qquad\qquad \underline{2x^3 - 4x^2 + 6x} \\
\qquad\qquad\qquad\qquad -2x^2 - 6x + 12 \\
\qquad\qquad\qquad\qquad \underline{-2x^2 + 4x - 6} \\
\qquad\qquad\qquad\qquad\quad -10x + 18
\end{array}
$$

III. Exercises

Answer the following.

1) Give the quotient of each of the following.

a) $30x^3y^5$ divided by $-5x^2y^5$

b) $\dfrac{13x^3 - 26x^5 - 39x^7}{13x^3}$

c) Divide $7x + x^3 – 6$ by $x – 2$

2) If I spent $(x^3 + 5x^2 – 2x – 24)$ pesos for $(x^2 + x – 6)$ pencils, how much does each pencil cost?

3) If 5 is the number needed to be multiplied by 9 to get 45, what polynomial is needed to be multiplied to $x + 3$ to get $2x^2 + 3x – 9$?

4) The length of the rectangle is x cm and its area is $(x^3 – x)$ cm^2. What is the measure of its width?

Summary:

In this lesson, you have learned about dividing polynomials first using the Tiles then using the long way of dividing.

Lesson 25: Special Products **Time: 3.5 hours**

Prerequisite Concepts: Multiplication and Division of Polynomials

About the Lesson: This is a very important lesson. The applications come much later but the skills will always be useful from here on.

Objectives:

In this lesson, you are expected to:

find (a) inductively, using models and (b) algebraically the

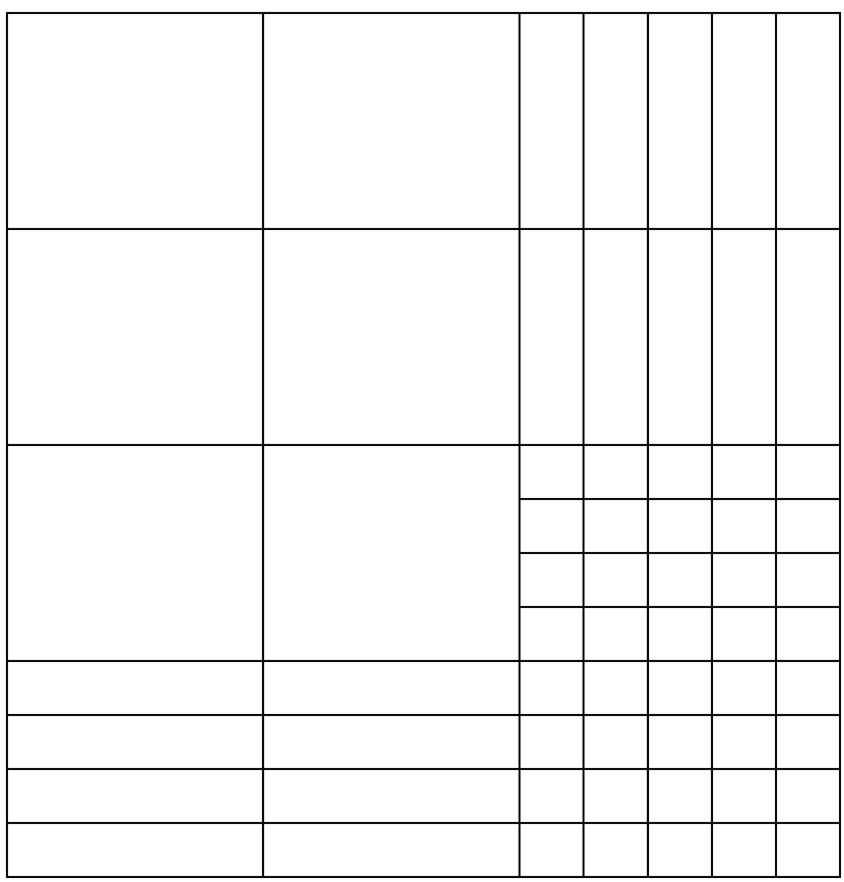
1. product of two binomials
2. product of a sum and difference of two terms
3. square of a binomial
4. cube of a binomial
5. product of a binomial and a trinomial

Lesson Proper:

A. *Product of two binomials*

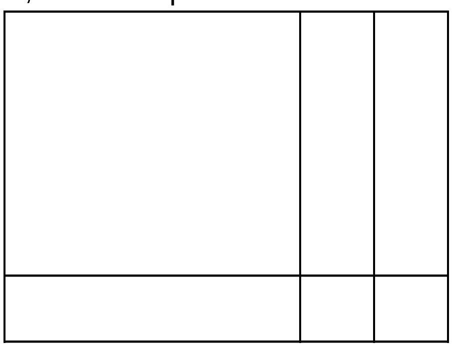
I. Activity

Prepare three sets of algebra tiles by cutting them out from a page of newspaper or art paper. If you are using newspaper, color the tiles from the first set black, the second set red and the third set yellow.



Problem:
1. What is the area of a square whose sides are 2cm?
2. What is the area of a rectangle with a length of 3cm and a width of 2cm?
3. Demonstrate the area of the figures using algebra tiles.

Problem:
1. What are the areas of the different kinds of algebra tiles?
2. Form a rectangle with a length of $x + 2$ and a width of $x + 1$ using the algebra tiles. What is the area of the rectangle?

Solution:
1. x^2, x and 1 square units.
2. 

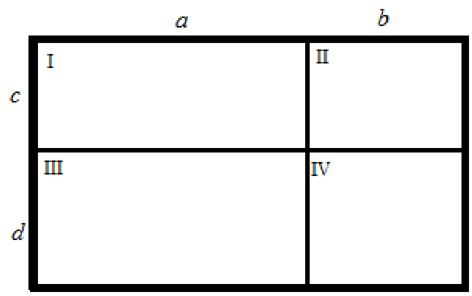
   The area is the sum of all the areas of the algebra tiles.
   Area = $x^2 + x + x + x + 1 + 1 = x^2 + 3x + 2$

Problem:
1. Use algebra tiles to find the product of the following:
   a. $(x+2)(x+3)$
   b. $(2x+1)(x+4)$
   c. $(2x+1)(2x+3)$
2. How can you represent the difference $x – 1$ using algebra tiles?

Problem:
1. Use algebra tiles to find the product of the following:
   a. $(x-1)(x-2)$
   b. $(2x-1)(x-1)$
   c. $(x-2)(x+3)$
   d. $(2x-1)(x+4)$

II. Questions to Ponder

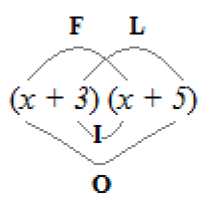
1. Using the concept learned in algebra tiles what is the area of the rectangle shown below?



2. Derive a general formula for the product of two binomials $(a+b)(c+d)$.

The area of the rectangle is equivalent to the product of $(a+b)(c+d)$ which is $ac+ad+bc+cd$. This is the general formula for the product of two binomials $(a+b)(c+d)$. This general form is sometimes called the **FOIL** method where the letters of **FOIL** stand for first, outside, inside, and last.

Example: Find the product of (x + 3) (x + 5)



First:	x . x = x^2
Outside:	x . 5 = 5x
Inside:	3 . x = 3x
Last:	3 . 5 = 15

(x + 3) (x + 5) = x^2 + 5x + 3x + 15 = x^2 + 8x + 15

III. Exercises

Find the product using the **FOIL** method. Write your answers on the spaces provided:

1. (x + 2) (x + 7)
2. (x + 4) (x + 8)
3. (x – 2) (x – 4)
4. (x – 5) (x + 1)
5. (2x + 3) (x + 5)
6. (3x – 2) (4x + 1)
7. (x^2 + 4) (2x – 1)
8. ($5x^3$ + 2x) (x^2 – 5)
9. (4x + 3y) (2x + y)
10. (7x – 8y) (3x + 5y)

B. Product of a sum and difference of two terms

I. Activity

1. Use algebra tiles to find the product of the following:
   a. $(x + 1)(x - 1)$
   b. $(x + 3)(x - 3)$
   c. $(2x - 1)(2x + 1)$
   d. $(2x - 3)(2x + 3)$
2. Use the **FOIL** method to find the products of the above numbers.

II. Questions to Ponder

1. What are the products?
2. What is the common characteristic of the factors in the activity?
3. Is there a pattern for the products for these kinds of factors? Give the rule.

Concepts to Remember

The factors in the activity are called the sum and difference of two terms. Each binomial factor is made up of two terms. One factor is the **sum** of the terms and the other factor being their **difference**. The general form is $(a + b)(a - b)$.

The product of the sum and difference of two terms is given by the general formula
$(a + b)(a - b) = a^2 - b^2$.

III. Exercises

Find the product of each of the following:

1. $(x - 5)(x + 5)$
2. $(x + 2)(x - 2)$
3. $(3x - 1)(3x + 1)$
4. $(2x + 3)(2x - 3)$
5. $(x + y^2)(x - y^2)$
6. $(x^2 - 10)(x^2 + 10)$
7. $(4xy + 3z^3)(4xy - 3z^3)$
8. $(3x^3 - 4)(3x^3 + 4)$
9. $[(x + y) - 1][(x + y) + 1]$
10. $(2x + y - z)(2x + y + z)$

C. Square of a binomial

I. Activity

1. Using algebra tiles, find the product of the following:
   a. $(x + 3)(x + 3)$
   b. $(x - 2)(x - 2)$
   c. $(2x + 1)(2x + 1)$
   d. $(2x - 1)(2x - 1)$

2. Use the **FOIL** method to find their products.

II. Questions to Ponder

1. Find another method of expressing the product of the given binomials.
2. What is the general formula for the square of a binomial?
3. How many terms are there? Will this be the case for all squares of binomials? Why?
4. What is the difference between the square of the sum of two terms from the square of the difference of the same two terms?

Concepts to Remember

The square of a binomial $(a \pm b)^2$ is the product of a binomial when multiplied to itself. The square of a binomial has a general formula, $(a \pm b)^2 = a^2 \pm 2ab + b^2$.

III. Exercises

Find the squares of the following binomials.

1. $(x + 5)^2$
2. $(x - 5)^2$
3. $(x + 4)^2$
4. $(x – 4)^2$
5. $(2x + 3)^2$
6. $(3x - 2)^2$
7. $(4 – 5x)^2$
8. $(1 + 9x)^2$
9. $(x^2 + 3y)^2$
10. $(3x^3 – 4y^2)^2$

D. Cube of a binomial

I. Activity

A. The cube of the binomial (x + 1) can be expressed as $(x + 1)^3$. This is equivalent to (x + 1)(x + 1)(x + 1).

1. Show that $(x + 1)^2 = x^2 + 2x + 1$.
2. How are you going to use the above expression to find $(x + 1)^3$?
3. What is the expanded form of $(x + 1)^3$?

B. Use the techniques outlined above, to find the following:

1. $(x + 2)^2$
2. $(x – 1)^2$
3. $(x – 2)^2$

II. Questions to Ponder

1. How many terms are there in each of the cubes of binomials?
2. Compare your answers in numbers 1 and 2?
   a. What are similar with the first term? How are they different?

b. What are similar with the second terms? How are they different?
c. What are similar with the third terms? How are they different?
d. What are similar with the fourth terms? How are they different?

3. Craft a rule for finding the cube of the binomial in the form $(x + a)^3$. Use this rule to find $(x + 3)^3$. Check by using the method outlined in the activity.
4. Compare numbers 1 and 3 and numbers 2 and 4.
   a. What are the similarities for each of these pairs?
   b. What are their differences?
5. Craft a rule for finding the cube of a binomial in the form $(x – a)^3$. Use this rule to find $(x – 4)^3$.
6. Use the method outlined in the activity to find $(2x + 5)^3$. Can you apply the rule you made in number 3 for getting the cube of this binomial? If not, modify your rule and use it to find
   $(4x + 1)^3$.

Concepts to Remember

The cube of a binomial has the general form, $(a \pm b)^3 = a^3 \pm 3a^2b + 3ab^2 \pm b^3$.

III. Exercises

Expand.

1. $(x+5)^3$
2. $(x-5)^3$
3. $(x+7)^3$
4. $(x-6)^3$
5. $(2x+1)^3$
6. $(3x-2)^3$
7. $(x^2-1)^3$
8. $(x+3y)^3$
9. $(4xy+3)^3$
10. $(2p-3q^2)^3$

E. Product of a binomial and a trinomial

I. Activity

In the previous activity, we have tried multiplying a trinomial with a binomial. The resulting product then had four terms. But, the product of a trinomial and a binomial **does not always** give a product of four terms.

1. Find the product of $x^2 - x + 1$ and $x + 1$.
2. How many terms are in the product?

3. What trinomial should be multiplied to $x-1$ to get x^3-1?
4. Is there a trinomial that can be multiplied to x – 1 to get $x^3 + 1$?
5. Using the methods outlined in the previous problems, what should be multiplied to x + 2 to get
   $x^3 + 8$? Multiplied to x – 3 to get $x^3 – 27$?

II. Questions to Ponder

1. What factors should be multiplied to get the product $x^3 + a^3$? $x^3 – a^3$?
2. What factors should be multiplied to get $27x^3 + 8$?

Concepts to Remember

The product of a trinomial and a binomial can be expressed as the sum or difference of two cubes if they are in the following form.

$$(a^2 - ab + b^2)(a+b) = a^3 + b^3$$
$$(a^2 + ab + b^2)(a-b) = a^3 - b^3$$

III. Exercises

A. Find the product.

1. $(x^2 - 3x + 9)(x+3)$
2. $(x^2 + 4x + 16)(x-4)$
3. $(x^2 - 6x + 36)(x+6)$
4. $(x^2 + 10x + 100)(x-10)$
5. $(4x^2 + 10x + 25)(2x-5)$
6. $(9x^2 + 12x + 16)(3x-4)$

B. What should be multiplied to the following to get a sum/difference of two cubes? Give the product.

1. $(x - 7)$
2. $(x + 8)$
3. $(4x + 1)$
4. $(5x - 3)$
5. $(x^2 + 2x + 4)$
6. $(x^2 - 11x + 121)$
7. $(100x^2 + 30x + 9)$
8. $(9x^2 - 21x + 49)$

Summary: You learned plenty of special products and techniques in solving problems that require special products.