# 9

# Mathematics

## Learner's Material
## Unit 1





**Department of Education**
**Republic of the Philippines**

**Mathematics – Grade 9**
**Learner's Material**
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LEARNING MODULE

MATH 9

# MODULE NO. 1: QUADRATIC EQUATIONS AND INEQUALITIES

**INTRODUCTION AND FOCUS QUESTIONS**:

Was there any point in your life when you asked yourself about the different real life quantities such as costs of goods or services, incomes, profits, yields and losses, amount of particular things, speed, area, and many others? Have you ever realized that these quantities can be mathematically represented to come up with practical decisions?




Find out the answers to these questions and determine the vast applications of quadratic equations and quadratic inequalities through this module.

**LESSONS AND COVERAGE:**

In this module, you will examine the above questions when you take the following lessons:

*Lesson 1* – **ILLUSTRATIONS OF QUADRATIC EQUATIONS**

*Lesson 2* – **SOLVING QUADRATIC EQUATIONS**

- **EXTRACTING SQUARE ROOTS**
- **FACTORING**
- **COMPLETING THE SQUARE**
- **QUADRATIC FORMULA**

*Lesson 3 –* **NATURE OF ROOTS OF QUADRATIC EQUATIONS**

*Lesson 4 –* **SUM AND PRODUCT OF ROOTS OF QUADRATIC EQUATIONS**

*Lesson 5 –* **EQUATIONS TRANSFORMABLE TO QUADRATIC EQUATIONS (INCLUDING RATIONAL ALGEBRAIC EQUATIONS)**

*Lesson 6 –* **APPLICATIONS OF QUADRATIC EQUATIONS AND RATIONAL ALGEBRAIC EQUATIONS**

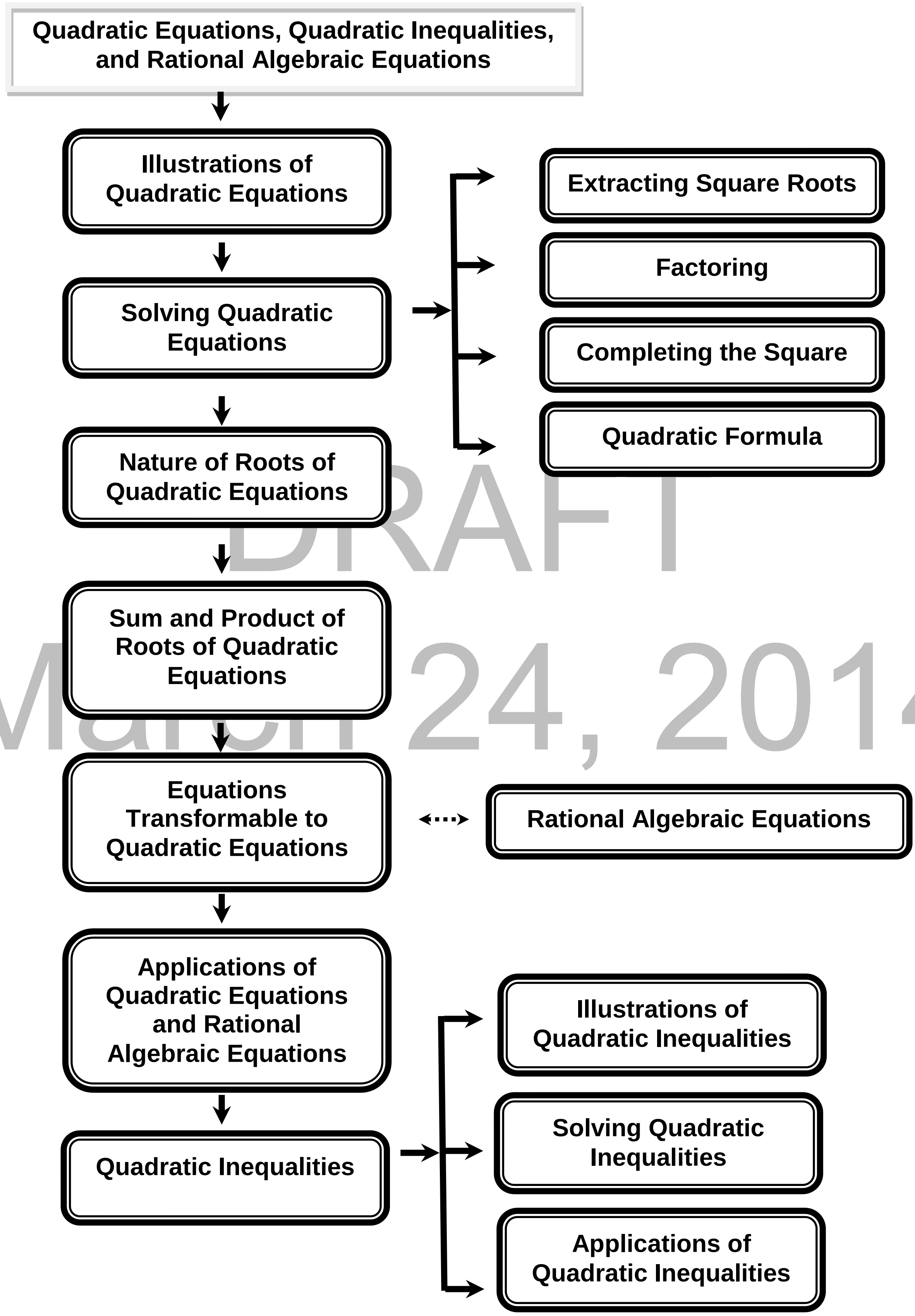
*Lesson 7 –* **QUADRATIC INEQUALITIES**

In these lessons, you will learn to:

| | |
|---|---|
| *Lesson 1* | • illustrate quadratic equations; |
| *Lesson 2* | • solve quadratic equations by: (a) extracting square roots; (b) factoring; (c) completing the square; (d) using the quadratic formula; |
| *Lesson 3* | • characterize the roots of a quadratic equation using the discriminant; |
| *Lesson 4* | • describe the relationship between the coefficients and the roots of a quadratic equation; |
| *Lesson 5* | • solve equations transformable to quadratic equations (including rational algebraic equations); |
| *Lesson 6* | • solve problems involving quadratic equations and rational algebraic equations; |
| *Lesson 7* | • illustrate quadratic inequalities;<br>• solve quadratic inequalities; and<br>• solve problems involving quadratic inequalities. |

**MODULE MAP:**

*Here is a simple map of the lessons that will be covered in this module:*



# PRE-ASSESSMENT

**Part I**

*Directions:* Find out how much you already know about this module. Choose the letter that you think best answers the question. Please answer all items. Take note of the items that you were not able to answer correctly and find the right answer as you go through this module.

1. It is a polynomial equation of degree two that can be written in the form $ax^2 + bx + c = 0$, where a, b, and c are real numbers and a ≠ 0.
   A. Linear Equation
   B. Linear Inequality
   C. Quadratic Equation
   D. Quadratic Inequality

2. Which of the following is a quadratic equation?
   A. $2r^2 + 4r - 1$
   B. $3t - 7 = 12$
   C. $s^2 + 5s - 14 = 0$
   D. $2x^2 - 7x \geq 3$

3. In the quadratic equation $3x^2 + 7x - 4 = 0$, which is the quadratic term?
   A. $x^2$
   B. $7x$
   C. $3x^2$
   D. $-4$



4. Which of the following rational algebraic equations is transformable to a quadratic equation?
   A. $\frac{w+1}{2} - \frac{w+2}{4} = 7$
   B. $\frac{2}{p} + \frac{3}{p+1} = 5$
   C. $\frac{2q-1}{3} + \frac{1}{2} = \frac{3q}{4}$
   D. $\frac{3}{m-2} + \frac{4}{m+2} = \frac{7}{m}$

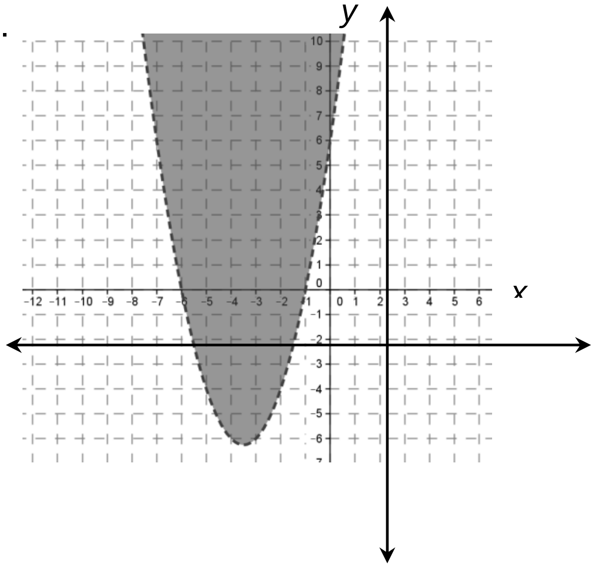
5. How many real roots does the quadratic equation $x^2 + 5x + 7 = 0$ have?
   A. 0
   B. 1
   C. 2
   D. 3

6. The roots of a quadratic equation are -5 and 3. Which of the following quadratic equations has these roots?
   A. $x^2 - 8x + 15 = 0$
   B. $x^2 + 8x + 15 = 0$
   C. $x^2 - 2x - 15 = 0$
   D. $x^2 + 2x - 15 = 0$

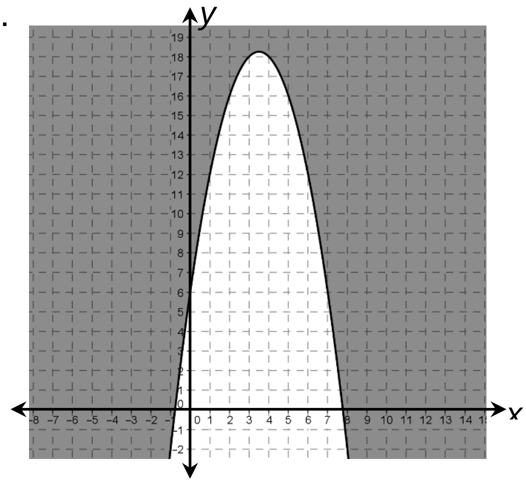
7. Which of the following mathematical statements is a quadratic inequality?
   A. $2r^2 - 3r - 5 = 0$
   B. $7h + 12 < 0$
   C. $3t^2 + 7t - 2 \geq 0$
   D. $s^2 + 8s + 15 = 0$

8. Which of the following shows the graph of $y \geq x^2 + 7x + 6$?

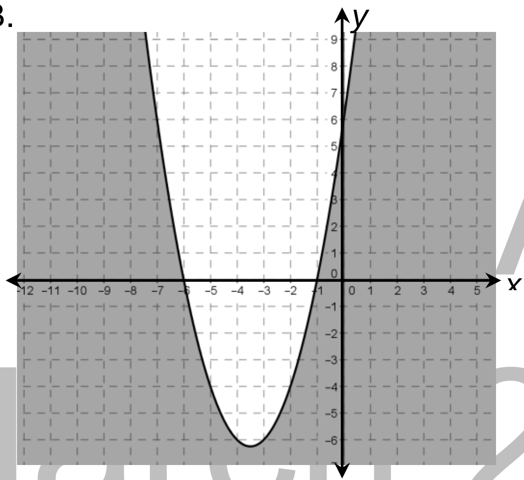
A.



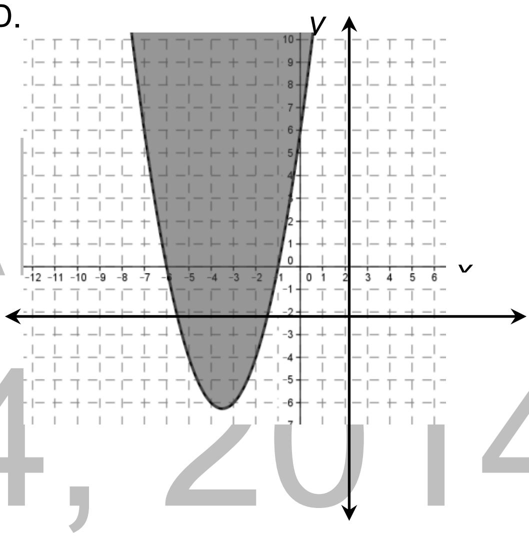

C.




B.




D.






9. Which of the following values of *x* make the equation $x^2 + 7x - 18 = 0$ true?

I. -9 II. 2 III. 9

A. I and II B. II and III C. I and III D. I, II, and III

10. Which of the following quadratic equations has no real roots?

A. $2x^2 + 4x = 3$
C. $3s^2 - 2s = -5$
B. $t^2 - 8t - 4 = 0$
D. $-2r^2 + r + 7 = 0$

11. What is the nature of the roots of the quadratic equation if the value of its discriminant is zero?

A. The roots are not real.
C. The roots are rational and not equal.
B. The roots are irrational and not equal.
D. The roots are rational and equal.

12. One of the roots of $2x^2 - 13x + 20 = 0$ is 4. What is the other root?

A. $-\frac{2}{5}$ B. $-\frac{5}{2}$ C. $\frac{2}{5}$ D. $\frac{5}{2}$

13. What are the roots of the quadratic equation $x^2 + x - 56 = 0$?

A. 2 and -1 B. 8 and -7 C. -8 and 7 D. 3 and -2

14. What is the sum of the roots of the quadratic equation $2x^2 + 6x - 14 = 0$?

A. -7 B. -3 C. 6 D. 14

15. Which of the following quadratic equations can be solved easily by extracting square roots?

A. $x^2 - 7x + 12 = 0$ C. $4t^2 - 9 = 0$

B. $2w^2 + 7w - 3 = 0$ D. $3v^2 - 2v - 8 = 0$

16. Which of the following coordinates of points belong to the solution set of the inequality $y < 2x^2 + 5x - 1$?

A. (-3,2) B. (-2,9) C. (1,6) D. (3,1)



17. A 3 cm by 3 cm square piece of cardboard was cut from a bigger square cardboard. The area of the remaining cardboard was 40 $cm^2$. If *s* represents the length of the bigger cardboard, which of the following expressions give the area of the remaining piece?

A. $s - 9$ B. $s^2 + 9$ C. $s^2 - 9$ D. $s^2 + 40$



18. The length of a wall is 12 m more than its width. If the area of the wall is less than 50 $m^2$, which of the following could be its length?

A. 3 m B. 4 m C. 15 m D. 16 m

19. The length of a garden is 5 m longer than its width and the area is 14 $m^2$. How long is the garden?

A. 9 m B. 7 m C. 5 m D. 2 m

20. A car travels 20 kph faster than a truck. The car covers 480 km in two hours less than the time it takes the truck to travel the same distance. How fast does the car travel?

A. 44 kph B. 60 kph C. 80 kph D. 140 kph

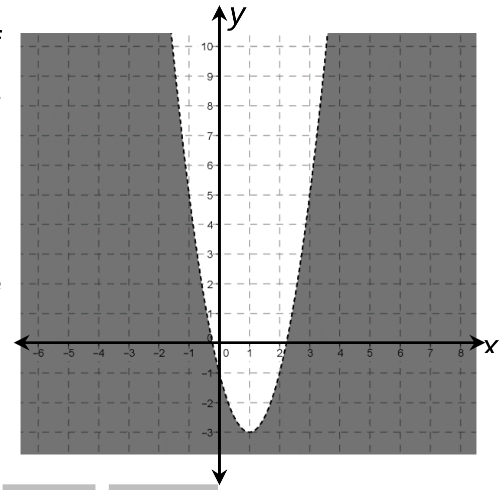
21. A 12 cm by 16 cm picture is mounted with border of uniform width on a rectangular frame. If the total area of the border is 288 $cm^2$, what is the length of the side of the frame?

A. 8 cm B. 16 cm C. 20 cm D. 24 cm

22. SamSon Electronics Company would like to come up with an LED TV such that its screen is 560 square inches larger than the present ones. Suppose the length of the screen of the larger TV is 6 inches longer than its width and the area of the smaller TV is 520 square inches. What is the length of the screen of the larger LED TV?

A. 26 in. B. 30 in C. 33 in. D. 36 in.

23. The figure on the right shows the graph of $y < 2x^2 - 4x - 1$. Which of the following is true about the solution set of the inequality?




I. The coordinates of all points on the shaded region belong to the solution set of the inequality.
II. The coordinates of all points along the parabola as shown by the broken line belong to the solution set of the inequality.
III. The coordinates of all points along the parabola as shown by the broken line do not belong to the solution set of the inequality.

A. I and II B. I and III C. II and III D. I, II, and III

24. It takes Mary 3 hours more to do a job than it takes Jane. If they work together, they can finish the same job in 2 hours. How long would it take Mary to finish the job alone?

A. 3 hours B. 5 hours C. 6 hours D. 8 hours

25. An open box is to be formed out of a rectangular piece of cardboard whose length is 12 cm longer than its width. To form the box, a square of side 5 cm will be removed from each corner of the cardboard. Then the edges of the remaining cardboard will be turned up. If the box is to hold at most 1900 cm$^3$, what mathematical statement would represent the given situation?

A. $x^2 - 12x \le 360$ C. $x^2 - 8x \le 400$

B. $x^2 - 12x \le 380$ D. $x^2 + 8x \le 400$

26. The length of a garden is 2 m more than twice its width and its area is 24 m$^2$. Which of the following equations represents the given situation?

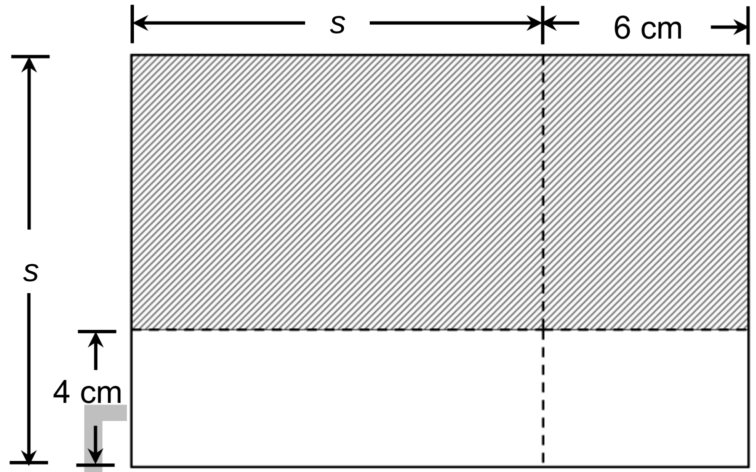
A. $x^2 + x = 12$ C. $x^2 + x = 24$

B. $x^2 + 2x = 12$ D. $x^2 + 2x = 24$

27. From 2004 through 2012, the average weekly income of an employee in a certain company is estimated by the quadratic expression $0.16n^2 + 5.44n + 2240$ , where $n$ is the number of years after 2004. In what year was the average weekly income of an employee equal to Php2,271.20?

A. 2007 B. 2008 C. 2009 D. 2010

28. In the figure below, the area of the shaded region is 144 cm$^2$. What is the length of the longer side of the figure?




A. 8 cm B. 12 cm C. 14 cm D. 18 cm

**Part II**

*Directions:* Read and understand the situation below then answer or perform what are asked.

*Mrs. Villareal was asked by her principal to transfer her Grade 9 class to a new classroom that was recently built. The room however still does not have fixtures such as teacher's table, bulletin boards, divan, bookshelves, and cabinets. To help her acquire these fixtures, she informed the parents of her students about these classroom needs. The parents decided to donate construction materials such as wood, plywood, nails, paints, and many others.*

*After all the materials have been received, she asked her students to make the designs of the different classroom needs. Each group of students was assigned to do the design of a particular fixture. The designs that the students will prepare shall be used by the carpenter in constructing the tables, chairs, bulletin boards, divan, bookshelves, and cabinets.*

1. Suppose you are one of the students of Mrs. Villareal. How will you prepare the design of one of the fixtures?

2. Make a design of the fixture assigned to your group.

3. Illustrate every part or portion of the fixture including their measurements.

4. Using the design of the fixture made, determine all the mathematics concepts or principles involved.

5. Formulate problems involving these mathematics concepts or principles.

6. Write the expressions, equations, or inequalities that describe the situations or problems.

7. Solve the equations, the inequalities, and the problems formulated.

Rubric for Design

| 4 | 3 | 2 | 1 |
|---|---|---|---|
| The design is accurately made, presentable, and appropriate. | The design is accurately made and appropriate. | The design is not accurately made but appropriate. | The design is made but not appropriate. |

Rubric for Equations Formulated and Solved

| 4 | 3 | 2 | 1 |
|---|---|---|---|
| Equations and inequalities are properly formulated and solved correctly. | Equations and inequalities are properly formulated but not all are solved correctly. | Equations and inequalities are properly formulated but are not solved correctly. | Equations and inequalities are properly formulated but are not solved. |

Rubric on Problems Formulated and Solved

| Score | Descriptors |
|---|---|
| 6 | Poses a more complex problem with 2 or more correct possible solutions and communicates ideas unmistakably, shows in-depth comprehension of the pertinent concepts and/or processes and provides explanations wherever appropriate. |
| 5 | Poses a more complex problem and finishes all significant parts of the solution and communicates ideas unmistakably, shows in-depth comprehension of the pertinent concepts and/or processes. |
| 4 | Poses a complex problem and finishes all significant parts of the solution and communicates ideas unmistakably, shows in-depth comprehension of the pertinent concepts and/or processes. |
| 3 | Poses a complex problem and finishes most significant parts of the solution and communicates ideas unmistakably, shows comprehension of major concepts although neglects or misinterprets less significant ideas or details. |
| 2 | Poses a problem and finishes some significant parts of the solution and communicates ideas unmistakably but shows gaps on theoretical comprehension. |
| 1 | Poses a problem but demonstrates minor comprehension, not being able to develop an approach. |

Source: D.O. #73 s. 2012

**LEARNING GOALS AND TARGETS:**

After going through this module, you should be able to demonstrate understanding of key concepts of quadratic equations, quadratic inequalities, and rational algebraic equations, formulate real-life problems involving these concepts, and solve these using a variety of strategies. Furthermore, you should be able to investigate mathematical relationships in various situations involving quadratic equations and quadratic inequalities.

## LESSON NO. 1: ILLUSTRATIONS OF QUADRATIC EQUATIONS

**What to KNOW:**

Start Lesson 1 of this module by assessing your knowledge of the different mathematics concepts previously studied and your skills in performing mathematical operations. These knowledge and skills will help you in understanding quadratic equations. As you go through this lesson, think of this important question: *"How are quadratic equations used in solving real-life problems and in making decisions?"* To find the answer, perform each activity. If you find any difficulty in answering the exercises, seek the assistance of your teacher or peers or refer to the modules you have gone over earlier. You may check your work with your teacher.



**Activity 1: Do You Remember These Products?**

*Directions*: Find each indicated product then answer the questions that follow.

1. $3(x^2+7)$
2. $2s(s-4)$
3. $(w+7)(w+3)$
4. $(x+9)(x-2)$
5. $(2t-1)(t+5)$
6. $(x+4)(x+4)$
7. $(2r-5)(2r-5)$
8. $(3-4m)^2$
9. $(2h+7)(2h-7)$
10. $(8-3x)(8+3x)$

*Questions:*

a. How did you find each product?

b. In finding each product, what mathematics concepts or principles did you apply? Explain how you applied these mathematics concepts or principles.

c. How would you describe the products obtained?

Are the products polynomials? If YES, what common characteristics do these polynomials have?

> Were you able to find and describe the products of some polynomials? Were you able to recall and apply the different mathematics concepts or principles in finding each product? Why do you think there is a need to perform such mathematical tasks? You will find this out as you go through this lesson.

**Activity 2: Another Kind of Equation!**

*Directions*: Below are different equations. Use these equations to answer the questions that follow.

| | | | |
|---|---|---|---|
| $x^2 - 5x + 3 = 0$ | $9r^2 - 25 = 0$ | $c = 12n - 5$ | $9 - 4x = 15$ |
| $2s + 3t = -7$ | $\frac{1}{2}x^2 + 3x = 8$ | $6p - q = 10$ | $\frac{3}{4}h + 6 = 0$ |
| $8k - 3 = 12$ | $4m^2 + 4m + 1 = 0$ | $t^2 - 7t + 6 = 0$ | $r^2 = 144$ |



1. Which of the given equations are linear?
2. How do you describe linear equations?
3. Which of the given equations are not linear? Why?

   How are these equations different from those which are linear?

   What common characteristics do these equations have?

> In the activity you have just done, were you able to identify equations which are linear and which are not? Were you able to describe those equations which are not linear? These equations have common characteristics and you will learn more of these in the succeeding activities.

**Activity 3: A Real Step to Quadratic Equations**

*Directions:* Use the situation below to answer the questions that follow.

*Mrs. Jacinto asked a carpenter to construct a rectangular bulletin board for her classroom. She told the carpenter that the board's area must be 18 square feet.*

1. Draw a diagram to illustrate the bulletin board.
2. What are the possible dimensions of the bulletin board? Give at least 2 pairs of possible dimensions.
3. How did you determine the possible dimensions of the bulletin board?

4. Suppose the length of the board is 7 ft. longer than its width. What equation would represent the given situation?
5. How would you describe the equation formulated?
6. Do you think you can use the equation formulated to find the length and the width of the bulletin board? Justify your answer.

> How did you find the preceding activities? Are you ready to learn about quadratic equations? I'm sure you are!!! From the activities done, you were able to describe equations other than linear equations, and these are quadratic equations. You were able to find out how a particular quadratic equation is illustrated in real life. But how are quadratic equations used in solving real-life problems and in making decisions? You will find these out in the activities in the next section. Before doing these activities, read and understand first some important notes on quadratic equations and the examples presented.

A **quadratic equation** in one variable is a mathematical sentence of degree 2 that can be written in the following standard form.

$\mathbf{a}x^2 + \mathbf{b}x + \mathbf{c} = 0$, where **a**, **b**, and **c** are real numbers and $\mathbf{a} \neq 0$

> *Why do you think* **a** *must not be equal to zero in the equation* $\mathbf{a}x^2 + \mathbf{b}x + \mathbf{c} = 0$*?*

In the equation, $\mathbf{a}x^2$ is the quadratic term, $\mathbf{b}x$ is the linear term, and **c** is the constant term.

*Example 1:* $2x^2 + 5x - 3 = 0$ is a quadratic equation in standard form with a = 2, b = 5, and c = -3.

*Example 2:* $3x(x - 2) = 10$ is a quadratic equation. However, it is not written in standard form.

To write the equation in standard form, expand the product and make one side of the equation zero as shown below.

$$3x(x-2)=10 \quad \rightarrow \quad 3x^2-6x=10$$

$$3x^2-6x-10=10-10$$

$$3x^2-6x-10=0$$

The equation becomes $3x^2 - 6x - 10 = 0$, which is in standard form.

In the equation $3x^2 - 6x - 10 = 0$, a = 3, b = -6, and c = -10.

*Example 3:* The equation $(2x + 5)(x – 1) = -6$ is also a quadratic equation but is not written in standard form.

Just like in *Example 2*, the equation $(2x + 5)(x – 1) = -6$ can be written in standard form by expanding the product and making one side of the equation zero as shown below.

$$(2x+5)(x-1)=-6 \quad \rightarrow \quad 2x^2-2x+5x-5=-6$$

$$2x^2+3x-5=-6$$

$$2x^2+3x-5+6=-6+6$$

$$2x^2+3x+1=0$$

The equation becomes $2x^2 + 3x + 1 = 0$, which is in standard form.

In the equation $2x^2 + 3x + 1 = 0$, a = 2, b = 3, and c = 1.

When **b** = 0 in the equation $\mathbf{ax^2 + bx + c = 0}$, it results to a quadratic equation of the form $\mathbf{ax^2 + c = 0}$.

*Examples:* Equations such as $x^2 + 5 = 0$, $-2x^2 + 7 = 0$, and $16x^2 – 9 = 0$ are quadratic equations of the form $ax^2 + c = 0$. In each equation, the value of b = 0.

Learn more about Quadratic Equations through the WEB. You may open the following links.

http://library.thinkquest.org/20991/alg2/quad.html

http://math.tutorvista.com/algebra/quadratic-equation.html

http://www.algebra.com/algebra/homework/quadratic/lessons/quadform/

**What to PROCESS:**

Your goal in this section is to apply the key concepts of quadratic equations. Use the mathematical ideas and the examples presented in the preceding section to answer the activities provided.

**Activity 4: Quadratic or Not Quadratic?**

*Directions*: Identify which of the following equations are quadratic and which are not. If the equation is not quadratic, explain.

1. $3m + 8 = 15$
2. $x^2 - 5x + 10 = 0$
3. $12 - 4x = 0$
4. $2t^2 - 7t = 12$
5. $6 - 2x + 3x^2 = 0$
6. $25 - r^2 = 4r$
7. $3x(x - 2) = -7$
8. $\frac{1}{2}(h - 6) = 0$
9. $(x + 2)^2 = 0$
10. $(w - 8)(w + 5) = 14$

Were you able to identify which equations are quadratic? Some of the equations given are not quadratic equations. Were you able to explain why? I'm sure you did. In the next activity, you will identify the situations that illustrate quadratic equations and represent these by mathematical statements.



**Activity 5: Does It Illustrate Me?**

*Directions:* Tell whether or not each of the following situations illustrates quadratic equations. Justify your answer by representing each situation by a mathematical sentence.

1. The length of a swimming pool is 8 m longer than its width and the area is 105 $m^2$.



2. Edna paid at least Php1,200 for a pair of pants and a blouse. The cost of the pair of pants is Php600 more than the cost of the blouse.

3. A motorcycle driver travels 15 kph faster than a bicycle rider. The motorcycle driver covers 60 km in two hours less than the time it takes the bicycle rider to travel the same distance.

4. A realty developer sells residential lots for Php4,000 per square meter plus a processing fee of Php25,000. One of the lots the realty developer is selling costs Php625,000.

5. A garden 7 m by 12 m will be expanded by planting a border of flowers. The border will be of the same width around the entire garden and has an area of 92 $m^2$.

Did you find the activity challenging? Were you able to represent each situation by a mathematical statement? For sure you were able to identify the situations that can be represented by quadratic equations. In the next activity, you will write quadratic equations in standard form.

**Activity 6: Set Me To Your Standard!**

*Directions*: Write each quadratic equation in standard form, $\mathbf{a}\boldsymbol{x}^{\mathbf{2}} + \mathbf{b}\boldsymbol{x} + \mathbf{c} = \mathbf{0}$ then identify the values of **a**, **b**, and **c.** Answer the questions that follow.

1. $3x - 2x^2 = 7$
2. $5 - 2x^2 = 6x$
3. $(x+3)(x+4) = 0$
4. $(2x+7)(x-1) = 0$
5. $2x(x-3) = 15$
6. $(x+7)(x-7) = -3x$
7. $(x-4)^2 + 8 = 0$
8. $(x+2)^2 = 3(x+2)$
9. $(2x-1)^2 = (x+1)^2$
10. $2x(x+4) = (x-3)(x-3)$

*Questions:*

a. How did you write each quadratic equation in standard form?

b. What mathematics concepts or principles did you apply to write each quadratic equation in standard form? Discuss how you applied these mathematics concepts or principles.

c. Which quadratic equations did you find difficult to write in standard form? Why?

d. Compare your work with those of your classmates. Did you arrive at the same answers? If NOT, explain.



How was the activity you have just done? Was it easy for you to write quadratic equations in standard form? It was easy for sure!

In this section, the discussion was about quadratic equations, their forms and how they are illustrated in real life.

Go back to the previous section and compare your initial ideas with the discussion. How much of your initial ideas are found in the discussion? Which ideas are different and need revision?

Now that you know the important ideas about this topic, let's go deeper by moving on to the next section.

**What to REFLECT ON and FURTHER UNDERSTAND:**

Your goal in this section is to take a closer look at some aspects of the topic. You are going to think deeper and test further your understanding of quadratic equations. After doing the following activities, you should be able to answer this important question: *How are quadratic equations used in solving real-life problems and in making decisions?*

**Activity 7: Dig Deeper!**

*Directions*: Answer the following questions.

1. How are quadratic equations different from linear equations?

2. How do you write quadratic equations in standard form? Give at least 3 examples.

3. The following are the values of a, b, and c that Edna and Luisa got when they expressed $5-3x=2x^2$ in standard form.

   Edna: a = 2; b = 3; c = -5

   Luisa: a = -2; b = -3; c = 5

   Who do you think got the correct values of a, b, and c? Justify your answer.

4. Do you agree that the equation $4-3x=2x^2$ can be written in standard form in two different ways? Justify your answer.

5. The members of the school's Mathematics Club shared equal amounts for a new Digital Light Processing (DLP) projector amounting to Php25,000. If there had been 25 members more in the club, each would have contributed Php50 less.

   a. How are you going to represent the number of Mathematics Club members?

   b. What expression represents the amount each member will share?

   c. If there were 25 members more in the club, what expression would represent the amount each would share?

   d. What mathematical sentence would represent the given situation? Write this in standard form then describe.

In this section, the discussion was about your understanding of quadratic equations and how they are illustrated in real life.

What new realizations do you have about quadratic equations? How would you connect this to real life? How would you use this in making decisions?

Now that you have a deeper understanding of the topic, you are ready to do the tasks in the next section.

**What to TRANSFER:**

Your goal in this section is to apply your learning to real-life situations. You will be given a practical task which will demonstrate your understanding of quadratic equations.

**Activity 8: Where in the Real World?**

1. Give 5 examples of quadratic equations written in standard form. Identify the values of a, b, and c in each equation.

2. Name some objects or cite situations in real life where quadratic equations are illustrated. Formulate quadratic equations out of these objects or situations then describe each.

In this section, your task was to give examples of quadratic equations written in standard form and name some objects or cite real-life situations where quadratic equations are illustrated.

How did you find the performance task? How did the task help you realize the importance of the topic in real life?

**Summary/Synthesis/Generalization:**

This lesson was about quadratic equations and how they are illustrated in real life. The lesson provided you with opportunities to describe quadratic equations using practical situations and their mathematical representations. Moreover, you were given the chance to formulate quadratic equations as illustrated in some real-life situations. Your understanding of this lesson and other previously learned mathematics concepts and principles will facilitate your learning of the next lesson, Solving Quadratic Equations.

# LEARNING MODULE

# MATH 9

## LESSON NO. 2A: SOLVING QUADRATIC EQUATIONS BY EXTRACTING SQUARE ROOTS

### What to KNOW:

Start Lesson 2A of this module by assessing your knowledge of the different mathematics concepts previously studied and your skills in performing mathematical operations. These knowledge and skills will help you in solving quadratic equations by extracting square roots. As you go through this lesson, think of this important question: *"How does finding solutions of quadratic equations facilitate in solving real-life problems and in making decisions?"* To find the answer, perform each activity. If you find any difficulty in answering the exercises, seek the assistance of your teacher or peers or refer to the modules you have gone over earlier. You may check your answers with your teacher.



### Activity 1: Find My Roots!!!

*Directions:* Find the following square roots. Answer the questions that follow.

1. $\sqrt{16} =$
2. $-\sqrt{25} =$
3. $\sqrt{49} =$
4. $-\sqrt{64} =$
5. $\sqrt{121} =$
6. $-\sqrt{289} =$
7. $\sqrt{0.16} =$
8. $\pm\sqrt{36} =$
9. $\sqrt{\dfrac{16}{25}} =$
10. $\pm\sqrt{\dfrac{169}{256}} =$

*Questions:*

a. How did you find each square root?

b. How many square roots does a number have? Explain your answer.

c. Does a negative number have a square root? Why?

d. Describe the following numbers: $\sqrt{8}$, $-\sqrt{40}$, $\sqrt{60}$, and $-\sqrt{90}$

Are the numbers rational or irrational? Explain your answer.

How do you describe rational numbers? How about numbers that are irrational?

Were you able to find the square roots of some numbers? Did the activity provide you with an opportunity to strengthen your understanding of rational and irrational numbers? In the next activity, you will be solving linear equations. Just like finding square roots of numbers, solving linear equations is also a skill which you need to develop further in order for you understand the new lesson.

**Activity 2: What Would Make A Statement True?**

*Directions:* Solve each of the following equations in as many ways as you can. Answer
the questions that follow.

1. $x + 7 = 12$
2. $t - 4 = 10$
3. $r + 5 = -3$
4. $x - 10 = -2$
5. $2s = 16$
6. $-5x = 35$
7. $3h - 2 = 16$
8. $-7x = -28$
9. $3(x + 7) = 24$
10. $2(3k - 1) = 28$



*Questions:*

a. How did you solve each equation?

b. What mathematics concepts or principles did you apply to come up with the solution of each equation? Explain how you applied these.

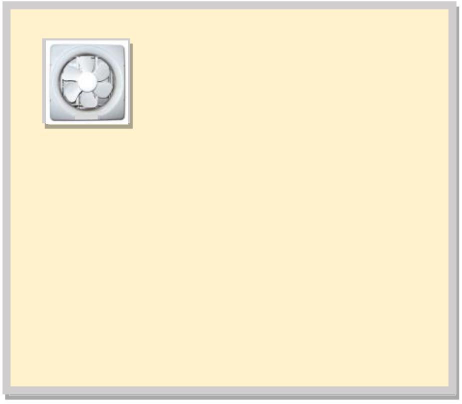
c. Compare the solutions you got with those of your classmates. Did you arrive at the same answers? If not, why?

d. Which equations did you find difficult to solve? Why?

How did you find the activity? Were you able to recall and apply the different mathematics concepts or principles in solving linear equations? I'm sure you were. In the next activity, you will be representing a situation using a mathematical sentence. Such mathematical sentence will be used to satisfy the conditions of the given situation.

**Activity 3: Air Out!!!**

*Directions*: Use the situation below to answer the questions that follow.



*Mr. Cayetano plans to install a new exhaust fan on his room's square-shaped wall. He asked a carpenter to make a square opening on the wall where the exhaust fan will be installed. The square opening must have an area of 0.25 $m^2$.*

1. Draw a diagram to illustrate the given situation.

2. How are you going to represent the length of a side of the square-shaped wall? How about its area?

3. Suppose the area of the remaining part of the wall after the carpenter has made the square opening is 6 $m^2$. What equation would describe the area of the remaining part of the wall?

4. How will you find the length of a side of the wall?



> The activity you have just done shows how a real-life situation can be represented by a mathematical sentence. Were you able to represent the given situation by an equation? Do you now have an idea on how to use the equation in finding the length of a side of the wall? To further give you ideas in solving the equation or other similar equations, perform the next activity.

**Activity 4: Learn to Solve Quadratic Equations!!!**

*Directions:* Use the quadratic equations below to answer the questions that follow.

| $x^2 = 36$ | $t^2 - 64 = 0$ | $2s^2 - 98 = 0$ |
|---|---|---|

1. Describe and compare the given equations. What statements can you make?

2. Solve each equation in as many ways as you can. Determine the values of each variable to make each equation true.

3. How did you know that the values of the variable really satisfy the equation?

4. Aside from the procedures that you followed in solving each equation, do you think there are other ways of solving it? Describe these ways if there are any.

Were you able to determine the values of the variable that make each equation true? Were you able to find other ways of solving each equation? Let us extend your understanding of quadratic equations and learn more about their solutions by performing the next activity.

**Activity 5: Anything Real or Nothing Real?**

*Directions:* Find the solutions of each of the following quadratic equations, then answer the questions that follow.

| $x^2 = 9$ | $r^2 = 0$ | $w^2 = -9$ |
|---|---|---|



1. How did you determine the solutions of each equation?

2. How many solutions does each equation have? Explain your answer.

3. What can you say about each quadratic equation based on the solutions obtained?



How did you find the preceding activities? Are you ready to learn about solving quadratic equations by extracting square roots? I'm sure you are!!! From the activities done, you were able to find the square roots of numbers, solve linear equations, represent a real-life situation by a mathematical sentence, and use different ways of solving a quadratic equation. But how does finding solutions of quadratic equations facilitate in solving real-life problems and in making decisions? You will find these out in the activities in the next section. Before doing these activities, read and understand first some important notes on solving quadratic equations by extracting square roots and the examples presented.

Quadratic equations that can be written in the form $\boldsymbol{x^2 = k}$ can be solved by applying the following properties:

1. If $k > 0$, then $x^2 = k$ has two real solutions or roots: $x = \pm\sqrt{k}$ .
2. If $k = 0$, then $x^2 = k$ has one real solution or root: $x = 0$ .
3. If $k < 0$, then $x^2 = k$ has no real solutions or roots.

The method of solving the quadratic equation $x^2 = k$ is called ***extracting square roots***.

*Example 1:* Find the solutions of the equation $x^2 - 16 = 0$ by extracting square roots.

Write the equation in the form $x^2 = k$.

$$x^2 - 16 = 0 \quad \rightarrow \quad x^2 - 16 + 16 = 0 + 16$$
$$x^2 = 16$$

Since 16 is greater than 0, then the first property above can be applied to find the values of $x$ that will make the equation $x^2 - 16 = 0$ true.

$$x^2 = 16 \quad \rightarrow \quad x = \pm\sqrt{16}$$
$$x = \pm 4$$

To check, substitute these values in the original equation.

| For $x = 4$: | For $x = -4$: |
|---|---|
| $x^2 - 16 = 0$ | $x^2 - 16 = 0$ |
| $4^2 - 16 \overset{?}{=} 0$ | $(-4)^2 - 16 \overset{?}{=} 0$ |
| $16 - 16 \overset{?}{=} 0$ | $16 - 16 \overset{?}{=} 0$ |
| $0 = 0$ | $0 = 0$ |

Both values of $x$ satisfy the given equation. So the equation $x^2 - 16 = 0$ is true when $x = 4$ or when $x = -4$.

*Answer:* The equation $x^2 - 16 = 0$ has two solutions: $\boldsymbol{x = 4}$ or $\boldsymbol{x = -4}$.

*Example 2:* Solve the equation $t^2 = 0$.

Since $t^2$ equals 0, then the equation has only one solution. That is, $t = 0$.

To check:
$$t^2 = 0$$
$$0^2 \overset{?}{=} 0$$
$$0 = 0$$

*Answer:* The equation $t^2 = 0$ has one solution: $\boldsymbol{t = 0}$.

*Example 3:* Find the roots of the equation $s^2 + 9 = 0$.

Write the equation in the form $x^2 = k$.

$$s^2 + 9 = 0 \quad \rightarrow \quad s^2 + 9 - 9 = 0 - 9$$
$$s^2 = -9$$

Since -9 is less than 0, then the equation $s^2 = -9$ has no real solutions or roots. There is no real number when squared gives -9.

*Answer:* The equation $s^2 + 9 = 0$ has no real solutions or roots.

*Example 4:* Find the solutions of the equation $(x - 4)^2 - 25 = 0$.

To solve $(x - 4)^2 - 25 = 0$, add 25 on both sides of the equation.

$$(x - 4)^2 - 25 + 25 = 0 + 25$$

The resulting equation is $(x - 4)^2 = 25$.

Solve the resulting equation.

$$(x - 4)^2 = 25 \quad \rightarrow \quad x - 4 = \pm\sqrt{25}$$
$$x - 4 = \pm 5$$



Solve for $x$ in the equation $x - 4 = \pm 5$.

$$x - 4 + 4 = \pm 5 + 4 \quad \rightarrow \quad x = \pm 5 + 4$$

The equation will result to two values of $x$.

| $x = 5 + 4$<br>$x = 9$ | $x = -5 + 4$<br>$x = -1$ |
|---|---|

Check the obtained values of $x$ against the original equation.

| For $x = 9$: | For $x = -1$: |
|---|---|
| $(x - 4)^2 - 25 = 0$ | $(x - 4)^2 - 25 = 0$ |
| $(9 - 4)^2 - 25 \stackrel{?}{=} 0$ | $(-1 - 4)^2 - 25 \stackrel{?}{=} 0$ |
| $5^2 - 25 \stackrel{?}{=} 0$ | $(-5)^2 - 25 \stackrel{?}{=} 0$ |
| $25 - 25 \stackrel{?}{=} 0$ | $25 - 25 \stackrel{?}{=} 0$ |
| $0 = 0$ | $0 = 0$ |

Both values of $x$ satisfy the given equation. So the equation $(x - 4)^2 - 25 = 0$ is true when $x = 9$ or when $x = -1$.

*Answer:* The equation $(x - 4)^2 - 25 = 0$ has two solutions: $\boldsymbol{x = 9}$ or $\boldsymbol{x = -1}$

Learn more about Solving Quadratic Equations by Extracting Square Roots through the WEB. You may open the following links.

http://2012books.lardbucket.org/books/beginning-algebra/s12-01-extracting-square-roots.html

http://www.purplemath.com/modules/solvquad2.htm

http://www.personal.kent.edu/~bosikiew/Algebra-handouts/solving-sqroot.pdf

**What to PROCESS:**

Your goal in this section is to apply previously learned mathematics concepts and principles in solving quadratic equations by extracting square roots. Use the mathematical ideas and the examples presented in the preceding section to answer the activities provided.



**Activity 6: Extract Me!!!**



*Directions:* Solve the following quadratic equations by extracting square roots. Answer the questions that follow.

1. $x^2 = 16$
2. $t^2 = 81$
3. $r^2 - 100 = 0$
4. $x^2 - 144 = 0$
5. $2s^2 = 50$
6. $4x^2 - 225 = 0$
7. $3h^2 - 147 = 0$
8. $(x - 4)^2 = 169$
9. $(k + 7)^2 = 289$
10. $(2s - 1)^2 = 225$

*Questions:*

a. How did you find the solutions of each equation?

b. What mathematics concepts or principles did you apply in finding the solutions? Explain how you applied these.

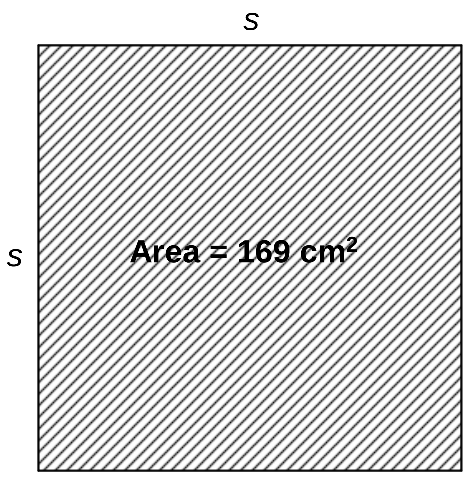
c. Compare your answers with those of your classmates. Did you arrive at the same solutions? If NOT, explain.

Was it easy for you to find the solutions of quadratic equations by extracting square roots? Did you apply the different mathematics concepts and principles in finding the solutions of each equation? I know you did!

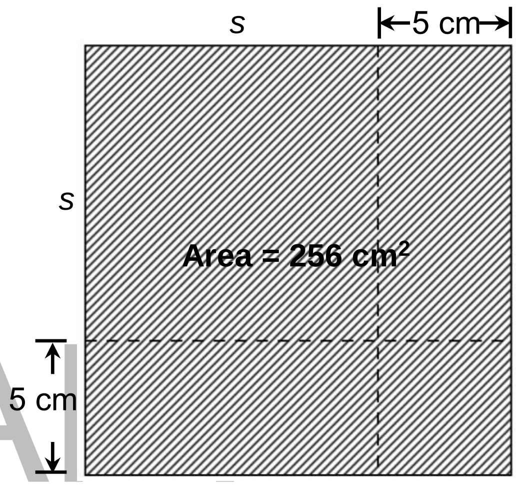
**Activity 7: What Does a Square Have?**

*Directions:* Write a quadratic equation that represents the area of each square. Then find the length of its side using the equation formulated. Answer the questions that follow.

1.




2.




*Questions:*

a. How did you come up with the equation that represents the area of each shaded region?

b. How did you find the length of side of each square?

c. Do all solutions to each equation represent the length of side of the square? Explain your answer.

In this section, the discussion was about solving quadratic equations by extracting square roots.

Go back to the previous section and compare your initial ideas with the discussion. How much of your initial ideas are found in the discussion? Which ideas are different and need revision?

Now that you know the important ideas about this topic, let's go deeper by moving on to the next section.

**What to REFLECT ON and FURTHER UNDERSTAND:**

Your goal in this section is to take a closer look at some aspects of the topic. You are going to think deeper and test further your understanding of solving quadratic equations by extracting square roots. After doing the following activities, you should be able to answer this important question: *How does finding solutions of quadratic equations facilitate in solving real-life problems and in making decisions?*

**Activity 8: Extract Then Describe Me!**

*Directions*: Solve each of the following quadratic equations by extracting square roots. Answer the questions that follow.

1. $3t^2 = 12$
2. $x^2 - 7 = 0$
3. $3r^2 = 18$
4. $x^2 = 150$
5. $x^2 = \frac{9}{16}$
6. $(s-4)^2 - 81 = 0$

*Questions:*

a. How did you find the roots of each equation?

b. Which equation did you find difficult to solve by extracting square roots? Why?

c. Which roots are rational? Which are not? Explain your answer.

d. How will you approximate those roots that are irrational?

Were you able to find and describe the roots of each equation? Were you able to approximate the roots that are irrational? I'm sure you did! Deepen further your understanding of solving quadratic equations by extracting square roots by doing the next activity.

**Activity 9: Intensify Your Understanding!**

*Directions*: Answer the following.

1. Do you agree that a quadratic equation has at most two solutions? Justify your answer and give examples.

2. Give examples of quadratic equations with (a) two real solutions, (b) one real solution, and (c) no real solution.

3. Sheryl says that the solutions of the quadratic equations $w^2 = 49$ and $w^2 + 49 = 0$ are the same. Do you agree with Sheryl? Justify your answer.

4. Mr. Cruz asked Emilio to construct a square table such that its area is 3 $m^2$. Is it possible for Emilio to construct such table using an ordinary tape measure? Explain your answer.

5. A 9 $ft.^2$ square painting is mounted with border on a square frame. If the total area of the border is 3.25 $ft.^2$, what is the length of a side of the frame?

In this section, the discussion was about your understanding of solving quadratic equations by extracting square roots.

What new realizations do you have about solving quadratic equations by extracting square roots? How would you connect this to real life? How would you use this in making decisions?

Now that you have a deeper understanding of the topic, you are ready to do the tasks in the next section.

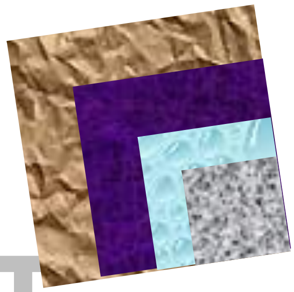
**What to TRANSFER:**

Your goal in this section is to apply your learning to real-life situations. You will be given a practical task in which you will demonstrate your understanding of solving quadratic equations by extracting square roots.

**Activity 10: What More Can I Do?**

*Directions:* Answer the following.

1. Describe quadratic equations with 2 solutions, 1 solution, and no solution. Give at least two examples for each.
2. Give at least five quadratic equations which can be solved by extracting square roots, then solve.
3. Collect square tiles of different sizes. Using these tiles, formulate quadratic equations that can be solved by extracting square roots. Find the solutions or roots of these equations.





How did you find the performance task? How did the task help you see the real world use of the topic?

**Summary/Synthesis/Generalization:**

This lesson was about solving quadratic equations by extracting square roots. The lesson provided you with opportunities to describe quadratic equations and solve these by extracting square roots. You were also able to find out how such equations are illustrated in real life. Moreover, you were given the chance to demonstrate your understanding of the lesson by doing a practical task. Your understanding of this lesson and other previously learned mathematics concepts and principles will enable you to learn about the wide applications of quadratic equations in real life.

**LEARNING MODULE**

**MATH 9**

## LESSON NO. 2B: SOLVING QUADRATIC EQUATIONS BY FACTORING

### What to KNOW:

Start Lesson 2B of this module by assessing your knowledge of the different mathematics concepts previously studied and your skills in performing mathematical operations. These knowledge and skills will help you in understanding solving quadratic equations by factoring. As you go through this lesson, think of this important question: *“How does finding solutions of quadratic equations facilitate in solving real-life problems and in making decisions?”* To find the answer, perform each activity. If you find any difficulty in answering the exercises, seek the assistance of your teacher or peers or refer to the modules you have gone over earlier. You may check your answers with your teacher.



### Activity 1: What Made Me?

*Directions*: Factor each of the following polynomials. Answer the questions that follow.

1. $2x^2 - 8x$
2. $-3s^2 + 9s$
3. $4x + 20x^2$
4. $5t - 10t^2$
5. $s^2 + 8s + 12$
6. $x^2 - 10x + 21$
7. $x^2 + 5x - 6$
8. $4r^2 + 20r + 25$
9. $9t^2 - 4$
10. $2x^2 + 3x - 14$

*Questions:*

a. How did you factor each polynomial?

b. What factoring technique did you use to come up with the factors of each polynomial? Explain how you used this technique.

c. How would you know if the factors you got are the correct ones?

d. Which of the polynomials did you find difficult to factor? Why?

How did you find the activity? Were you able to recall and apply the different mathematics concepts or principles in factoring polynomials? I'm sure you were. In the next activity, you will be representing a situation using a mathematical sentence. This mathematical sentence will be used to satisfy the conditions of the given situation.

**Activity 2: The Manhole**

*Directions*: Use the situation below to answer the questions that follow.

*A rectangular metal manhole with an area of 0.5 $m^2$ is situated along a cemented pathway. The length of the pathway is 8 m longer than its width.*

1. Draw a diagram to illustrate the given situation.

2. How are you going to represent the length and the width of the pathway? How about its area?

3. What expression would represent the area of the cemented portion of the pathway?

4. Suppose the area of the cemented portion of the pathway is 19.5 $m^2$. What equation would describe its area?

5. How will you find the length and the width of the pathway?

The activity you have just done shows how a real-life situation can be represented by a mathematical sentence. Were you able to represent the given situation by an equation? Do you now have an idea on how to use the equation in finding the length and the width of the pathway? To further give you ideas in solving the equation or other similar equations, perform the next activity.

**Activity 3: Why is the Product Zero?**

*Directions*: Use the equations below to answer the following questions.

| | | |
|---|---|---|
| $x + 7 = 0$ | $x - 4 = 0$ | $(x + 7)(x - 4) = 0$ |

1. How would you compare the three equations?

2. What value(s) of $x$ would make each equation true?

3. How would you know if the value of $x$ that you got satisfies each equation?

4. Compare the solutions of the given equations. What statement can you make?

5. Are the solutions of $x + 7 = 0$ and $x - 4 = 0$ the same as the solutions of $(x + 7)(x - 4) = 0$? Why?

6. How would you interpret the meaning of the equation $(x + 7)(x - 4) = 0$?



How did you find the preceding activities? Are you ready to learn about solving quadratic equations by factoring? I'm sure you are!!! From the activities done, you were able to find the factors of polynomials, represent a real-life situation by a mathematical statement, and interpret zero product. But how does finding solutions of quadratic equations facilitate in solving real-life problems and in making decisions? You will find these out in the activities in the next section. Before doing these activities, read and understand first some important notes on solving quadratic equations by factoring and the examples presented.

Some quadratic equations can be solved easily by factoring. To solve such quadratic equations, the following procedure can be followed.

1. Transform the quadratic equation into standard form if necessary.

2. Factor the quadratic expression.

3. Apply the zero product property by setting each factor of the quadratic expression equal to 0.

**Zero Product Property**

If the product of two real numbers is zero, then either of the two is equal to zero or both numbers are equal to zero.

4. Solve each resulting equation.

5. Check the values of the variable obtained by substituting each in the original equation.

*Example 1:* Find the solutions of $x^2 + 9x = -8$ by factoring.

a. Transform the equation into standard form $ax^2 + bx + c = 0$.

$x^2 + 9x = -8 \quad \rightarrow \quad x^2 + 9x + 8 = 0$

b. Factor the quadratic expression $x^2 + 9x + 8$.

$x^2 + 9x + 8 = 0 \quad \rightarrow \quad (x + 1)(x + 8) = 0$

c. Apply the zero product property by setting each factor of the quadratic expression equal to 0.

$(x + 1)(x + 8) = 0 \quad \rightarrow \quad x + 1 = 0; \quad x + 8 = 0$



d. Solve each resulting equation.

$x + 1 = 0 \quad \rightarrow \quad x + 1 - 1 = 0 - 1$
$\boldsymbol{x = -1}$

$x + 8 = 0 \quad \rightarrow \quad x + 8 - 8 = 0 - 8$
$\boldsymbol{x = -8}$

e. Check the values of the variable obtained by substituting each in the equation $x^2 + 9x = -8$.

For $x = -1$:

$$x^2 + 9x = -8$$
$$(-1)^2 + 9(-1) \stackrel{?}{=} -8$$
$$1 - 9 \stackrel{?}{=} -8$$
$$-8 = -8$$

For $x = -8$:

$$x^2 + 9x = -8$$
$$(-8)^2 + 9(-8) \stackrel{?}{=} -8$$
$$64 - 72 \stackrel{?}{=} -8$$
$$-8 = -8$$

Both values of $x$ satisfy the given equation. So the equation $x^2 + 9x = -8$ is true when $x = -1$ or when $x = -8$.

*Answer:* The equation $x^2 + 9x = -8$ has two solutions: $\boldsymbol{x = -1}$ or $\boldsymbol{x = -8}$

*Example 2:* Solve $9x^2 - 4 = 0$ by factoring.

To solve the equation, factor the quadratic expression $9x^2 - 4$.

$$9x^2 - 4 = 0 \quad \rightarrow \quad (3x + 2)(3x - 2) = 0$$

Set each factor to 0.

$$3x + 2 = 0; \quad 3x - 2 = 0$$

Solve each resulting equation.

| | |
|---|---|
| $3x + 2 = 0 \rightarrow 3x + 2 - 2 = 0 - 2$ <br> $3x = -2$ <br> $\frac{3x}{3} = \frac{-2}{3}$ <br> $x = -\frac{2}{3}$ | $3x - 2 = 0 \rightarrow 3x - 2 + 2 = 0 + 2$ <br> $3x = 2$ <br> $\frac{3x}{3} = \frac{2}{3}$ <br> $x = \frac{2}{3}$ |

Check the values of the variable obtained by substituting each in the equation $9x^2 - 4 = 0$.

| | |
|---|---|
| For $x = \frac{-2}{3}$: <br> $9x^2 - 4 = 0$ <br> $9\left(\frac{-2}{3}\right)^2 - 4 \stackrel{?}{=} 0$ <br> $9\left(\frac{4}{9}\right) - 4 \stackrel{?}{=} 0$ <br> $4 - 4 \stackrel{?}{=} 0$ <br> $0 = 0$ | For $x = \frac{2}{3}$: <br> $9x^2 - 4 = 0$ <br> $9\left(\frac{2}{3}\right)^2 - 4 \stackrel{?}{=} 0$ <br> $9\left(\frac{4}{9}\right) - 4 \stackrel{?}{=} 0$ <br> $4 - 4 \stackrel{?}{=} 0$ <br> $0 = 0$ |

Both values of $x$ satisfy the given equation. So the equation $9x^2 - 4 = 0$ is true when $x = \frac{-2}{3}$ or when $x = \frac{2}{3}$.

*Answer:* The equation $9x^2 - 4 = 0$ has two solutions: $\boldsymbol{x = \frac{-2}{3}}$ or $\boldsymbol{x = \frac{2}{3}}$

Learn more about Solving Quadratic Equations by Factoring through the WEB. You may open the following links.

http://2012books.lardbucket.org/books/beginning-algebra/s09-06-solving-equations-by-factoring.html

http://www.purplemath.com/modules/solvquad.htm

http://www.webmath.com/quadtri.html

http://www.mathwarehouse.com/quadratic/solve-quadratic-equation-by-factoring.php

**What to PROCESS:**

Your goal in this section is to apply previously learned mathematics concepts and principles in solving quadratic equations by factoring. Use the mathematical ideas and the examples presented in the preceding section to answer the activities provided.



**Activity 4: Factor Then Solve!**

*Directions*: Solve the following quadratic equations by factoring. Answer the questions that follow.

1. $x^2 + 7x = 0$
2. $6s^2 + 18s = 0$
3. $t^2 + 8t + 16 = 0$
4. $x^2 - 10x + 25 = 0$
5. $h^2 + 6h = 16$
6. $x^2 - 14 = 5x$
7. $11r + 15 = -2r^2$
8. $x^2 - 25 = 0$
9. $81 - 4x^2 = 0$
10. $4s^2 + 9 = 12s$

*Questions:*

d. How did you find the solutions of each equation?

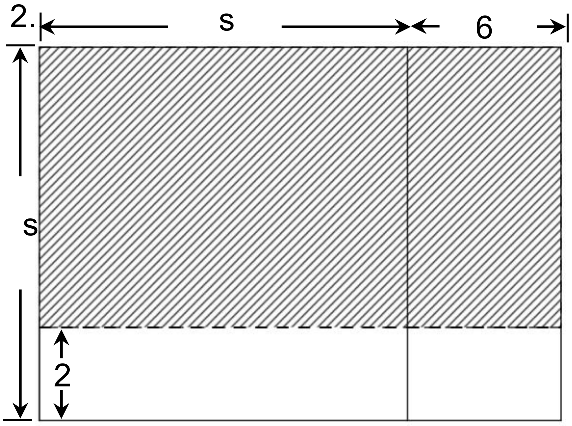
e. What mathematics concepts or principles did you apply in finding the solutions? Explain how you applied these.

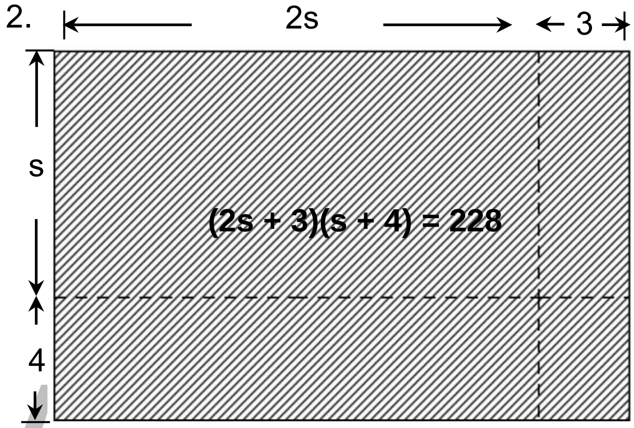
f. Compare your answers with those of your classmates. Did you arrive at the same solutions? If NOT, explain.

Was it easy for you to find the solutions of quadratic equations by factoring? Did you apply the different mathematics concepts and principles in finding the solutions of each equation? I know you did!

**Activity 5: What Must Be My Length and Width?**

*Directions:* The quadratic equation given describes the area of the shaded region of each figure. Use the equation to find the length and width of the figure. Answer the questions that follow.






*Questions:*

a. How did you find the length and width of each figure?
b. Can all solutions to each equation be used to determine the length and width of each figure? Explain your answer.



In this section, the discussion was about solving quadratic equations by factoring.

Go back to the previous section and compare your initial ideas with the discussion. How much of your initial ideas are found in the discussion? Which ideas are different and need revision?

Now that you know the important ideas about this topic, let's go deeper by moving on to the next section.

**What to REFLECT ON and FURTHER UNDERSTAND:**

Your goal in this section is to take a closer look at some aspects of the topic. You are going to think deeper and test further your understanding of solving quadratic equations by factoring. After doing the following activities, you should be able to answer this important question: *How does finding solutions of quadratic equations facilitate in solving real-life problems and in making decisions?*

**Activity 6: How Well Did I Understand?**

*Directions*: Answer each of the following.

1. Which of the following quadratic equations may be solved more appropriately by factoring? Explain your answer.

   a. $2x^2 = 72$ c. $w^2 - 64 = 0$

   b. $t^2 + 12t + 36 = 0$ d. $2s^2 + 8s - 10 = 0$

2. Patricia says that it's more appropriate to use the method of factoring than extracting square roots in solving the quadratic equation $4x^2 - 9 = 0$. Do you agree with Patricia? Explain your answer.

3. Do you agree that not all quadratic equations can be solved by factoring? Justify your answer by giving examples.

4. Find the solutions of each of the following quadratic equations by factoring. Explain how you arrived at you answer.

   a. $(x + 3)^2 = 25$ c. $(2t - 3)^2 = 2t^2 + 5t - 26$

   b. $(s + 4)^2 = -2s$ d. $3(x + 2)^2 = 2x^2 + 3x - 8$

5. Do you agree that $x^2 + 5x - 14 = 0$ and $14 - 5x - x^2 = 0$ have the same solutions? Justify your answer.

6. Show that the equation $(x - 4)^2 = 9$ can be solved both by factoring and extracting square roots.

7. A computer manufacturing company would like to come up with a new laptop computer such that its monitor is 80 square inches smaller than the present ones. Suppose the length of the monitor of the larger computer is 5 inches longer than its width and the area of the smaller computer is 70 square inches. What are the dimensions of the monitor of the larger computer?

In this section, the discussion was about your understanding of solving quadratic equations by factoring.

What new insights do you have about solving quadratic equations by factoring? How would you connect this to real life? How would you use this in making decisions?

Now that you have a deeper understanding of the topic, you are ready to do the tasks in the next section.

**What to TRANSFER:**

Your goal in this section is to apply your learning to real-life situations. You will be given a practical task which will demonstrate your understanding of solving quadratic equations by factoring.

**Activity 7: Meet My Demands!!!**

*Directions:* Answer the following.

*Mr. Lakandula would like to increase his production of milkfish (bangus) due to its high demand in the market. He is thinking of making a larger fishpond in his 8000 sq m lot near a river. Help Mr. Lakandula by making a sketch plan of the fishpond to be made. Out of the given situation and the sketch plan made, formulate as many quadratic equations then solve by factoring. You may use the rubric in the next page to rate your work.*

Rubric for Sketch Plan and Equations Formulated and Solved

| 4 | 3 | 2 | 1 |
|---|---|---|---|
| The sketch plan is accurately made, presentable, and appropriate. | The sketch plan is accurately made and appropriate. | The sketch plan is not accurately made but appropriate. | The sketch plan is made but not appropriate. |
| Quadratic equations are accurately formulated and solved correctly. | Quadratic equations are accurately formulated but not all are solved correctly. | Quadratic equations are accurately formulated but are not solved correctly. | Quadratic equations are accurately formulated but are not solved. |



How did you find the performance task? How did the task help you see the real world use of the topic?

**Summary/Synthesis/Generalization:**

This lesson was about solving quadratic equations by factoring. The lesson provided you with opportunities to describe quadratic equations and solve these by factoring. You were able to find out also how such equations are illustrated in real life. Moreover, you were given the chance to demonstrate your understanding of the lesson by doing a practical task. Your understanding of this lesson and other previously learned mathematics concepts and principles will facilitate your learning of the wide applications of quadratic equations in real life.

## LEARNING MODULE

## MATH 9

### LESSON NO. 2C: SOLVING QUADRATIC EQUATIONS BY COMPLETING THE SQUARE

**What to KNOW:**

Start Lesson 2C of this module by assessing your knowledge of the different mathematics concepts previously studied and your skills in performing mathematical operations. These knowledge and skills will help you in understanding Solving Quadratic Equations by Completing the Square. As you go through this lesson, think of this important question: *"How does finding solutions of quadratic equations facilitate in solving real-life problems and in making decisions?"* To find the answer, perform each activity. If you find any difficulty in answering the exercises, seek the assistance of your teacher or peers or refer to the modules you have gone over earlier. You may check your answers with your teacher.



**Activity 1: How Many Solutions Do I Have?**

*Directions*: Find the solution/s of each of the following equations. Answer the questions that follow.

1. $x + 12 = 17$
2. $s + 15 = -9$
3. $r - 25 = 12$
4. $x - \frac{5}{6} = 3$
5. $t + \frac{4}{7} = 5$
6. $x - \frac{3}{4} = \frac{1}{2}$
7. $(x + 10)^2 = 36$
8. $(w - 9)^2 = 12$
9. $\left(k + \frac{1}{2}\right)^2 = \frac{9}{16}$
10. $\left(h - \frac{3}{5}\right)^2 = \frac{1}{2}$

*Questions:*

a. How did you find the solution(s) of each equation?

b. Which of the equations has only one solution? Why?

c. Which of the equations has two solutions? Why?

d. Which of the equations has solutions that are irrational? Why?

e. Were you able to simplify those solutions that are irrational? Why?

f. How did you write those irrational solutions?

> How did you find the activity? Were you able to recall and apply the different mathematics concepts or principles in finding the solution/s of each equation? I'm sure you did! In the next activity, you will be expressing a perfect square trinomial as a square of a binomial. I know that you already have an idea on how to do this. This activity will help you in solving quadratic equations by completing the square.



**Activity 2: Perfect Square Trinomial to Square of a Binomial**

*Directions*: Express each of the following perfect square trinomials as a square of a binomial. Answer the questions that follow.

1. $x^2 + 4x + 4$
2. $t^2 + 12t + 36$
3. $s^2 + 10s + 25$
4. $x^2 - 16x + 64$
5. $h^2 - 14h + 49$
6. $x^2 + 18x + 81$
7. $t^2 + \frac{2}{3}t + \frac{1}{9}$
8. $r^2 - 7r + \frac{49}{4}$
9. $s^2 + \frac{3}{4}s + \frac{9}{64}$
10. $w^2 - 5w + \frac{25}{4}$

*Questions:*

a. How do you describe a perfect square trinomial?

b. How did you express each perfect square trinomial as the square of a binomial?

c. What mathematics concepts or principles did you apply to come up with your answer? Explain how you applied these.

d. Compare your answer with those of your classmates. Did you get the same answer? If NOT, explain.

e. Observe the terms of each trinomial. How is the third term related to the coefficient of the middle term?

f. Is there an easy way of expressing a perfect square trinomial as a square of a binomial? If there is any, explain how.

> Were you able to express each perfect square trinomial as a square of a binomial? I'm sure you did! Let us further strengthen your knowledge and skills in mathematics particularly in writing perfect square trinomials by doing the next activity.



**Activity 3: Make It Perfect!!!**

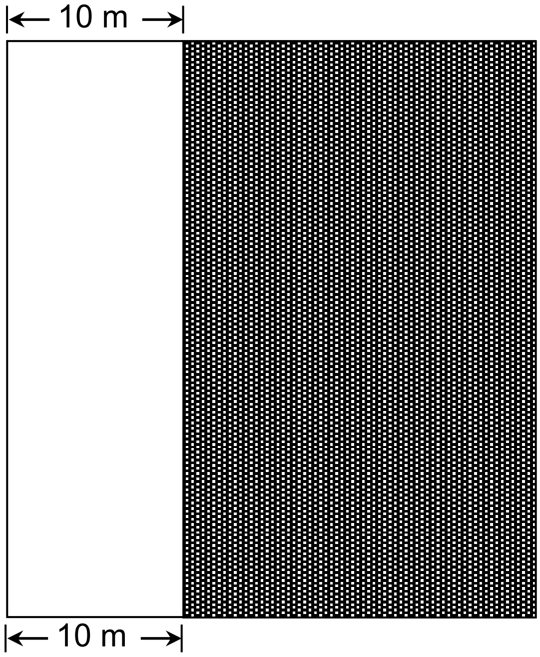
*Directions*: Determine a number that must be added to make each of the following a perfect square trinomial. Explain how you arrived at your answer.

1. $x^2 + 2x + \_\_\_\_$
2. $t^2 + 20t + \_\_\_\_$
3. $r^2 - 16r + \_\_\_\_$
4. $r^2 + 24r + \_\_\_\_$
5. $x^2 - 30x + \_\_\_\_$
6. $x^2 + 11x + \_\_\_\_$
7. $x^2 - 15x + \_\_\_\_$
8. $w^2 + 21w + \_\_\_\_$
9. $s^2 + \frac{2}{3}s + \_\_\_\_$
10. $h^2 - \frac{3}{4}h + \_\_\_\_$

> Was it easy for you to determine the number that must be added to the terms of a polynomial to make it a perfect square trinomial? Were you able to figure out how it can be easily done? In the next activity, you will be representing a situation using a mathematical sentence. Such a mathematical sentence will be used to satisfy the conditions of the given situation.

**Activity 4: Finish the Contract!**

*Directions:* The shaded region of the diagram at the right shows the portion of a square-shaped car park that is already cemented. The area of the cemented part is 600 $m^2$. Use the diagram to answer the following questions.




1. How would you represent the length of the side of the car park? How about the width of the cemented portion?

2. What equation would represent the area of the cemented part of the car park?

3. Using the equation formulated, how are you going to find the length of a side of the car park?



How did you find the preceding activities? Are you ready to learn about solving quadratic equations by completing the square? I'm sure you are!!! From the activities done, you were able to solve equations, express a perfect square trinomial as a square of a binomial, write perfect square trinomials, and represent a real-life situation by a mathematical sentence. But how does finding solutions of quadratic equations facilitate in solving real-life problems and in making decisions? You will find these out in the activities in the next section. Before doing these activities, read and understand first some important notes on Solving Quadratic Equations by Completing the Square and the examples presented.

Extracting square roots and factoring are usually used to solve quadratic equations of the form $ax^2 - c = 0$. If the factors of the quadratic expression of $ax^2 + bx + c = 0$ are determined, then it is more convenient to use factoring to solve it.

Another method of solving quadratic equation is by completing the square. This method involves transforming the quadratic equation $\mathbf{a}\boldsymbol{x}^2 + \mathbf{b}\boldsymbol{x} + \mathbf{c} = \mathbf{0}$ into the form $(\boldsymbol{x} - \boldsymbol{h})^2 = \boldsymbol{k}$, where $\boldsymbol{k} \geq \mathbf{0}$. Can you tell why the value of $\boldsymbol{k}$ should be positive?

To solve the quadratic equation $\mathbf{a}\boldsymbol{x}^2 + \mathbf{b}\boldsymbol{x} + \mathbf{c} = \mathbf{0}$ by completing the square, the following steps can be followed:

1. Divide both sides of the equation by **a** then simplify.

2. Write the equation such that the terms with variables are on the left side of the equation and the constant term is on the right side.

3. Add the square of one-half of the coefficient of $x$ on both sides of the resulting equation. The left side of the equation becomes a perfect square trinomial.
4. Express the perfect square trinomial on the left side of the equation as a square of a binomial.
5. Solve the resulting quadratic equation by extracting the square root.
6. Solve the resulting linear equations.
7. Check the solutions obtained against the original equation.

*Example 1:* Solve the quadratic equation $2x^2 + 8x - 10 = 0$ by completing the square.

Divide both sides of the equation by 2 then simplify.

$$2x^2 + 8x - 10 = 0 \quad \rightarrow \quad \frac{2x^2 + 8x - 10}{2} = \frac{0}{2}$$
$$x^2 + 4x - 5 = 0$$



Add 5 to both sides of the equation then simplify.

$$x^2 + 4x - 5 = 0 \quad \rightarrow \quad x^2 + 4x - 5 + 5 = 0 + 5$$
$$x^2 + 4x = 5$$

Add to both sides of the equation the square of one-half of 4.

$$\boxed{\frac{1}{2}(4) = 2 \quad \rightarrow \quad 2^2 = 4}$$

$$x^2 + 4x = 5 \quad \rightarrow \quad x^2 + 4x + 4 = 5 + 4$$
$$x^2 + 4x + 4 = 9$$

Express $x^2 + 4x + 4$ as a square of a binomial.

$$x^2 + 4x + 4 = 9 \quad \rightarrow \quad (x + 2)^2 = 9$$

Solve $(x + 2)^2 = 9$ by extracting the square root.

$$(x + 2)^2 = 9 \quad \rightarrow \quad x + 2 = \pm\sqrt{9}$$
$$x + 2 = \pm 3$$

Solve the resulting linear equations.

| | |
|---|---|
| $x + 2 = 3$<br>$x + 2 - 2 = 3 - 2$<br>$\mathbf{x = 1}$ | $x + 2 = -3$<br>$x + 2 - 2 = -3 - 2$<br>$\mathbf{x = -5}$ |

Check the solutions obtained against the original equation
$2x^2 + 8x - 10 = 0$.

| For $x = 1$: | For $x = -5$: |
|---|---|
| $2x^2 + 8x - 10 = 0$ | $2x^2 + 8x - 10 = 0$ |
| $2(1)^2 + 8(1) - 10 \overset{?}{=} 0$ | $2(-5)^2 + 8(-5) - 10 \overset{?}{=} 0$ |
| $2(1) + 8 - 10 \overset{?}{=} 0$ | $2(25) - 40 - 10 \overset{?}{=} 0$ |
| $2 + 8 - 10 \overset{?}{=} 0$ | $50 - 40 - 10 \overset{?}{=} 0$ |
| $0 = 0$ | $0 = 0$ |

Both values of $x$ satisfy the given equation. So the equation $2x^2 + 8x - 10 = 0$ is true when $x = 1$ or when $x = -5$.

*Answer:* The equation $2x^2 + 8x - 10 = 0$ has two solutions:
***x* = 1** or ***x* = -5**

*Example 2:* Find the solutions of the equation $x^2 + 3x - 18 = 0$ by completing the square.

Add 18 to both sides of the equation then simplify.

$$x^2 + 3x - 18 = 0 \quad \rightarrow \quad x^2 + 3x - 18 + 18 = 0 + 18$$

$$x^2 + 3x = 18$$

Add to both sides of the equation the square of one-half of 3.

$$\boxed{\frac{1}{2}(3) = \frac{3}{2} \quad \rightarrow \quad \left(\frac{3}{2}\right)^2 = \frac{9}{4}}$$

$$x^2 + 3x = 18 \rightarrow \quad x^2 + 3x + \frac{9}{4} = 18 + \frac{9}{4}$$

$$x^2 + 3x + \frac{9}{4} = \frac{72}{4} + \frac{9}{4} \quad \rightarrow \quad x^2 + 3x + \frac{9}{4} = \frac{81}{4}$$

Express $x^2 + 3x + \frac{9}{4}$ as a square of a binomial.

$$x^2 + 3x + \frac{9}{4} = \frac{81}{4} \quad \rightarrow \quad \left(x + \frac{3}{2}\right)^2 = \frac{81}{4}$$

Solve $\left(x + \frac{3}{2}\right)^2 = \frac{81}{4}$ by extracting the square root.

$$\left(x+\frac{3}{2}\right)^2=\frac{81}{4} \quad \rightarrow \quad x+\frac{3}{2}=\pm\sqrt{\frac{81}{4}}$$

$$x+\frac{3}{2}=\pm\frac{9}{2}$$

Solve the resulting linear equations.

| | |
|---|---|
| $x+\frac{3}{2}=\frac{9}{2}$<br>$x+\frac{3}{2}-\frac{3}{2}=\frac{9}{2}-\frac{3}{2}$<br>$x=\frac{6}{2}$<br>***x* = 3** | $x+\frac{3}{2}=-\frac{9}{2}$<br>$x+\frac{3}{2}-\frac{3}{2}=-\frac{9}{2}-\frac{3}{2}$<br>$x=-\frac{12}{2}$<br>***x* = -6** |

Check the solutions obtained against the equation $x^2+3x-18=0$.

| | |
|---|---|
| For $x$ = 3:<br>$x^2+3x-18=0$<br>$(3)^2+3(3)-18\overset{?}{=}0$<br>$9+9-18\overset{?}{=}0$<br>$0=0$ | For $x$ = -6:<br>$x^2+3x-18=0$<br>$(-6)^2+3(-6)-18\overset{?}{=}0$<br>$36-18-18\overset{?}{=}0$<br>$0=0$ |



Both values of $x$ satisfy the given equation. So the equation $x^2+3x-18=0$ is true when $x$ = 3 or when $x$ = -6.

*Answer:* The equation $x^2+3x-18=0$ has two solutions: ***x* = 3** or ***x* = -6**

*Example 3:* Find the solutions of $x^2 - 6x - 41 = 0$ by completing the square.

Add 41 to both sides of the equation then simplify.

$$x^2 - 6x - 41 = 0 \quad \rightarrow \quad x^2 - 6x - 41 + 41 = 0 + 41$$

$$x^2 - 6x = 41$$

Add to both sides of the equation the square of one-half of -6.

$$\frac{1}{2}(-6) = -3 \quad \rightarrow \quad (-3)^2 = 9$$

$x^2 - 6x = 41 \quad \rightarrow \quad x^2 - 6x + 9 = 41 + 9$

$x^2 - 6x + 9 = 50$

Express $x^2 - 6x + 9$ as a square of a binomial.

$x^2 - 6x + 9 = 50 \quad \rightarrow \quad (x - 3)^2 = 50$

Solve $(x - 3)^2 = 50$ by extracting the square root.

$(x - 3)^2 = 50 \quad \rightarrow \quad x - 3 = \pm\sqrt{50}$

$\pm\sqrt{50}$ can be expressed as $\pm\sqrt{25 \cdot 2}$ or $\pm\sqrt{25} \cdot \sqrt{2}$. Notice that 25 is a perfect square. So, $\pm\sqrt{25} \cdot \sqrt{2}$ can be simplified further to $\pm 5 \cdot \sqrt{2}$.

Hence, $x - 3 = \pm\sqrt{50}$ is the same as $x - 3 = \pm 5\sqrt{2}$.

Solve the resulting linear equations.

$$x - 3 = 5\sqrt{2}$$
$$x - 3 + 3 = 5\sqrt{2} + 3$$
$$\boldsymbol{x = 3 + 5\sqrt{2}}$$

$$x - 3 = -5\sqrt{2}$$
$$x - 3 + 3 = -5\sqrt{2} + 3$$
$$\boldsymbol{x = 3 - 5\sqrt{2}}$$

Check the solutions obtained against the equation $x^2 - 6x - 41 = 0$.

For $x = 3 + 5\sqrt{2}$:

$$x^2 - 6x - 41 = 0$$
$$\left(3 + 5\sqrt{2}\right)^2 - 6\left(3 + 5\sqrt{2}\right) - 41 \stackrel{?}{=} 0$$
$$9 + 30\sqrt{2} + 50 - 18 - 30\sqrt{2} - 41 \stackrel{?}{=} 0$$
$$0 = 0$$

For $x = 3 - 5\sqrt{2}$:

$$x^2 - 6x - 41 = 0$$

$$\left(3 - 5\sqrt{2}\right)^2 - 6\left(3 - 5\sqrt{2}\right) - 41 \stackrel{?}{=} 0$$

$$9 - 30\sqrt{2} + 50 - 18 + 30\sqrt{2} - 41 \stackrel{?}{=} 0$$

$$0 = 0$$

Both values of $x$ satisfy the given equation. So the equation $x^2 - 6x - 40 = 0$ is true when $x = 3 + 5\sqrt{2}$ or when $x = 3 - 5\sqrt{2}$.

*Answer:* The equation $x^2 - 6x - 40 = 0$ has two solutions: $\boldsymbol{x = 3 + 5\sqrt{2}}$ or $\boldsymbol{x = 3 - 5\sqrt{2}}$

Learn more about Solving Quadratic Equations by Completing the Square through the WEB. You may open the following links.

http://www.purplemath.com/modules/sqrquad.htm

http://2012books.lardbucket.org/books/beginning-algebra/s12-02-completing-the-square.html

http://www.regentsprep.org/Regents/math/algtrig/ATE12/indexATE12.htm

http://www.mathsisfun.com/algebra/completing-square.html



**What to PROCESS:**

Your goal in this section is to apply the key concepts of solving quadratic equations by completing the square. Use the mathematical ideas and the examples presented in the preceding section to answer the activities provided.

**Activity 5: Complete Me!**

*Directions*: Find the solutions of each of the following quadratic equations by completing the square. Answer the questions that follow.

1. $x^2 - 2x = 3$
2. $s^2 + 4s - 21 = 0$
3. $t^2 + 10t + 9 = 0$
4. $x^2 + 14x = 32$
5. $r^2 - 10r = -17$
6. $4x^2 - 32x = -28$
7. $x^2 - 5x - 6 = 0$
8. $m^2 + 7m - \frac{51}{4} = 0$
9. $r^2 + 4r = -1$
10. $w^2 + 6w - 11 = 0$

*Questions:*

a. How did you find the solutions of each equation?

b. What mathematics concepts or principles did you apply in finding the solutions? Explain how you applied these.

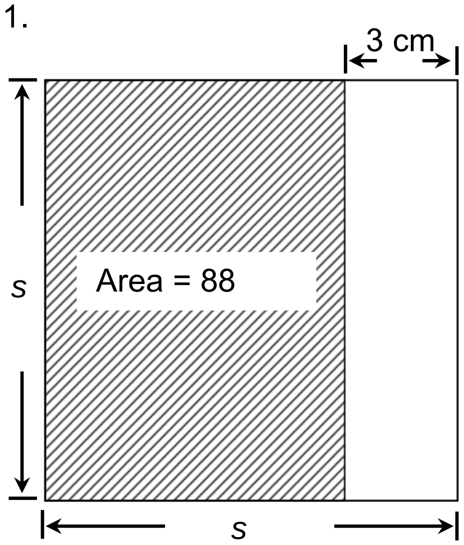
c. Compare your answers with those of your classmates. Did you arrive at the same answers? If NOT, explain.

Was it easy for you to find the solutions of quadratic equations by completing the square? Did you apply the different mathematics concepts and principles in finding the solutions of each equation? I know you did!

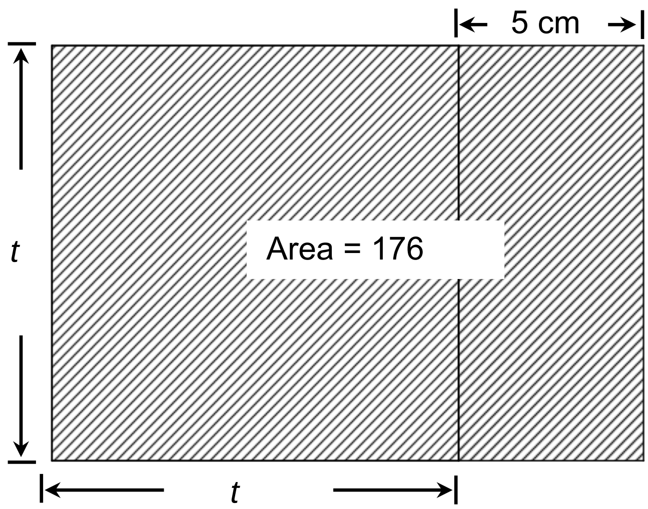
**Activity 6: Represent then Solve!**

*Directions*: Using each figure, write a quadratic equation that represents the area of the shaded region. Then find the solutions to the equation by completing the square. Answer the questions that follow.

1.




2.




*Questions:*

a. How did you come up with the equation that represents the area of each shaded region?

b. How did you find the solution/s of each equation?

c. Do all solutions to each equation represent a particular measure of each figure? Explain your answer.



Were you able to come up with the right representations of the area of the shaded region of each figure? Were you able to solve the equations formulated and obtain the appropriate measure that would describe each figure?

In this section, the discussion was about solving quadratic equations by completing the square.

Go back to the previous section and compare your initial ideas with the discussion. How much of your initial ideas are found in the discussion? Which ideas are different and need revision?

Now that you know the important ideas about this topic, let's go deeper by moving on to the next section.

**What to REFLECT ON and FURTHER UNDERSTAND:**

> Your goal in this section is to take a closer look at some aspects of the topic. You are going to think deeper and test further your understanding of solving quadratic equations by completing the square. After doing the following activities, you should be able to answer this important question: *How does finding solutions of quadratic equations facilitate in solving real-life problems and in making decisions?*

**Activity 7: What Solving Quadratic Equations by Completing the Square Means to Me…**

*Directions*: Answer the following.

1. Karen wants to use completing the square in solving the quadratic equation $4x^2 - 25 = 0$. Can she use it in finding the solutions of the equation? Explain why or why not.

2. Do you agree that any quadratic equation can be solved by completing the square? Justify your answer.

3. If you are to choose between completing the square and factoring in finding the solutions of each of the following equations, which would you choose? Explain and solve the equation using your preferred method.

   a. $4x^2 - 20x = 11$  b. $x^2 + 7x + 12 = 0$

4. Gregorio solved the equation $2x^2 - 8x + 15 = 0$. The first few parts of his solution are shown below.

   $2x^2 - 8x + 15 = 0 \rightarrow 2x^2 - 8x + 15 - 15 = 0 - 15$

   $2x^2 - 8x = -15$

   $\boxed{\frac{1}{2}(-8) = -4\ ;\ (-4)^2 = 16}$

   $2x^2 - 8x + 16 = -15 + 16$

   Do you think Gregorio arrived at the correct solutions of the equation? Justify your answer.

5. An open box is to be formed out of a rectangular piece of cardboard whose length is 8 cm longer than its width. To form the box, a square of side 4 cm will be removed from each corner of the cardboard. Then the edges of the remaining cardboard will be turned up.
   a. Draw a diagram to illustrate the given situation.

   b. How would you represent the dimensions of the cardboard?

   c. What expressions represent the length, width, and height of the box?

d. If the box is to hold 448 cm$^3$, what mathematical sentence would represent the given situation?

e. Using the mathematical sentence formulated, how are you going to find the dimensions of the rectangular piece of cardboard?

f. What are the dimensions of the rectangular piece of cardboard?

g. What is the length of the box? How about its width and height?

6. From 2006 through 2012, the average weekly income of an employee in a certain company is estimated by the quadratic expression $0.18n^2 + 6.48n + 3240$ , where $n$ is the number of years after 2006. In what year did the average weekly income of an employee become Php3268.80?

> In this section, the discussion was about your understanding of solving quadratic equations by completing the square.
>
> What new insights do you have about solving quadratic equations by completing the square? How would you connect this to real life? How would you use this in making decisions?

> Now that you have a deeper understanding of the topic, you are ready to do the tasks in the next section.



**What to TRANSFER:**

> Your goal in this section is to apply your learning to real-life situations. You will be given a practical task which will demonstrate your understanding of solving quadratic equations by completing the square.

**Activity 8: Design Packaging Boxes!!!**

*Directions:* Perform the following.

A. Designing Open Boxes
   1. Make sketch plans of 5 rectangular open boxes such that:
      a. the heights of the boxes are the same; and

      b. the boxes can hold 240 cm$^3$, 270 cm$^3$, 504 cm$^3$, 810 cm$^3$, and 468 cm$^3$, respectively.

   2. Write a quadratic equation that would represent the volume of each box.

3. Solve each quadratic equation by completing the square to determine the dimensions of the materials to be used in constructing each box.

B. Designing Covers of the Open Boxes
   1. Make sketch plans of covers of the open boxes in Part A such that:
      a. the heights of the covers are the same; and

      b. the base of each cover is rectangular.

   2. Write a quadratic equation that would represent the volume of the box's cover.

   3. Solve each quadratic equation by completing the square to determine the dimensions of the materials to be used in constructing the box's cover.

Rubric for Sketch Plan and Equations Formulated and Solved

| 4 | 3 | 2 | 1 |
|---|---|---|---|
| The sketch plan is accurately made, presentable, and appropriate. | The sketch plan is accurately made and appropriate. | The sketch plan is not accurately made but appropriate. | The sketch plan is made but not appropriate. |
| Quadratic equations are accurately formulated and solved correctly. | Quadratic equations are accurately formulated but not all are solved correctly. | Quadratic equations are accurately formulated but are not solved correctly. | Quadratic equations are accurately formulated but are not solved. |



How did you find the performance task? How did the task help you see the real world use of the topic?

**Summary/Synthesis/Generalization:**

This lesson was about solving quadratic equations by completing the square. The lesson provided you with opportunities to describe quadratic equations and solve these by completing the square. You were able to find out also how such equations are illustrated in real life. Moreover, you were given the chance to demonstrate your understanding of the lesson by doing a practical task. Your understanding of this lesson and other previously learned mathematics concepts and principles will facilitate your learning of the wide applications of quadratic equations in real life.

**LEARNING MODULE**

**MATH 9**

## LESSON NO. 2D: SOLVING QUADRATIC EQUATIONS BY USING THE QUADRATIC FORMULA

### What to KNOW:

Start Lesson 2D of this module by assessing your knowledge of the different mathematics concepts previously studied and your skills in performing mathematical operations. These knowledge and skills will help you in understanding solving quadratic equations by using the quadratic formula. As you go through this lesson, think of this important question: *"How does finding solutions of quadratic equations facilitate in solving real-life problems and in making decisions?"* To find the answer, perform each activity. If you find any difficulty in answering the exercises, seek the assistance of your teacher or peers or refer to the modules you have gone over earlier. You may check your answers with your teacher.



### Activity 1: It's Good to be Simple!

*Directions*: Work with a partner in simplifying each of the following expressions. Answer the questions that follow.

1. $\dfrac{6+\sqrt{9}}{2(3)}$

2. $\dfrac{6-\sqrt{9}}{2(3)}$

3. $\dfrac{-6+\sqrt{18}}{2(2)}$

4. $\dfrac{-9-\sqrt{24}}{2(2)}$

5. $\dfrac{-8+\sqrt{64-28}}{2(-3)}$

6. $\dfrac{-6-\sqrt{36-20}}{2(1)}$

7. $\dfrac{5+\sqrt{25+100}}{2(4)}$

8. $\dfrac{-10+\sqrt{10^2-52}}{2(3)}$

9. $\dfrac{-4-\sqrt{4^2+16}}{2(4)}$

10. $\dfrac{-5+\sqrt{5^2-4(3)(-2)}}{2(3)}$

*Questions:*

a. How would you describe the expressions given?

b. How did you simplify each expression?

c. Which expression did you find difficult to simplify? Why?

d. Compare your work with those of your classmates. Did you arrive at the same answer?

How did you find the activity? Were you able to simplify the expressions? I'm sure you did! In the next activity, you will be writing quadratic equations in standard form. You need this skill for you to solve quadratic equations by using the quadratic formula.

**Activity 2: Follow the Standards!**

*Directions*: Write the following quadratic equations in standard form, $ax^2 + bx + c = 0$. Then identify the values of a, b, and c. Answer the questions that follow.



1. $2x^2 + 9x = 10$
2. $-2x^2 = 2 - 7x$
3. $6x - 1 = 2x^2$
4. $10 + 7x - 3x^2 = 0$
5. $2x(x - 6) = 5$
6. $x(5 - 2x) + 15 = 0$
7. $(x + 4)(x + 12) = 0$
8. $(x - 6)(x - 9) = 0$
9. $(3x + 7)(x - 1) = 0$
10. $3(x - 5)^2 + 10 = 0$

*Questions:*

a. How did you write each quadratic equation in standard form?

b. How do you describe a quadratic equation that is written in standard form?

c. Are there different ways of writing a quadratic equation in standard form? Justify your answer.

Were you able to write each quadratic equation in standard form? Were you able to determine the values of a, b, and c? If you did, then extend further your understanding of the real-life applications of quadratic equations by doing the next activity.

**Activity 3: Why do the Gardens Have to be Adjacent?**

*Directions:* Use the situation below to answer the questions that follow.

*Mr. Bonifacio would like to enclose his two adjacent rectangular gardens with 70.5 m of fencing materials. The gardens are of the same size and their total area is 180 m$^2$.*

1. How would you represent the dimensions of each garden?

2. What mathematical sentence would represent the length of fencing material to be used in enclosing the two gardens?

   How about the mathematical sentence that would represent the total area of the two gardens?

3. How will you find the dimensions of each garden?

4. What equation will you use in finding the dimensions of each garden?

5. How would you describe the equation formulated in item 4?

   How are you going to find the solutions of this equation?

6. Do you think the methods of solving quadratic equations that you already learned can be used to solve the equation formulated in item 4? Why?

Did the activity you just performed capture your interest? Were you able to formulate mathematical sentence that will lead you in finding the measures of the unknown quantities? In the next activity, you will be given the opportunity to derive a general mathematical sentence which you can use in solving quadratic equations.

**Activity 4: Lead Me to the Formula!**

*Directions*: Work in groups of 4 in finding the solutions of the following quadratic equation by completing the square. Answer the questions that follow.

$$\boxed{2x^2 + 9x + 10 = 0}$$

1. How did you use completing the square in solving the given equation? Show the complete details of your work.

2. What are the solutions of the given equation?

3. How would you describe the solutions obtained?

4. Compare your work with those of other groups. Did you obtain the same solutions? If NOT, explain.

5. In the equation $2x^2 + 9x + 10 = 0$, what would be the resulting equation if 2, 9, and 10 were replaced by a, b, and c, respectively?

6. Using the resulting equation in item 5, how are you going to find the value of $x$ if you follow the same procedure in finding the solutions of $2x^2 + 9x + 10 = 0$?

   What equation or formula would give the value of $x$?

7. Do you think the equation or formula that would give the value of $x$ can be used in solving other quadratic equations? Justify your answer by giving examples.

How did you find the preceding activities? Are you ready to learn about solving quadratic equations by using the quadratic formula? I'm sure you are!!! From the activities done, you were able to solve equations, express a perfect square trinomial as a square of a binomial, write perfect square trinomials, and represent a real-life situation by a mathematical sentence. But how does finding solutions of quadratic equations facilitate in solving real-life problems and in making decisions? You will find these out in the activities in the next section. Before doing these activities, read and understand first some important notes on Solving Quadratic Equations by using the Quadratic Formula and the examples presented.

The solutions of any quadratic equation a$x^2$ + b$x$ + c = 0 can be determined using the quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, a ≠ 0. This formula can be derived by applying the method of completing the square as shown on the next page.

| | |
|---|---|
| $ax^2 + bx + c = 0 \rightarrow ax^2 + bx = -c$ | Why? |
| $\frac{ax^2 + bx}{a} = \frac{-c}{a} \rightarrow x^2 + \frac{bx}{a} = \frac{-c}{a}$ | Why? |
| $\frac{1}{2}\left(\frac{b}{a}\right) = \frac{b}{2a}$; $\left(\frac{b}{2a}\right)^2 = \frac{b^2}{4a^2}$ | Why? |
| $x^2 + \frac{bx}{a} + \frac{b^2}{4a^2} = \frac{-c}{a} + \frac{b^2}{4a^2}$ | Why? |
| $\left(x + \frac{b}{2a}\right)^2 = \frac{-4ac + b^2}{4a^2} = \frac{b^2 - 4ac}{4a^2}$ | Why? |
| $x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}} \rightarrow x + \frac{b}{2a} = \frac{\pm\sqrt{b^2 - 4ac}}{2a}$ | Why? |
| $x = \frac{\pm\sqrt{b^2 - 4ac}}{2a} - \frac{b}{2a}$ | Why? |
| $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ | Why? |



To solve any quadratic equation **a$x^2$ + b$x$ + c = 0** using the quadratic formula, determine the values of a, b, and c, then substitute these in the equation $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. Simplify the result if possible then check the solutions obtained against the original equation.

*Example 1:* Find the solutions of the equation 2$x^2$ + 3$x$ = 27 using the quadratic formula.

Write the equation in standard form.

$$2x^2 + 3x = 27 \rightarrow \quad 2x^2 + 3x - 27 = 0$$

Determine the values of a, b, and c.

$$2x^2 + 3x - 27 = 0 \quad \rightarrow \quad a = 2;\ b = 3;\ c = -27$$

Substitute the values of a, b, and c in the quadratic formula.

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a} \rightarrow x=\frac{-(3)\pm\sqrt{(3)^2-4(2)(-27)}}{2(2)}$$

Simplify the result.

$$x=\frac{-(3)\pm\sqrt{(3)^2-4(2)(-27)}}{2(2)} \rightarrow x=\frac{-3\pm\sqrt{9+216}}{4}$$

$$x=\frac{-3\pm\sqrt{225}}{4}$$

$$x=\frac{-3\pm 15}{4}$$

$$\boxed{x=\frac{-3+15}{4}=\frac{12}{4} \quad x=3} \qquad \boxed{x=\frac{-3-15}{4}=\frac{-18}{4} \quad x=-\frac{9}{2}}$$



Check the solutions obtained against the equation $2x^2 + 3x = 27$.

When $x = 3$:

$$2(3)^2+3(3)\stackrel{?}{=}27 \rightarrow 2(9)+3(3)\stackrel{?}{=}27$$

$$18+9\stackrel{?}{=}27$$

$$27=27$$

When $x=-\frac{9}{2}$:

$$2\left(-\frac{9}{2}\right)^2+3\left(-\frac{9}{2}\right)\stackrel{?}{=}27 \rightarrow 2\left(\frac{81}{4}\right)+3\left(-\frac{9}{2}\right)\stackrel{?}{=}27$$

$$\frac{81}{2}-\frac{27}{2}\stackrel{?}{=}27$$

$$\frac{54}{2}\stackrel{?}{=}27$$

$$27=27$$

Both values of $x$ satisfy the given equation. So the equation $2x^2+3x=27$ is true when $x = 3$ or when $x=-\frac{9}{2}$.

Answer: The equation $2x^2 + 3x = 27$ has two solutions: $x = 3$ or $x = -\frac{9}{2}$.

Learn more about Solving Quadratic Equations by Using the Quadratic Formula through the WEB. You may open the following links.

http://2012books.lardbucket.org/books/beginning-algebra/s12-03-quadratic-formula.html

http://www.regentsprep.org/Regents/math/algtrig/ATE3/quadformula.htm

http://www.purplemath.com/modules/quadform.htm

http://www.algebrahelp.com/lessons/equations/quadratic/

**What to PROCESS:**



Your goal in this section is to apply the key concepts of solving quadratic equations by using the quadratic formula. Use the mathematical ideas and the examples presented in the preceding section to answer the activities provided.



**Activity 5: Is the Formula Effective?**

*Directions*: Find the solutions of each of the following quadratic equations using the quadratic formula. Answer the questions that follow.

1. $x^2 + 10x + 9 = 0$
2. $x^2 - 12x + 35 = 0$
3. $x^2 + 5x - 14 = 0$
4. $x^2 - 4x + 12 = 0$
5. $x^2 + 7x = 4$
6. $2x^2 + 7x + 9 = 0$
7. $4x^2 - 4x + 1 = 0$
8. $3x^2 - 4x = 0$
9. $9x^2 - 72 = 0$
10. $2x^2 + 4x = 3$

*Questions:*

a. How did you use the quadratic formula in finding the solution/s of each equation?

b. How many solutions does each equation have?

c. Is there any equation whose solutions are equal? If there is any, describe the equation.

d. Is there any equation with zero as one of the solutions? Describe the equation if there is any.

e. Compare your answers with those of your classmates. Did you arrive at the same solutions? If NOT, explain.

> Was it easy for you to find the solutions of quadratic equations by using the quadratic formula? Were you able to simplify the solutions obtained? I know you did!

**Activity 6: Cut to Fit!**

*Directions:* Read and understand the situation below then answer the questions that follow.

*Mr. Bonifacio cuts different sizes of rectangular plywood to be used in the furniture that he makes. Some of these rectangular plywood are described below.*

*Plywood 1: The length of the plywood is twice its width and the area is 4.5 sq ft.*

*Plywood 2: The length of the plywood is 1.4 ft. less than twice its width and the area is 16 sq ft.*

*Plywood 3: The perimeter of the plywood is 10 ft. and its area is 6 sq ft.*

*Questions:*

1. What quadratic equation represents the area of each piece of plywood? Write the equation in terms of the width of the plywood.

2. Write each quadratic equation formulated in item 1 in standard form. Then determine the values of a, b, and c.

3. Solve each quadratic equation using the quadratic formula.

4. Which of the solutions or roots obtained represents the width of each plywood? Explain your answer.

5. What is the length of each piece of plywood? Explain how you arrived at your answer.

Were you able to come up with the quadratic equation that represents the area of each piece of plywood? Were you able to determine the length and width of each piece of plywood?

In this section, the discussion was about solving quadratic equations by using the quadratic formula.

Go back to the previous section and compare your initial ideas with the discussion. How much of your initial ideas are found in the discussion? Which ideas are different and need revision?

Now that you know the important ideas about this topic, let's go deeper by moving on to the next section.

**What to REFLECT ON and FURTHER UNDERSTAND:**

Your goal in this section is to take a closer look at some aspects of the topic. You are going to think deeper and test further your understanding of solving quadratic equations by using the quadratic formula. After doing the following activities, you should be able to answer this important question: *How does finding solutions of quadratic equations facilitate in solving real-life problems and in making decisions?*



**Activity 7: Make the Most Out of It!**

*Directions*: Answer the following.

1. The values of a, b, and c of a quadratic equation written in standard form are -2, 8, and 3, respectively. Another quadratic equation has 2, -8, and -3 as the values of a, b, and c, respectively. Do you agree that the two equations have the same solutions? Justify your answer.

2. How are you going to use the quadratic formula in determining whether a quadratic equation has no real solutions? Give at least two examples of quadratic equations with no real solutions.

3. Find the solutions of the following quadratic equations using the quadratic formula. Tell whether the solutions are real numbers or not real numbers. Explain your answer.
   a. $x^2 + 2x + 9 = 0$
   b. $2x^2 + 4x + 7 = 0$
   c. $(2x - 5)^2 - 4 = 0$
   d. $(3x + 2)^2 = 3x + 10$
4. Do you think the quadratic formula is more appropriate to use in solving quadratic equations? Explain then give examples to support your answer.

5. If you are to solve each of the following quadratic equations, which method would you use and why? Explain your answer.
   a. $9x^2 = 225$
   b. $4x^2 - 121 = 0$
   c. $x^2 + 11x + 30 = 0$
   d. $2x^2 + x - 28 = 0$
   e. $4x^2 + 16x + 15 = 0$
   f. $4x^2 + 4x - 15 = 0$

6. The length of a car park is 120 m longer than its width. The area of the car park is 6400 $m^2$.
   a. How would you represent the width of the car park?
      How about its length?
   b. What equation represents the area of the car park?
   c. How would you use the equation representing the area of the car park in finding its length and width?
   d. What is the length of the car park? How about its width? Explain how you arrived at your answer.
   e. Suppose the area of the car park is doubled, would its length and width also double? Justify your answer.

7. The length of a rectangular table is 0.6 m more than twice its width and its area is 4.6 $m^2$. What are the dimensions of the table?

8. Grace constructed an open box with a square base out of 192 $cm^2$ material. The height of the box is 4 cm. What is the length of the side of the base of the box?

9. A car travels 30 kph faster than a truck. The car covers 540 km in three hours less than the time it takes the truck to travel the same distance. What is the speed of the car? What about the truck?

In this section, the discussion was about your understanding of solving quadratic equations by using the quadratic formula.

What new insights do you have about solving quadratic equations by using the quadratic formula? How would you connect this to real life? How would you use this in making decisions?

Now that you have a deeper understanding of the topic, you are ready to do the tasks in the next section.

**What to TRANSFER:**

Your goal in this section is to apply your learning to real-life situations. You will be given a practical task which will demonstrate your understanding.

**Activity 8: Show Me The Best Floor Plan!!!**

*Directions:* Use the situation below to answer the questions that follow.

*Mr. Luna would like to construct a new house with a floor area of 72 m². He asked an architect to prepare a floor plan that shows the following:*

| | |
|---|---|
| *c. 2 bedrooms* | *d. Comfort room* |
| *d. Living room* | *e. Kitchen* |
| *e. Dining room* | *f. Laundry Area* |

1. Suppose you were the architect asked by Mr. Luna to prepare a floor plan. How will you do it? Draw the floor plan.

2. Formulate as many quadratic equations using the floor plan that you prepared. Solve the equations using the quadratic formula.



Rubric for Sketch Plan and Equations Formulated and Solved

| 4 | 3 | 2 | 1 |
|---|---|---|---|
| The sketch plan is accurately made, presentable, and appropriate. | The sketch plan is accurately made and appropriate. | The sketch plan is not accurately made but appropriate. | The sketch plan is made but not appropriate. |
| Quadratic equations are accurately formulated and solved correctly. | Quadratic equations are accurately formulated but not all are solved correctly. | Quadratic equations are accurately formulated but are not solved correctly. | Quadratic equations are accurately formulated but are not solved. |

How did you find the performance task? How did the task help you see the real world use of the topic?

**Summary/Synthesis/Generalization:**

This lesson was about solving quadratic equations by using the quadratic formula. The lesson provided you opportunities to describe quadratic equations and solve these by using the quadratic formula. You were able to find out also how such equations are illustrated in real life. Moreover, you were given the chance to demonstrate your understanding of the lesson by doing a practical task. Your understanding of this lesson and other previously learned mathematics concepts and principles will facilitate your learning of the wide applications of quadratic equations in real life.

## LEARNING MODULE
## MATH 9

### LESSON NO. 3: THE NATURE OF THE ROOTS OF A QUADRATIC EQUATION

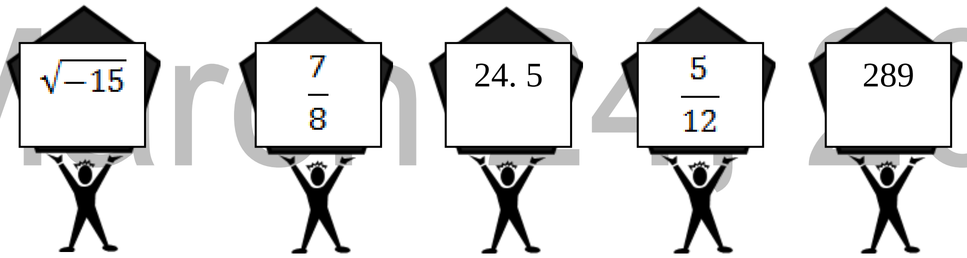
**What to KNOW:**

Start lesson 3 of this module by assessing your knowledge of the different mathematics concepts previously studied and your skills in performing mathematical operations. These knowledge and skills will help you in understanding the nature of roots of quadratic equations. As you go through this lesson, think of this important question: *"How does the nature of roots of quadratic equation facilitate in understanding the conditions of real life situations?"* To find the answer, perform each activity. If you find any difficulty in answering the exercises, seek the assistance of your teacher or peers or refer to the modules you have gone over earlier. You may check your answers with your teacher.

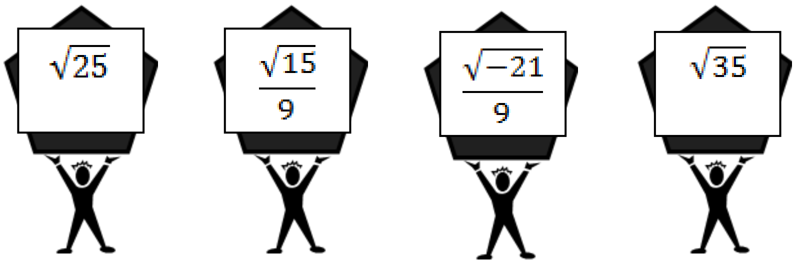


**Activity 1: Which are Real? Which are Not?**

*Directions:* Refer to the numbers below to answer the questions that follow.




I L O V E




M A T H

*Questions:*

1. Which of the numbers above are familiar to you? Why? Describe these numbers.
2. Which of the numbers are real? Which are not real?
3. Which of the numbers are rational? irrational? Explain your answer.
4. Which of the numbers are perfect squares? not perfect squares?
5. How do you describe numbers that are perfect squares?

> Were you able to classify the given numbers as real or not real, and as rational or irrational? In the next activity, you will determine the values of a, b, and c when the quadratic equation is written in the form $ax^2 + bx + c = 0$. You have done this activity in the previous lessons so I am sure you are already familiar with this.

**Activity 2: Math in A, B, C?**



*Directions:* Write the following quadratic equations in standard form, $ax^2 + bx + c = 0$, then identify the values of a, b, and c. Answer the questions that follow.

$$ax^2 + bx + c = 0$$

1. $x^2 + 5x = 4$ ________________ a = ____ b = ____ c = _____
2. $-4x^2 = 8x - 3$ ________________ a = ____ b = ____ c = _____
3. $10x - 1 = 4x^2$ ________________ a = ____ b = ____ c = _____
4. $15 + 8x - 3x^2 = 0$ ________________ a = ____ b = ____ c = _____
5. $3x(x - 14) = 12$ ________________ a = ____ b = ____ c = _____

*Questions:*

a. How did you write each quadratic equation in standard form?

b. Aside from your answer, do you think there is another way of writing each quadratic equation in standard form? If YES, show then determine the values of a, b, and c.

> Were you able to rewrite the given quadratic equations in the form $ax^2+bx+c = 0$? Were you able to find the value of a, b and c? In the next activity, you will find the value of $b^2 - 4ac$. This value will be your basis in describing the roots of a quadratic equation.

**Activity 3: What's My Value?**

*Directions:* Evaluate the expression $b^2 - 4ac$ given the following values of a, b, and c.

1. a = 1, b = 5, c = 4
2. a = 2, b = 1, c = -21
3. a = 4, b = 4, c = 1
4. a = 1, b = - 2, c = -2
5. a = 9, b = 0, c = 16

> Were you able to evaluate the expression $b^2 - 4ac$ given the values of a, b and c? What do you think is the importance of the expression $b^2 - 4ac$ in determining the nature of the roots of quadratic equation? You will find this out as you perform the succeeding activities.



**Activity 4: Find my Equation and Roots**

*Directions:* Using the values of a, b and c in Activity 3, write the quadratic equation $ax^2 + bx + c = 0$.Then find the roots of each resulting equation.



| | $ax^2 + bx + c = 0$ | Roots |
|---|---|---|
| 1. | ______________________ | _______________ |
| 2. | ______________________ | _______________ |
| 3. | ______________________ | _______________ |
| 4. | ______________________ | _______________ |
| 5. | ______________________ | _______________ |

> Were you able to write the quadratic equation given the values of a, b and c? Were you able to find the roots of the resulting quadratic equation? In the next activity, you will describe the nature of the roots of quadratic equation using the value of $b^2 - 4ac$.

**Activity 5: Place Me on the Table!**

*Directions:* Answer the following.

1. Complete the table below using your answers in activities 3 and 4.

| Equation | $b^2 - 4ac$ | Roots |
|---|---|---|
| 1. | | |
| 2. | | |
| 3. | | |
| 4. | | |
| 5. | | |

2. How would you describe the roots of quadratic equation when the value of $b^2 - 4ac$ is 0? positive and perfect square? positive but not perfect square? negative?

3. Which quadratic equation has roots that are real numbers and equal? rational numbers? Irrational numbers? not real numbers?

4. How do you determine the quadratic equation having roots that are real numbers and equal? rational numbers? irrational numbers? not real numbers?



Were you able to relate the value of $b^2 - 4ac$ to the nature of the roots of the quadratic equation? In the next activity, you will find out how the discriminant of the quadratic equation is illustrated in real-life situations.

**Activity 6: Let's Shoot that Ball!**

*Directions:* Study the situation below and answer the questions that follow.

*A basketball player throws a ball vertically with an initial velocity of 100ft/sec. The distance of the ball from the ground after t seconds is given by the expression $100t - 16t^2$.*

1. What is the distance of the ball from the ground after 6 seconds?

2. After how many seconds does the ball reach a distance of 50 ft. from the ground?

3. How many seconds will it take for the ball to fall to the ground?

4. Do you think the ball can reach the height of 160 ft.? Why? Why not?

How did you find the preceding activities? Are you ready to learn more about the nature of the roots of quadratic equations? From the activities you have done, you were able to determine the nature of roots of quadratic equations, whether they are real numbers, not real numbers, rational or irrational numbers. Moreover, you were able to find out how quadratic equations are illustrated in real life. Find out more about the applications of quadratic equations by performing the activities in the next section. Before doing these activities, read and understand first some important notes on Quadratic Equations and the examples presented.

The value of the expression $b^2 - 4ac$ is called the ***discriminant*** of the quadratic equation $ax^2 + bx + c = 0$. This value can be used to describe the nature of the roots of a quadratic equation. It can be zero, positive and perfect square, positive but not perfect square or negative.

1. When $b^2 - 4ac$ is equal to zero, then the roots are real numbers and are equal.

*Example:* Describe the roots of $x^2 - 4x + 4 = 0$.



The values of a, b, and c in the equation are the following.

$a = 1 \qquad b = -4 \qquad c = 4$

Substitute these values of a, b and c in the expression $b^2 - 4ac$.

$$\begin{aligned} b^2 - 4ac &= (-4)^2 - 4(1)(4) \\ &= 16 - 16 \\ &= 0 \end{aligned}$$

Since the value of $b^2 - 4ac$ is zero, we can say that the roots of the quadratic equation $x^2 - 4x + 4 = 0$ are real numbers and are equal.

This can be checked by determining the roots of $x^2 - 4x + 4 = 0$ using any of the methods of solving quadratic equation.

If the quadratic formula is used, the roots that can be obtained are the following.

$$x = \frac{-(-4) + \sqrt{(-4)^2 - 4(1)(4)}}{2(1)}$$
$$x = \frac{4 + \sqrt{16 - 16}}{2} = \frac{4 + \sqrt{0}}{2}$$
$$x = \frac{4 + 0}{2} = \frac{4}{2}$$
$$x = 2$$

$$x = \frac{-(-4) - \sqrt{(-4)^2 - 4(1)(4)}}{2(1)}$$
$$x = \frac{4 - \sqrt{16 - 16}}{2} = \frac{4 - \sqrt{0}}{2}$$
$$x = \frac{4 - 0}{2} = \frac{4}{2}$$
$$x = 2$$

The roots of the quadratic equation $x^2 - 4x + 4 = 0$ are real numbers and are equal.

2. When $b^2 - 4ac$ is greater than zero and a perfect square, then the roots are rational numbers but are not equal.

*Example:* Determine the nature of the roots of $x^2 + 7x + 10 = 0$.

In the equation, the values of a, b, and c are 1, 7, and 10, respectively. Use these values to evaluate $b^2 - 4ac$.

$$\begin{aligned} b^2 - 4ac &= (7)^2 - 4(1)(10) \\ &= 49 - 40 \\ &= 9 \end{aligned}$$

Since the value of $b^2 - 4ac$ is greater than zero and a perfect square, then the roots of the quadratic equation $x^2 + 7x + 10 = 0$ are rational numbers but are not equal.

To check, solve for the roots of $x^2 + 7x + 10 = 0$.

$$x = \frac{-7 + \sqrt{9}}{2} = \frac{-7 + 3}{2}$$
$$x = \frac{-4}{2}$$
$$x = -2$$

$$x = \frac{-7 - \sqrt{9}}{2} = \frac{-7 - 3}{2}$$
$$x = \frac{-10}{2}$$
$$x = -5$$

The roots of the quadratic equation $x^2 + 7x + 10 = 0$ are rational numbers but are not equal.

3. When $b^2$ – 4ac is greater than zero but not a perfect square, then the roots are irrational numbers and are not equal.

*Example:* Describe the roots of $x^2 + 6x + 3 = 0$.

Evaluate the expression $b^2 - 4ac$ using the values a, b, and c.

In the equation, the values of a, b, and c are 1, 6, and 3, respectively.

$$\begin{aligned} b^2 - 4ac &= (6)^2 - 4(1)(3) \\ &= 36 - 12 \\ &= 24 \end{aligned}$$

Since the value of $b^2$ – 4ac is greater than zero but not a perfect square, then the roots of the quadratic equation $x^2 + 6x + 3 = 0$ are irrational numbers and are not equal.

To check, solve for the roots of $x^2 + 6x + 3 = 0$.

$$x = \frac{-6 + \sqrt{24}}{2} = \frac{-6 + 2\sqrt{6}}{2}$$
$$x = -3 + \sqrt{6}$$

$$x = \frac{-6 - \sqrt{24}}{2} = \frac{-6 - 2\sqrt{6}}{2}$$
$$x = -3 - \sqrt{6}$$

The roots of the quadratic equation $x^2 + 6x + 3 = 0$ are irrational numbers and are not equal.

4. When $b^2$ – 4ac is less than zero, then the equation has no real roots.

*Example:* Determine the nature of the roots of $x^2 + 2x + 5 = 0$.

In the equation, the values of a, b, and c are 1, 2, and 5, respectively. Use these values to evaluate $b^2$ – 4ac.

$$\begin{aligned} b^2 - 4ac &= (2)^2 - 4(1)(5) \\ &= 4 - 20 \\ &= -16 \end{aligned}$$

Since the value of $b^2$ – 4ac is less than zero, then the quadratic equation $x^2 + 2x + 5 = 0$ has no real roots.

To check, solve for the roots of $x^2 + 2x + 5 = 0$.

$$x = \frac{-2 + \sqrt{2^2 - 4(1)(5)}}{2(1)}$$
$$x = \frac{-2 + \sqrt{4 - 20}}{2}$$
$$x = \frac{-2 + \sqrt{-16}}{2}$$

$$x = \frac{-2 - \sqrt{2^2 - 4(1)(5)}}{2(1)}$$
$$x = \frac{-2 - \sqrt{4 - 20}}{2}$$
$$x = \frac{-2 - \sqrt{-16}}{2}$$

The roots of the quadratic equation $x^2 + 2x + 5 = 0$ are not real numbers.

Learn more about the Nature of the Roots of Quadratic Equations through the WEB. You may open the following links.

http://www.analyzemath.com/Equations/Quadratic-1.html

http://www.regentsprep.org/Regents/math/algtrig/ATE3/discriminant.htm

http://www.icoachmath.com/math_dictionary/discriminant.html



Now that you have learned about the discriminant and how it determines the nature of the roots of a quadratic equation, you are ready to perform the succeeding activities.

**What to PROCESS:**

Your goal in this section is to apply the key concepts of the discriminant of the quadratic equation. Use the mathematical ideas and examples presented in the preceding section to answer the activities provided.

**Activity 7: What is My Nature?**

*Directions:* Determine the nature of the roots of the following quadratic equations using the discriminant. Answer the questions that follow.

1. $x^2 + 6x + 9 = 0$ discriminant : ______ nature of the roots: _________
2. $x^2 + 9x + 20 = 0$ discriminant: ______ nature of the roots: _________
3. $2x^2 – 10x + 8 = 0$ discriminant: ______ nature of the roots: _________
4. $x^2 + 5x + 10 = 0$ discriminant: ______ nature of the roots: _________
5. $x^2 + 6x + 3 = 0$ discriminant: ______ nature of the roots: _________
6. $2x^2 + 6x + 4 = 0$ discriminant: ______ nature of the roots: _________
7. $3x^2 – 5x = -4$ discriminant: ______ nature of the roots: _________
8. $9x^2 – 6x = -9$ discriminant: ______ nature of the roots: _________
9. $10x^2 – 4x = 8$ discriminant: ______ nature of the roots: _________
10. $3x^2 – 2x – 5 = 0$ discriminant: ______ nature of the roots: _________



*Questions:*

a. How did you determine the nature of the roots of each quadratic equation?
b. When do you say that the roots of a quadratic equation are real or not real numbers? rational or irrational numbers? equal or not equal?
c. How does the knowledge of the discriminant help you in determining the nature of the roots of any quadratic equation?



Were you able to determine the nature of the roots of any quadratic equation? I know you did!

**Activity 8: Let's Make a Table!**

*Directions*: Study the situation below and answer the questions that follow.

*Mang Jose wants to make a table which has an area of 6 m$^2$. The length of the table has to be 1 m longer than the width.*

a. If the width of the table is *p* meters, what will be its length?
b. Form a quadratic equation that represents the situation.
c. Without actually computing for the roots, determine whether the dimensions of the table are rational numbers. Explain.
d. Give the dimensions of the table.

Was it easy for you to determine the nature of the roots of the quadratic equation? Try to compare your initial ideas with the discussion in this section. How much of your initial ideas were found in this section? Which ideas are different and need revision?

Now that you know the important ideas about the topic, let's go deeper by moving on to the next section. In the next section, you will develop further your understanding of the nature of the roots of quadratic equations.

## What to REFLECT ON and FURTHER UNDERSTAND

Your goal in this section is to take a closer look at some aspects of the topic. You are going to think deeper and test further your understanding of the nature of the roots of quadratic equations. After doing the following activities, you should be able to answer this important question: *How is the concept of the discriminant of a quadratic equation used in solving real-life problems?*



### Activity 9: How Well Did I Understand the Lesson?

*Directions:* Answer the following questions

1. Describe the roots of a quadratic equation when the discriminant is
   a. zero.
   c. positive but not perfect square.
   b. positive perfect square.
   d. negative.

   Give examples for each.

2. How do you determine the nature of the roots of quadratic equation?

3. Danica says that the quadratic equation $2x^2 + 5x - 4 = 0$ has two possible solutions because the value of its discriminant is positive. Do you agree with Danica? Justify your answer.

4. When the quadratic expression $ax^2 + bx + c$ is a perfect square trinomial, do you agree that the value of its discriminant is zero? Justify your answer by giving at least two examples.

5. You and a friend are camping. You want to hang your food pack from a branch 20 ft. from the ground. You will attach a rope to a stick and throw it over the branch. Your friend can throw the stick upward with an initial velocity of 29 feet per second. The distance of the stick after $t$ seconds from an initial height of 6 feet is given by the expression $-16t^2 + 29t + 6$.

   a. Form and describe the equation representing the situation. How did you come up with the equation?

   b. With the given conditions, will the stick reach the branch when thrown? Justify your answer.

In this section, the discussion was about your understanding of the nature of the roots of quadratic equations.

What new insights do you have about the nature of the roots of quadratic equations? How would you connect this to real life? How would you use this in making decisions?

Now that you have a deeper understanding of the topic, you are ready to do the tasks in the next section.



**What to TRANSFER:**

Your goal in this section is to apply your learning to real life situations. You will be given tasks which will demonstrate your understanding of the discriminant of a quadratic equation.

**Activity 10: Will It or Will It Not?**

*Direction:* Answer the following.

1. When a basketball player shoots a ball from his hand at an initial height of 2 m with an initial upward velocity of 10 meters per second, the height of the ball can be modeled by the quadratic expression $-4.9t^2 + 10t + 2$ after $t$ seconds.

   a. What will be the height of the ball after 2 seconds?

b. How long will it take the ball to reach the height of 4.5 m?

How long will it take to touch the ground?

c. Do you think the ball can reach the height of 12 m? Why?

d. Will the ball hit the ring if the ring is 3 m high?

e. Write a similar situation but with varied initial height when the ball is thrown with an initial upward velocity. Then model the path of the ball by a quadratic expression.

f. Using the situation and the quadratic expression you have written in item e, formulate and solve problems involving the height of the ball when it is thrown after a given time.

2. Cite two more real-life situations where the discriminant of a quadratic equation is being applied or illustrated.



In this section, your task was to solve problems involving the discriminant of a quadratic equation. You were also asked to cite real-life situations where the discriminant of a quadratic equation is applied or illustrated.

How did you find the performance task? How did the task help you see the real-world use of the topic?



**SUMMARY/ SYNTHESIS/ GENERALIZATION**

This lesson was about the nature of the roots of quadratic equations. The lesson provided you with opportunities to describe the nature of the roots of quadratic equations using the discriminant even without solving the equation. More importantly, you were able to find out how the discriminant of a quadratic equation is illustrated in real-life situations. Your understanding of this lesson and other previously learned mathematical concepts and principles will facilitate your understanding of the succeeding lessons.

## LESSON NO. 4: THE SUM AND THE PRODUCT OF ROOTS OF QUADRATIC EQUATIONS

### What to KNOW:

Start lesson 4 of this module by assessing your knowledge of the different mathematics concepts and principles previously studied and your skills in performing mathematical operations. These knowledge and skills will help you in understanding the sum and product of the roots of quadratic equations. As you go through this lesson, think of this important question: *"How do the sum and product of roots of quadratic equation facilitate in understanding the required conditions of real-life situations?"* To find the answer, perform each activity. If you find any difficulty in answering the exercises, seek the assistance of your teacher or peers or refer to the modules you have gone over earlier. You may check your answers with your teacher.

### Activity 1:Let's do Addition and Multiplication!



*Directions:* Perform the indicated operation then answer the questions that follow.

1. 7 + 15 =
2. -9 + 14 =
3. -6 + (-17) =
4. $\left(\frac{-3}{8}\right)+\frac{1}{2}=$
5. $\left(\frac{-5}{6}\right)+\left(\frac{-2}{3}\right)=$
6. (8)(15) =
7. (-4)(7) =
8. (-6)(-12)=
9. $\left(\frac{-3}{7}\right)\left(\frac{2}{5}\right)=$
10. $\left(\frac{-4}{5}\right)\left(\frac{-3}{8}\right)=$

*Questions:*

a. How did you determine the result of each operation?

b. What mathematics concepts and principles did you apply to arrive at each result?

c. Compare your answers with those of your classmates. Did you arrive at the same answers? If NOT, explain why.

Were you able to perform each indicated operation correctly? In the next activity, you will strengthen further your skills in finding the roots of quadratic equations.

**Activity 2: Find My Roots!**

*Directions:* Find the roots of each of the following quadratic equations using any method. Answer the questions that follow.

1. $x^2 + 3x + 2 = 0$
2. $s^2 - 5s + 6 = 0$
3. $r^2 + 2r - 8 = 0$
4. $t^2 + 12t + 36 = 0$
5. $4x^2 + 16x + 15 = 0$
6. $15h^2 - 7h - 2 = 0$
7. $12s^2 - 5s - 3 = 0$
8. $6t^2 - 7t - 3 = 0$
9. $3m^2 - 8m - 4 = 0$
10. $2w^2 - 3w - 20 = 0$

*Questions:*

a. How did you find the roots of each quadratic equation?

   Which method of solving quadratic equations did you use in finding the roots?

b. Which quadratic equation did you find difficult to solve? Why?

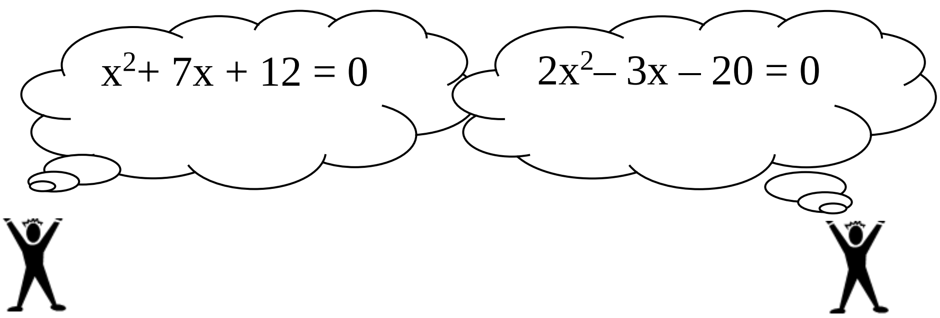
c. Compare your answers with those of your classmates. Did you arrive at the same answers? If NOT, explain why.



Were you able to find the roots of each quadratic equation? In the next activity, you will evaluate the sum and product of the roots and its relation to the coefficients of the quadratic equation.

**Activity 3: Relate Me to My Roots!**

*Directions*: Use the quadratic equations below to answer the questions that follow. You may work in groups of 4.



1. What are the values of a, b, and c in each equation?
   a. $x^2 + 7x + 12 = 0$; a = _____ b = _____ c = _____

   b. $2x^2 - 3x - 20 = 0$; a = _____ b = _____ c = _____

2. Determine the roots of each quadratic equation using any method.

   a. $x^2 + 7x + 12 = 0$; $x_1$ = _______ $x_2$ = _______

   b. $2x^2 - 3x - 20 = 0$; $x_1$ = _______ $x_2$ = _______

3. Complete the following table.

| Quadratic Equation | Sum of Roots | Product of Roots |
|---|---|---|
| $x^2 + 7x + 12 = 0$ | | |
| $2x^2 - 3x - 20 = 0$ | | |

4. What do you observe about the sum and the product of the roots of each quadratic equation in relation to the values of a, b, and c?

5. Do you think a quadratic equation can be determined given its roots or solutions? Justify your answer by giving 3 examples.

6. Do you think a quadratic equation can be determined given the sum and product of its roots? Justify your answer by giving 3 examples.



Were you able to relate the values of a, b, and c of each quadratic equation with the sum and product of its roots?

**Activity 4: What the Sum and Product Mean to Me…**

*Directions:* Study the situation below and answer the questions that follow.

*A rectangular garden has an area of 132 m² and a perimeter of 46 m.*

*Questions:*

1. What equation would describe the area of the garden? Write the equation in terms of the width of the garden.

2. What can you say about the equation formulated in item 1?

3. Find the roots of the equation formulated in item 1. What do the roots represent?

4. What is the sum of the roots? How is this related to the perimeter?

5. What is the product of the roots? How is this related to area?

Were you able to relate the sum and product of the roots of a quadratic equation with its values of a, b, and c? Suppose you are asked to find the quadratic equation given the sum and product of its roots, how will you do it? You will be able to answer this as you perform the succeeding activities. However, before performing these activities, read and understand first some important notes on the sum and product of the roots of quadratic equations and the examples presented.



We now discuss how the sum and product of the roots of the quadratic equation $ax^2 + bx + c = 0$ can be determined using the coefficients a, b, and c.

Remember that the roots of a quadratic equation can be determined using the quadratic formula, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. From the quadratic formula, let $x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$ and $x_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$ be the roots. Let us now find the sum and the product of these roots.

**Sum of the Roots of Quadratic Equation**

$$x_1 + x_2 = \frac{-b+\sqrt{b^2-4ac}}{2a} + \frac{-b-\sqrt{b^2-4ac}}{2a}$$

$$x_1 + x_2 = \frac{-b+\sqrt{b^2-4ac}-b-\sqrt{b^2-4ac}}{2a}$$

$$x_1 + x_2 = \frac{-2b}{2a} \quad \rightarrow \quad x_1 + x_2 = \frac{-b}{a}$$

The sum of the roots of quadratic equation is $\frac{-b}{a}$.

**Product of the Roots of Quadratic Equation**



$$x_1 \cdot x_2 = \left(\frac{-b+\sqrt{b^2-4ac}}{2a}\right)\left(\frac{-b-\sqrt{b^2-4ac}}{2a}\right)$$

$$x_1 \cdot x_2 = \frac{(-b)^2-\left(\sqrt{b^2-4ac}\right)^2}{(2a)^2} \quad \rightarrow \quad x_1 \cdot x_2 = \frac{b^2-b^2+4ac}{4a^2}$$

$$x_1 \cdot x_2 = \frac{4ac}{4a^2} \quad \rightarrow \quad x_1 \cdot x_2 = \frac{c}{a}$$

The product of the roots of quadratic equation is $\frac{c}{a}$.

*Example 1:* Find the sum and the product of the roots of $2x^2 + 8x - 10 = 0$.

The values of a, b, and c in the equation are 2, 8, and -10, respectively.

Sum of the roots = $\frac{-b}{a} \quad \rightarrow \quad \frac{-b}{a} = \frac{-(8)}{2} = -4$

The sum of the roots of $2x^2 + 8x - 10 = 0$ is $-4$.

Product of the roots = $\frac{c}{a}$ → $\frac{c}{a} = \frac{-10}{2} = -5$

The product of the roots of $2x^2 + 8x - 10 = 0$ is $-5$.

To check, find the roots of $2x^2 + 8x - 10 = 0$ using any of the methods of solving quadratic equations. Then determine the sum and the product of the roots that will be obtained.

The roots of the equation are 1 and -5. Find the sum and the product of these roots

Let $x_1 = 1$ and $x_2 = -5$.

Sum of the roots: $x_1 + x_2 = 1 + (-5) = -4$

Product of the roots: $x_1 \cdot x_2 = (1)(-5) = -5$



Therefore, the sum and the product of the roots of $2x^2 + 8x - 10 = 0$ are $-4$ and $-5$, respectively.



*Example 2:* Use the values of a, b, and c in finding the roots of the quadratic equation $x^2 + 7x - 18 = 0$.

The values of a, b, and c in the equation are 1, 7, and -18, respectively. Use these values to find the sum and the product of the roots of the equation.

Sum of the roots = $\frac{-b}{a}$ → $\frac{-b}{a} = \frac{-(7)}{1} = -7$

The sum of the roots of $x^2 + 7x - 18 = 0$ is $-7$.

Product of the roots = $\frac{c}{a}$ → $\frac{c}{a} = \frac{-18}{1} = -18$

The product of the roots of $x^2 + 7x - 18 = 0$ is $-18$.

If $x_1$ and $x_2$ are the roots of the quadratic equation $x^2 + 7x - 18 = 0$, then the sum and the product of its roots are as follows:

| | |
|---|---|
| Sum of the roots: | $x_1 + x_2 = -7$ |
| Product of the roots: | $x_1 \cdot x_2 = -18$ |

By inspection, the two numbers that give a sum of -7 and a product of -18 are -9 and 2.

To check, let $x_1 = -9$ and $x_2 = 2$ then find their sum and product.

Sum: $x_1 + x_2 = -7$

$$-9 + 2 \stackrel{?}{=} -7$$

$$-7 = -7$$

$x_1 + x_2 = -7$ is true for $x_1 = -9$ and $x_2 = 2$.

Product: $x_1 \cdot x_2 = -18$

$$(-9)(2) \stackrel{?}{=} -18$$

$$-18 = -18$$

$x_1 \cdot x_2 = -18$ is true for $x_1 = -9$ and $x_2 = 2$.



Therefore, the roots of the quadratic equation $x^2 + 7x - 18 = 0$ are: $x = -9$ or $x = 2$. These values of $x$ make the equation true.

Learn more about the Sum and the Product of Roots of Quadratic Equations through the WEB. You may open the following links.

http://www.regentsprep.org/Regents/math/algtrig/ATE4/natureofroots.htm

http://www.youtube.com/watch?v=l7FI4T19uIA

http://www.athometuition.com/QuadraticEquationsFormula.php

http://www.math-help-ace.com/Quadratic-Equation-Solver.html

Now that you learned about the sum and product of the roots of quadratic equations, you may now try the activities in the next sections.

## What to PROCESS

Your goal in this section is to apply previously learned mathematics concepts and principles in writing and in determining the roots of quadratic equations. Use the mathematical ideas and the examples presented in the preceding section to answer the activities provided.

### Activity 5: This is My Sum and this is My Product. Who Am I?

*Directions:* Use the values of a, b, and c of each of the following quadratic equations in determining the sum and the product of its roots. Verify your answers by obtaining the roots of the equation. Answer the questions that follow.

1. $x^2 + 4x + 3 = 0$ Sum: _____ Product: _____ Roots: _____________
2. $6x^2 + 12x - 18 = 0$ Sum: _____ Product: _____ Roots: _____________
3. $x^2 + 4x - 21 = 0$ Sum: _____ Product: _____ Roots: _____________
4. $2x^2 + 3x - 2 = 0$ Sum: _____ Product: _____ Roots: _____________
5. $3x^2 - 10x - 8 = 0$ Sum: _____ Product: _____ Roots: _____________
6. $4x^2 + 8x + 3 = 0$ Sum: _____ Product: _____ Roots: _____________
7. $9x^2 - 6x = 8$ Sum: _____ Product: _____ Roots: _____________
8. $8x^2 = 6x + 9$ Sum: _____ Product: _____ Roots: _____________
9. $10x^2 - 19x + 6 = 0$ Sum: _____ Product: _____ Roots: _____________
10. $2x^2 - 3x = 0$ Sum: _____ Product: _____ Roots: _____________

*Questions:*

a. How did you determine the sum and the product of the roots of each quadratic equation?

b. Do you think it is always convenient to use the values of a, b, and c of a quadratic equation in determining its roots? Explain your answer.

c. What do you think is the significance of knowing the sum and the product of the roots of quadratic equations?

Was it easy for you to determine the sum and the product of the roots of quadratic equations? Were you able to find out the importance of knowing these concepts? In the next activity, you will determine the quadratic equation given its roots.

**Activity 6: Here Are The Roots. Where is the Trunk?**

*Directions:* Write the quadratic equation in the form $ax^2 + bx + c = 0$ given the following roots. Answer the questions that follow.

1. 5 and 9
2. 8 and 10
3. 6 and 3
4. -8 and -10
5. -3 and 15
6. -9 and 0
7. 2.5 and 4.5
8. -3 and -3
9. $-\frac{5}{6}$ and $-\frac{1}{6}$
10. $-\frac{2}{3}$ and $\frac{3}{4}$



*Questions:*

a. How did you determine the quadratic equation given its roots?

b. What mathematics concepts or principles did you apply to arrive at the equation?

c. Are there other ways of getting the quadratic equation given the roots? If there are any, explain and give examples.

d. Compare your answers with those of your classmates. Did you arrive at the same answers? If NOT, explain.

Were you able to determine the quadratic equation given its roots? Did you use the sum and the product of the roots to determine the quadratic equation? I know you did! Let us now find out how the sum and the product of roots are illustrated in real life. Perform the next activity.

**Activity 7: Fence my Lot!**

*Directions:* Read and understand the situation below to answer the questions that follow.

*Mang Juan owns a rectangular lot. The perimeter of the lot is 90 m and its area is 450 m$^2$.*

*Questions:*

1. What equation represents the perimeter of the lot? How about the equation that represents its area?

2. How is the given situation related to the lesson, the sum and the product of roots of quadratic equation?

3. Using your idea of the sum and product of roots of quadratic equation, how would you determine the length and the width of the rectangular lot?

4. What are the dimensions of the rectangular lot?



In this section, the discussion was about the sum and product of the roots of quadratic equation $ax^2 + bx + c = 0$ and how they are related to the values of a, b, and c.

Go back to the previous section and compare your initial ideas with the discussion. How much of your initial ideas are found in the discussion? Which ideas are different and need revision?

Now that you know the important ideas about the topic, let's go deeper by moving on to the next section.

**What to REFLECT ON and FURTHER UNDERSTAND:**

> You goal in this section is to take a closer look at some aspects of the topic. You are going to think deeper and test further your understanding of the sum and product of roots of quadratic equations. After doing the following activities, you should be able to answer this important question: *"How do the sum and product of roots of quadratic equation facilitate in understanding the required conditions of real-life situations?"*

**Activity 8: Think of These Further!**

*Directions*: Answer the following.

1. The following are two different ways of determining a quadratic equation whose roots are 5 and 12.

**Method 1:** $x = 5$ or $x = 12$ → $x - 5 = 0$ or $x - 12 = 0$ *Why?*

$(x - 5)(x - 12) = 0$ *Why?*

Quadratic Equation: $x^2 - 17x + 60 = 0$ *Why?*

**Method 2:** $x_1 = 5$ or $x_2 = 12$

Sum of the Roots: $x_1 + x_2 = 5 + 12 = 17$

$x_1 + x_2 = \frac{-b}{a}$

$\frac{-b}{a} = 17$ *Why?*

$\frac{b}{a} = -17$ *Why?*

Product of the Roots: $x_1 \cdot x_2 = (5)(12) = 60$

$x_1 \cdot x_2 = \frac{c}{a}$

$\frac{c}{a} = 60$ *Why?*

Quadratic Equation: $ax^2 + bx + c = 0$ → $x^2 + \frac{b}{a}x + \frac{c}{a} = 0$ *Why?*

$x^2 - 17x + 60 = 0$ *Why?*

a. Describe each method of finding the quadratic equation.

b. Which method of determining the quadratic equation do you think is easier to follow? Why?

c. What do you think are the advantages and disadvantages of each method used in determining the quadratic equation? Explain and give 3 examples.

2. Suppose the sum of the roots of a quadratic equation is given, do you think you can determine the equation? Justify your answer.

3. The sum of the roots of a quadratic equation is -5. If one of the roots is 7, how would you determine the equation? Write the equation.

4. Suppose the product of the roots of a quadratic equation is given, do you think you can determine the equation? Justify your answer.

5. The product of the roots of a quadratic equation is 51. If one of the roots is -17, what could be the equation?

6. The perimeter of a rectangular bulletin board is 20 ft. If the area of the board is 21 ft.$^2$, what are its length and width?



In this section, the discussion was about your understanding of the sum and product of roots of quadratic equations.

What new insights do you have about the sum and product of roots of quadratic equations? How would you connect this to real life? How would you use this in making decisions?

Now that you have a deeper understanding of the topic, you are ready to do the tasks in the next section.

**What to TRANSFER**

Your goal in this section is to apply your learning to real-life situations. You will be given a practical task in which you will demonstrate your understanding.

**Activity 9: Let's Make a Scrap Book!**

*Direction:* Work in a group of 3 and make a scrap book that contains all the things you have learned in this lesson. This includes the following:

1. A journal on how to determine a quadratic equation given the roots, or given the sum and product of roots;

2. At least 5 examples of finding the quadratic equations given the roots, or given the sum and product of the roots, and;

3. Three pictures showing the applications of sum and product of roots of quadratic equations in real life. Describe how quadratic equations are illustrated in the pictures.

In this section, your task was to make a journal on how to determine the quadratic equation given its roots and to cite three real-life situations that illustrate the applications of quadratic equations.

How did you find the performance tasks? How did the tasks help you see the real-world use of the topic?



**SUMMARY/SYNTHESIS/GENERALIZATION:**



This lesson was about the Sum and Product of Roots of Quadratic Equations. In this lesson, you were able to relate the sum and product of the roots of quadratic equation $ax^2 + bx + c = 0$ with its values of a, b, and c. Furthermore, this lesson has given you an opportunity to find the quadratic equation given the roots. Your understanding of this lesson and other previously learned mathematics concepts and principles will facilitate your learning of the succeeding lessons.

## LESSON NO. 5: EQUATIONS TRANSFORMABLE TO QUADRATIC EQUATIONS

### What to KNOW:

Start lesson 5 of this module by assessing your knowledge of the different mathematics concepts and principles previously studied and your skills in performing mathematical operations. These knowledge and skills will help you in understanding the solution of equations that are transformable to quadratic equations. As you go through this lesson, think of this important question: *"How does finding solutions of quadratic equations facilitate in solving real-life problems?"* To find the answer, perform each activity. If you find any difficulty in answering the exercises, seek the assistance of your teacher or peers or refer to the modules you have gone over earlier. You may check your answers with your teacher.

### Activity 1: Let's Recall



*Directions:* Find the solution/s of the following quadratic equations. Answer the questions that follow.

1. $x^2 - 4x + 4 = 0$
2. $s^2 - 3s - 10 = 0$
3. $r^2 + 5r - 14 = 0$
4. $2m^2 + 5m + 2 = 0$
5. $2n^2 + 2n - 12 = 0$
6. $3p^2 + 7p + 4 = 0$

*Questions:*

a. How did you find the solutions of each equation?

   What method of solving quadratic equations did you use to find the roots of each?

b. Compare your answers with those of your classmates. Did you arrive at the same answers? If NOT, explain.

Were you able to find the solution/s of the quadratic equations? In the next activity, you will add or subtract rational algebraic expressions and express the results in simplest forms. These mathematical skills are necessary for you to solve equations that are transformable to quadratic equations.

**Activity 2: Let's Add and Subtract!**

*Direction:* Perform the indicated operation then express your answer in simplest form. Answer the questions that follow.

1. $\frac{1}{x}+\frac{2x}{5}$

2. $\frac{4}{x}-\frac{2x-1}{5}$

3. $\frac{2x}{3}+\frac{x+1}{x}$

4. $\frac{x+1}{2x}-\frac{x+2}{3x}$

5. $\frac{x-5}{2x}+\frac{x+1}{x-2}$

6. $\frac{x}{x+1}-\frac{2}{x+2}$

*Questions:*

a. How did you find the sum or the difference of the rational algebraic expressions?

b. What mathematics concepts or principles did you apply in adding or subtracting the rational algebraic expressions?

c. How did you simplify the resulting expressions?



Were you able to add or subtract the rational expressions and simplify the results? Suppose you were given a rational algebra equation, how would you find its solution/s? You will learn this in the succeeding activities.

**Activity 3: How Long Does It Take To Finish Your Job?**

*Directions:* Read and understand the situation below, then answer the questions that follow.

*Mary and Carol are doing a math project. Carol can do the work twice as fast as Mary. If they work together, they can finish the project in 4 hours. How long does it take Mary working alone to do the same project?*

*Questions:*

1. If Mary can finish the job in $x$ hours alone, how many hours will it take Carol to do the same job alone?

2. How would you represent the amount of work that Mary can finish in 1 hour? How about the amount of work that Carol can finish in 1 hour?

3. If they work together, what equation would represent the amount of work they can finish in 1 hour?

4. How would you describe the equation formulated in item 3?

5. How would you solve the equation formulated? What mathematics concepts and principles are you going to use?

How did you find the preceding activities? Are you ready to learn more about rational algebraic equations? From the activities done, you were able to simplify rational algebraic expressions. Also, you were able to represent quantities in real-life using rational algebraic expressions and equations. But how are quadratic equations used in solving real-life problems? You will find this out in the activities in the next section. Before doing these activities, read and understand first some important notes on equations that are transformable to quadratic equations and the examples presented.



There are equations that are transformable to quadratic equations. These equations may be given in different forms. Hence, the procedures in transforming these equations to quadratic equations may also be different.

Once the equations are transformed to quadratic equations, then they can be solved using the techniques learned in previous lessons. The different methods of solving quadratic equations, such as extracting square roots, factoring, completing the square, and using the quadratic formula, can be used to solve these transformed equations.

***Solving Quadratic Equations That Are Not Written In Standard Form***

*Example 1:* Solve $x(x-5) = 36$.

This is a quadratic equation that is not written in standard form.

To write the quadratic equation in standard form, simplify the expression $x(x-5)$.

$x(x-5) = 36 \rightarrow \quad x^2 - 5x = 36$

Write the resulting quadratic equation in standard form.

$x^2 - 5x = 36 \rightarrow \quad x^2 - 5x - 36 = 0$

Use any of the four methods of solving quadratic equations in finding the solutions of the equation $x^2 - 5x - 36 = 0$.

Try factoring in finding the roots of the equation.

$x^2 - 5x - 36 = 0 \quad \rightarrow \quad (x-9)(x+4) = 0$

$x - 9 = 0$ or $x + 4 = 0$ *Why?*

$x = 9$ or $x = -4$ *Why?*



Check whether the obtained values of $x$ make the equation $x(x-5) = 36$ true.

If the obtained values of $x$ make the equation $x(x-5) = 36$ true, then the solutions of the equation are: $x = 9$ or $x = -4$.

*Example 2:* Find the roots of the equation $(x+5)^2 + (x-2)^2 = 37$.

The given equation is a quadratic equation but it is not written in standard form. Transform this equation to standard form, then solve it using any of the methods of solving quadratic equations.

$(x+5)^2 + (x-2)^2 = 37 \rightarrow \quad x^2 + 10x + 25 + x^2 - 4x + 4 = 37$ *Why?*

$x^2 + x^2 + 10x - 4x + 25 + 4 = 37$ *Why?*

$2x^2 + 6x + 29 = 37$ *Why?*

$2x^2 + 6x - 8 = 0$ *Why?*

$2x^2 + 6x - 8 = 0 \quad \rightarrow \quad (2x-2)(x+4) = 0$ *Why?*

$2x - 2 = 0$ or $x + 4 = 0$ *Why?*

$x = 1$ or $x = -4$ *Why?*

The solutions of the equation are: $x = 1$ or $x = -4$. These values of $x$ make the equation $(x+5)^2 + (x-2)^2 = 37$ true.

***Solving Rational Algebraic Equations Transformable to Quadratic Equations***

*Example 3:* Solve the rational algebraic equation $\frac{6}{x} + \frac{x-3}{4} = 2$.

The given rational algebraic equation can be transformed to a quadratic equation. To solve the equation, the following procedure can be followed.

a. Multiply both sides of the equation by the Least Common Multiple (LCM) of all denominators. In the given equation, the LCM is 4$x$.

$$\frac{6}{x} + \frac{x-3}{4} = 2 \quad \rightarrow \quad 4x\left(\frac{6}{x} + \frac{x-3}{4}\right) = 4x(2)$$

$$24 + x^2 - 3x = 8x \qquad \textit{Why?}$$



b. Write the resulting quadratic equation in standard form.

$$24 + x^2 - 3x = 8x \quad \rightarrow \quad x^2 - 11x + 24 = 0 \qquad \textit{Why?}$$

c. Find the roots of the resulting equation using any of the methods of solving quadratic equations. Try factoring in finding the roots of the equation.

$$x^2 - 11x + 24 = 0 \quad \rightarrow \quad (x-3)(x-8) = 0 \qquad \textit{Why?}$$

$$x - 3 = 0 \text{ or } x - 8 = 0 \qquad \textit{Why?}$$

$$x = 3 \text{ or } x = 8 \qquad \textit{Why?}$$

Check whether the obtained values of $x$ make the equation $\frac{6}{x} + \frac{x-3}{4} = 2$ true.

If the obtained values of $x$ make the equation $\frac{6}{x} + \frac{x-3}{4} = 2$ true, then the solutions of the equation are: $x = 3$ or $x = 8$.

*Example 4:* Find the roots of $x+\frac{8}{x-2}=1+\frac{4x}{x-2}$.

The equation $x+\frac{8}{x-2}=1+\frac{4x}{x-2}$ is a rational algebraic equation that can be written in the form $ax^2+bx+c=0$.

To find the roots of the equation, you can follow the same procedure as in the previous examples of solving rational algebraic equations.

a. Multiply both sides of the equation by the LCM of all denominators. In the given equation, the LCM is $x-3$.

$$x+\frac{8}{x-2}=1+\frac{4x}{x-2} \rightarrow (x-2)\left(x+\frac{8}{x-2}\right)=(x-2)\left(1+\frac{4x}{x-2}\right)$$

$$\frac{x^2-2x+8}{x-2}=\frac{x-2+4x}{x-2} \qquad \textit{Why?}$$

$$x^2-2x+8=x-2+4x \qquad \textit{Why?}$$

b. Write the resulting quadratic equation in standard form.

$$x^2-2x+8=x-2+4x \rightarrow \quad x^2-2x+8=5x-2 \qquad \textit{Why?}$$

$$x^2-7x+10=0 \qquad \textit{Why?}$$

c. Find the roots of the resulting equation using any of the methods of solving quadratic equations. Let us solve the equation by factoring.

$$x^2-7x+10=0 \quad \rightarrow \quad (x-5)(x-2)=0 \qquad \textit{Why?}$$

$$x-5=0 \text{ or } x-2=0 \qquad \textit{Why?}$$

$$x=5 \text{ or } x=2 \qquad \textit{Why?}$$

The equation $x^2-7x+10=0$ has two solutions, $x=5$ or $x=2$. Check whether the obtained values of $x$ make the equation $x+\frac{8}{x-2}=1+\frac{4x}{x-2}$ true.

For $x = 5$:

$$x + \frac{8}{x-2} = 1 + \frac{4x}{x-2} \rightarrow \quad 5 + \frac{8}{5-2} \overset{?}{=} 1 + \frac{4(5)}{5-2}$$

$$5 + \frac{8}{3} \overset{?}{=} 1 + \frac{20}{3}$$

$$\frac{15+8}{3} \overset{?}{=} \frac{3+20}{3}$$

$$\frac{23}{3} = \frac{23}{3}$$

The equation $x + \frac{8}{x-2} = 1 + \frac{4x}{x-2}$ is true when $x = 5$. Hence $x = 5$ is a solution.



For $x = 2$:

$$x + \frac{8}{x-2} = 1 + \frac{4x}{x-2} \rightarrow \quad 2 + \frac{8}{2-2} \overset{?}{=} 1 + \frac{4(2)}{2-2}$$

$$2 + \frac{8}{0} \overset{?}{=} 1 + \frac{8}{0}$$

Observe that at $x = 2$, the value of $\frac{8}{x-2}$ is undefined or does not exist. The same is true with $\frac{4x}{x-2}$. Hence, $x = 2$ is an extraneous root or solution of the equation $x + \frac{8}{x-2} = 1 + \frac{4x}{x-2}$. An ***extraneous root or solution*** is a solution of an equation derived from an original equation. However, it is not a solution of the original equation.

Learn more about Equations Transformable to Quadratic Equations through the WEB. You may open the following links.

http://www.analyzemath.com/Algebra2/Algebra2.html

http://mathvids.com/lesson/mathhelp/1437-solving-rational-equation-to-quadratic-1

http://www.analyzemath.com/Algebra2/algebra2_solutions.html

**What to PROCESS:**

Your goal in this section is to transform equations into quadratic equations and solve these. Use the mathematical ideas and examples presented in the preceding section to answer the activities provided.



**Activity 4: View Me In Another Way!**

*Directions:* Transform each of the following equations to a quadratic equation in the form $ax^2 + bx + c = 0$. Answer the questions that follow.

1. $x(x+5) = 2$
2. $(s+6)^2 = 15$
3. $(t+2)^2 + (t-3)^2 = 9$
4. $(2r+3)^2 + (r+4)^2 = 10$
5. $(m-4)^2 + (m-7)^2 = 15$
6. $\frac{2x^2}{5} + \frac{5x}{4} = 10$
7. $\frac{2}{t} - \frac{3t}{2} = 7$
8. $\frac{3}{x} + \frac{4}{2x} = x - 1$
9. $\frac{6}{s+5} + \frac{s-5}{2} = 3$
10. $\frac{2}{r-1} + \frac{4}{r+5} = 7$

*Questions:*

a. How did you transform each equation into quadratic equation? What mathematics concepts or principles did you apply?

b. Did you find any difficulty in transforming each equation into a quadratic equation? Explain.

c. Compare your answers with those of your classmates. Did you arrive at the same answer? If NOT, explain.

> Were you able to transform each equation into a quadratic equation? Why do you think there is a need for you to do such activity? Find this out in the next activity.

**Activity 5: What Must Be The Right Value?**

*Directions:* Find the solution/s of each of the following equations. Answer the questions that follow.

**Eq.1** $x(x-10)=-21$

**Eq.3** $\frac{1}{3x}+\frac{4x}{6}=1$

**Eq.2** $(x+1)^2+(x-3)^2=15$



*Questions:*

a. How did you solve each equation? What mathematics concepts or principles did you apply to solve each equation?

b. Which equation did you find difficult to solve? Why?

c. Compare your answers with those of your classmates. Did you arrive at the same answers? If NOT, explain.

d. Do you think there are other ways of solving each equation? Show these if there are any.

> Were you able to solve the given equations? Was it easy for you to transform those equations into quadratic equations? In the next activity, you will solve equations that are transformable to quadratic equations.

**Activity 6: Let's Be True!**

*Directions*: Find the solution set of the following.

1. $x(x+3)=28$
2. $3s(s-2)=12s$
3. $(t+1)^2+(t-8)^2=45$
4. $(3r+1)^2+(r+2)^2=65$
5. $\frac{(x+2)^2}{5}+\frac{(x-2)^2}{3}=\frac{16}{3}$
6. $\frac{1}{x}-\frac{x}{6}=\frac{2}{3}$
7. $\frac{4}{t-3}+\frac{t}{2}=-2$
8. $\frac{5}{4x}-\frac{x+2}{3}=x-1$
9. $\frac{s+2}{2s}-\frac{s+1}{4}=\frac{-1}{2}$
10. $\frac{2x}{x-5}+\frac{1}{x-3}=3$

> Were you able to find the solution of each equation above? Now it's time to apply those equations in solving real-life problems. In the next activity, you will solve real-life problems using your knowledge in solving rational algebraic equations.



**Activity 7: Let's Paint the House!**

*Directions:* Read and understand the situation below, then answer the questions that follow.

*Jessie and Mark are planning to paint a house together. Jessie thinks that if he works alone, it would take him 5 hours more than the time Mark takes to paint the entire house. Working together, they can complete the job in 6 hours.*

*Questions:*

1. If Mark can finish the job in *m* hours, how long will it take Jessie to finish the job?
2. How would you represent the amount of work that Mark can finish in 1 hour? How about the amount of work that Jessie can finish in 1 hour?
3. If they work together, what equation would represent the amount of work they can finish in 1 hour?
4. How would you describe the equation formulated in item 3?
5. How will you solve the equation formulated? What mathematics concepts and principles are you going to use?

In this section, the discussion was about the solutions of equations transformable to quadratic equations.

Go back to the previous section and compare your initial ideas with the discussion. How much of your initial ideas are found in the discussion? Which ideas are different and need revision?

Now that you know the important ideas about the topic, let's go deeper by moving on to the next section.

**What to REFLECT ON and FURTHER UNDERSTAND:**

You goal in this section is to take a closer look at some aspects of the topic. You are going to think deeper and test further your understanding of the solution of equations that are transformable to quadratic equations. After doing the following activities, you should be able to answer this important question: *How does the concept of quadratic equation used in solving real-life problems?*



**Activity 8: My Understanding of Equations Transformable to Quadratic Equations**

*Directions:* Answer the following.

1. How do you transform a rational algebraic equation into a quadratic equation? Explain and give examples.

2. How do you determine the solutions of quadratic equations? How about rational algebraic equations transformable to quadratic equations?

3. Suppose a quadratic equation is derived from a rational algebraic equation. How do you check if the solutions of the quadratic equation are also the solutions of the rational algebraic equation?

4. Which of the following equations have extraneous roots or solutions? Justify your answer.

   a. $\frac{1}{x}+\frac{1}{x+1}=\frac{7}{12}$

   b. $\frac{x^2-5x}{x-5}=15-2x$

   c. $\frac{3x^2-6}{8-x}=x-2$

   d. $\frac{3x+4}{5}-\frac{2}{x+3}=\frac{8}{5}$

5. In a water refilling station, the time that a pipe takes to fill a tank is 10 minutes more than the time that another pipe takes to fill the same tank. If the two pipes are opened at the same time, they can fill the tank in 12 minutes. How many minutes does each pipe take to fill the tank?

In this section, the discussion was about your understanding of equations transformable to quadratic equations. What new insights do you have about this lesson? How would you connect this to real life? How would you use this in making decisions?

Now that you have a deeper understanding of the topic, you are ready to do the tasks in the next section.

**What to TRANSFER:**



Your goal in this section is to apply your learning to real-life situations. You will be given tasks which will demonstrate your understanding.



**Activity 9: A Reality of Rational Algebraic Equation**

*Directions:* Cite a real-life situation where the concept of a rational algebraic equation transformable to a quadratic equation is being applied. Use the situation to answer the following questions.

1. How is the concept of a rational algebraic equation transformable to a quadratic equation applied in the situation?
2. What quantities are involved in the situation? Which of these quantities are known? How about the quantities that are unknown?
3. Formulate, then solve a problem out of the given situation.
4. What do the solutions obtained represent? Explain your answer.

***Rubric: Real-Life Situations Involving Rational Algebraic Equations Transformable to Quadratic Equations***

| 4 | 3 | 2 | 1 |
|---|---|---|---|
| The situation is clear, realistic and the use of a rational algebraic equation transformable to a quadratic equation and other mathematics concepts are properly illustrated. | The situation is clear but the use of a rational algebraic equation transformable to a quadratic equation and other mathematic concepts are not properly illustrated. | The situation is not so clear, and the use of a rational algebraic equation transformable to a quadratic equation is not illustrated. | The situation is not clear and the use of a rational algebraic equation transformable to a quadratic equation is not illustrated. |

**Rubric on Problems Formulated and Solved**

| Score | Descriptors |
|---|---|
| 6 | Poses a more complex problem with 2 or more correct possible solutions and communicates ideas unmistakably, shows in-depth comprehension of the pertinent concepts and/or processes and provides explanations wherever appropriate. |
| 5 | Poses a more complex problem and finishes all significant parts of the solution and communicates ideas unmistakably, shows in-depth comprehension of the pertinent concepts and/or processes. |
| 4 | Poses a complex problem and finishes all significant parts of the solution and communicates ideas unmistakably, shows in-depth comprehension of the pertinent concepts and/or processes. |
| 3 | Poses a complex problem and finishes most significant parts of the solution and communicates ideas unmistakably, shows comprehension of major concepts although neglects or misinterprets less significant ideas or details. |
| 2 | Poses a problem and finishes some significant parts of the solution and communicates ideas unmistakably but shows gaps on theoretical comprehension. |
| 1 | Poses a problem but demonstrates minor comprehension, not being able to develop an approach. |

Source: D.O. #73 s. 2012

In this section, your task was to cite a real-life situation where the concept of a rational algebraic equation transformable to a quadratic equation is illustrated.

How did you find the performance tasks? How did the tasks help you see the real-world use of the topic?

**Summary/Synthesis/Generalization:**

This lesson was about the solutions of equations that are transformable to quadratic equations including rational algebraic equations. This lesson provided you with opportunities to transform equations into the form $ax^2 + bx + c = 0$ and to solve these. Moreover, this lesson provided you with opportunities to solve real-life problems involving rational algebraic equations transformable to quadratic equations. Your understanding of this lesson and other previously learned mathematics concepts and principles will facilitate your understanding of the succeeding lessons.

## LESSON NO. 6: SOLVING PROBLEMS INVOLVING QUADRATIC EQUATIONS

### What to KNOW:

Start lesson 6 of this module by assessing your knowledge of the different mathematics concepts previously studied and your skills in performing mathematical operations. These knowledge and skills may help you in understanding the solutions to real-life problems involving quadratic equations. As you go through this lesson, think of this important question: *How are quadratic equations used in solving real-life problems and in making decisions?"* To find the answer, perform each activity. If you find any difficulty in answering the exercises, seek the assistance of your teacher or peers or refer to the modules you have gone over earlier. You may check your answers with your teacher.

### Activity 1: Find My Solutions!



*Directions*: Solve each of the following quadratic equations. Explain how you arrived at your answers.

| | |
|---|---|
| 1. $x(2x - 5) = 0$ | 6. $(3m + 4)(m - 5) = 0$ |
| 2. $2t(t - 8) = 0$ | 7. $k^2 - 4k - 45 = 0$ |
| 3. $6x(2x + 1) = 0$ | 8. $2t^2 - 7t - 49 = 0$ |
| 4. $(r + 2)(r + 13) = 0$ | 9. $3w^2 - 11w = 4$ |
| 5. $(h - 4)(h - 10) = 0$ | 10. $4u^2 + 4u = 15$ |

Were you able to find the solution of each quadratic equation? In the next activity, you will translate verbal phrases into mathematical expressions. This will help you solve real-life problems later on.

**Activity 2: Translate into…**

*Directions:* Use a variable to represent the unknown quantity then write an equation from the given information. Explain how you arrived at your answer.

1. The area of the concrete pathway is 350 $m^2$ and its perimeter pathway is 90 m. What is the length of the pathway?
2. A rectangular lot has an area of 240 $m^2$. What is the width of the lot if it requires 64 m of fencing materials to enclose it?

3. The area of a garden is 160 $m^2$. Suppose the length of the garden is 3 m more than twice its width. What is the length of the garden?

4. The length of a tarpaulin is 3 ft. more than thrice its width and its area is 126 $ft.^2$. What is the length of the tarpaulin?

5. Mario and Kenneth work in a car wash station. The time that Mario takes in washing a car alone is 20 minutes less than the time that Kenneth takes in washing the same car. If both of them work together in washing the car, it will take them 90 minutes. How long will it take each of them to wash the car?



Were you able to represent each situation by an equation? If YES, then you are ready to perform the next activity.



**Activity 3: What are My Dimensions?**

*Directions:* Use the situation below to answer the questions that follow.

*The length of a rectangular floor is 5 m longer than its width. The area of the floor is 84 $m^2$.*

*Questions:*

1. What expression represents the width of the floor? How about the expression that represents its length?

2. Formulate an equation relating the width, length and the area of the floor. Explain how you arrived at the mathematical sentence.

3. How would you describe the equation that you formulated?

4. Using the equation, how will you determine the length and width of the floor?
5. What is the width of the floor? How about its length?

6. How did you find the length and width of the floor?

> Were you able to find the width and length of the rectangular floor correctly? In the next activity, you will find out how to solve real-life problems using quadratic equations but before we proceed to the next activities, read and understand the following note in solving word and real-life problems.

The concept of quadratic equations is illustrated in many real-life situations. Problems that arise from these situations, such as those involving area, work, profits, and many others, can be solved by applying the different mathematics concepts and principles previously studied including quadratic equations and the different ways of solving them.



*Example 1:* A rectangular table has an area of 27 ft.$^2$ and a perimeter of 24 ft. What are the dimensions of the table?

The product of the length and width of the rectangular table represents its area. Hence, *length (l) times width (w)* = 27 or *lw* = 27.

Also, twice the sum of the length and the width of the table gives the perimeter. Hence, 2*l* + 2*w* = 24.

If we divide both sides of the equation $2l + 2w = 24$ by 2, then $l + w = 12$.

We can think of *lw* = 27 and $l + w = 12$ as the equations representing the product and sum of roots, respectively, of a quadratic equation.

Remember that if the sum and the product of the roots of a quadratic equation are given, the roots can be determined. This can be done by inspection or by using the equation $x^2 + \frac{b}{a}x + \frac{c}{a} = 0$, where $-\frac{b}{a}$ is the sum of the roots and $\frac{c}{a}$ is the product.

By inspection, the numbers whose product is 27 and whose sum is 12 are 3 and 9.

Product: $3 \cdot 9 = 27$

Sum: $3 + 9 = 12$

The roots of the quadratic equation then are 3 and 9. This implies that the width of the table is 3 ft. and its length is 9 ft.

Another method of finding the roots is to use the equation $x^2 + \frac{b}{a}x + \frac{c}{a} = 0$. Let $-\frac{b}{a} = 12$ or $\frac{b}{a} = -12$ and $\frac{c}{a} = 27$. Then substitute these values in the equation.

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0 \quad \rightarrow \quad x^2 + (-12)x + 27 = 0$$
$$\rightarrow \quad x^2 - 12x + 27 = 0$$



Solve the resulting equation $x^2 - 12x + 27 = 0$ using any of the methods of solving quadratic equation. Try factoring.



$$x^2 - 12x + 27 = 0 \quad \rightarrow \quad (x-3)(x-9) = 0$$
$$\rightarrow \quad x - 3 = 0 \text{ or } x - 9 = 0$$
$$\rightarrow \quad x = 3, \text{ or } x = 9$$

With the obtained roots of the quadratic equation, the dimensions of the table then are 3 ft. and 9 ft., respectively.

*Example 2:* An amusement park wants to place a new rectangular billboard to inform visitors of their new attractions. Suppose the length of the billboard to be placed is 4 m longer than its width and the area is 96 m². What will be the length and the width of the billboard?

If we represent the width, in meters, of the billboard by *x*, then its length is *x* + 4. Since the area of the billboard is 96 m², then $(x)(x+4) = 96$.

The equation $(x)(x+4) = 96$ is a quadratic equation that can be written in the form $ax^2 + bx + c = 0$.

$$(x)(x+4)=96 \quad \rightarrow \quad x^2+4x=96$$
$$x^2+4x-96=0$$

Solve the resulting equation.

$$x^2+4x-96=0 \quad \rightarrow \quad (x-8)(x+12)=0$$
$$x-8=0 \text{ or } x+12=0$$
$$x=8 \text{ or } x=-12$$

The equation $x^2+4x-96=0$ has two solutions: ***x* = 8** or ***x* = -12**.

However, we only consider the positive value of $x$ since the situation involves measure of length. Hence, the width of the billboard is 8 m and its length is 12 m.

Learn more about the Applications of Quadratic Equations through the WEB. You may open the following links.

http://www.mathsisfun.com/algebra/quadratic-equation-real-world.html

http://www.purplemath.com/modules/quadprob.htm

http://tutorial.math.lamar.edu/Classes/Alg/QuadraticApps.aspx

http://www.algebra.com/algebra/homework/quadratic/lessons/Using-quadratic-equations-to-solve-word-problems.lesson

http://www.slideshare.net/jchartiersjsd/quadratic-equation-word-problems

http://www.tulyn.com/algebra/quadratic-equations/wordproblems

http://www.uwlax.edu/faculty/hasenbank/archived/mth126fa08/notes/11.10%20-%20Quadratic%20Applications.pdf

http://www.pindling.org/Math/CA/By_Examples/1_4_Appls_Quadratic/1_4_Appls_Quadratic.html

Now that you have learned how to solve real-life problems involving quadratic equations, you may now try the activities in the next sections.

**What to PROCESS:**

Your goal in this section is to apply the key concepts of quadratic equations in solving real-life problems. Use the mathematical ideas and the examples presented in the preceding sections to answer the succeeding activities.

**Activity 4: Let Me Try!**

*Directions:* Answer each of the following.

1. A projectile that is fired vertically into the air with an initial velocity of 120 ft. per second can be modeled by the equation $s = 120t - 16t^2$. In the equation, $s$ is the distance in feet of the projectile above the ground after $t$ seconds.

   a. How long will it take for a projectile to reach 216 feet?

   b. Is it possible for the projectile to reach 900 feet? Justify your answer.



2. The length of a rectangular parking lot is 36 m longer than its width. The area of the parking lot is 5,152 $m^2$.

   a. How would you represent the width of the parking lot? How about its length?

   b. What equation represents the area of the parking lot?

   c. How would you use the equation representing the area of the parking lot in finding its length and width?

   d. What is the length of the parking lot? How about its width? Explain how you arrived at your answer.

   e. Suppose the area of the parking lot is doubled, would its length and width also double? Justify your answer.

3. The perimeter of a rectangular swimming pool is 86 m and its area is 450 $m^2$.

a. How would you represent the length and the width of the swimming pool?

b. What equation represents the perimeter of the swimming pool?
How about the equation that represents its area?

c. How would you find the length and the width of the swimming pool?

d. What is the length of the swimming pool? How about its width? Explain how you arrived at your answer.

e. How would you check if the dimensions of the swimming pool obtained satisfy the conditions of the given situation?

f. Suppose the dimensions of the swimming pool are both doubled, how would it affect its perimeter? How about its area?

In this section, the discussion was about solving real-life problems involving quadratic equations.

Go back to the previous section and compare your initial ideas with the discussion. How much of your initial ideas are found in the discussion? Which ideas are different and need revision?



Now that you know the important ideas about quadratic equations and their applications in real life, let's go deeper by moving on to the next section.

**What to REFLECT ON and FURTHER UNDERSTAND:**

Your goal in this section is to take a closer look at some aspects of the topic. You are going to think deeper and test further your understanding of the real-life applications of quadratic equations. After doing the following activities, you should be able to answer this important question: *How are quadratic equations used in solving real life problems and in making decisions?"*

**Activity 5: Find Those Missing!**

*Directions*: Solve the following problems. Explain how you arrived at your answers.

1. A rectangular garden has an area of 84 $m^2$ and a perimeter of 38 m. Find its length and width.

2. A children's park is 350 m long and 200 m wide. It is surrounded by a pathway of uniform width. Suppose the total area of the park and the pathway is 74,464 $m^2$. How wide is the pathway?

3. A car travels 20 kph faster than a truck. The car covers 350 km in two hours less than the time it takes the truck to travel the same distance. What is the speed of the car? How about the truck?

4. Jane and Maria can clean the house in 8 hours if they work together. The time that Jane takes in cleaning the house alone is 4 hours more than the time Maria takes in cleaning the same house. How long does it take Jane to clean the house alone? How about Maria?

5. If an amount of money *P* in pesos is invested at *r* percent compounded annually, it will grow to an amount $A = P(1 + r)^2$ in two years. Suppose Miss Madrigal wants her money amounting to Php200,000 to grow to Php228,980 in two years. At what rate must she invest her money?



In this section, the discussion was about your understanding of quadratic equations and their real-life applications.

What new insights do you have about the real-life applications of quadratic equations? How would you connect this to your daily life? How would you use this in making decisions?



Now that you have a deeper understanding of the topic, you are ready to do the tasks in the next section.

**What to TRANSFER:**

Your goal in this section is to apply your learning to real-life situations. You will be given tasks which will demonstrate your understanding of the lesson.

**Activity 6: Let's Draw!**

*Directions:* Make a design or sketch plan of a table than can be made out of ¾" x 4' x 8' plywood and 2" x 3" x 8' wood. Using the design or sketch plan, formulate problems that involve quadratic equations, then solve in as many ways as possible.

Rubric for Sketch Plan and Equations Formulated and Solved

| 4 | 3 | 2 | 1 |
|---|---|---|---|
| The sketch plan is accurately made, presentable, and appropriate. | The sketch plan is accurately made and appropriate. | The sketch plan is not accurately made but appropriate. | The sketch plan is made but not appropriate. |
| Quadratic equations are accurately formulated and solved correctly. | Quadratic equations are accurately formulated but not all are solved correctly. | Quadratic equations are accurately formulated but are not solved correctly. | Quadratic equations are accurately formulated but are not solved. |



**Activity 7: Play the Role of…**

*Directions:* Cite and role play a situation in real life where the concept of the quadratic equation is applied. Formulate and solve problems out of these situations.

Rubric for Real-Life Situation Involving Quadratic Equation

| 4 | 3 | 2 | 1 |
|---|---|---|---|
| The situation is clear, realistic and the use of the quadratic equation and other mathematical concepts are properly illustrated. | The situation is clear but the use of the quadratic equation and other mathematic concepts are not properly illustrated. | The situation is not so clear, and the use of the quadratic equation is not illustrated. | The situation is not clear and the use of the quadratic equation is not illustrated. |

Rubric on Problems Formulated and Solved

| Score | Descriptors |
| --- | --- |
| 6 | Poses a more complex problem with 2 or more correct possible solutions and communicates ideas unmistakably, shows in-depth comprehension of the pertinent concepts and/or processes and provides explanations wherever appropriate. |
| 5 | Poses a more complex problem and finishes all significant parts of the solution and communicates ideas unmistakably, shows in-depth comprehension of the pertinent concepts and/or processes. |
| 4 | Poses a complex problem and finishes all significant parts of the solution and communicates ideas unmistakably, shows in-depth comprehension of the pertinent concepts and/or processes. |
| 3 | Poses a complex problem and finishes most significant parts of the solution and communicates ideas unmistakably, shows comprehension of major concepts although neglects or misinterprets less significant ideas or details. |
| 2 | Poses a problem and finishes some significant parts of the solution and communicates ideas unmistakably but shows gaps on theoretical comprehension. |
| 1 | Poses a problem but demonstrates minor comprehension, not being able to develop an approach. |

Source: D.O. #73 s. 2012



How did you find the performance task? How did the task help you see the real world use of the topic?

**Summary/Synthesis/Generalization:**

This lesson was about solving real-life problems involving quadratic equations. The lesson provided you with opportunities to see the real-life applications of quadratic equations. Moreover, you were given opportunities to formulate and solve quadratic equations based on real-life situations. Your understanding of this lesson and other previously learned mathematics concepts and principles will facilitate your understanding of the succeeding lessons.

## LESSON NO. 7: QUADRATIC INEQUALITIES

### What to KNOW:

Start Lesson 7 of this module by assessing your knowledge of the different mathematics concepts previously studied and your skills in performing mathematical operations. These knowledge and skills will help you in understanding quadratic inequalities. As you go through this lesson, think of this important question: *"How are quadratic inequalities used in solving real-life problems and in making decisions?"* To find the answer, perform each activity. If you find any difficulty in answering the exercises, seek the assistance of your teacher or peers or refer to the modules you have gone over earlier. You may check your work with your teacher.

### Activity 1: What Makes Me True?

*Directions:* Find the solution/s of each of the following mathematical sentences. Answer the questions that follow.



1. $x + 5 > 8$
2. $r - 3 < 10$
3. $2s + 7 \geq 21$
4. $3t - 2 \leq 13$
5. $12 - 5m > -8$
6. $x^2 + 5x + 6 = 0$
7. $t^2 - 8t + 7 = 0$
8. $r^2 + 7r = 18$
9. $2h^2 - 5h - 12 = 0$
10. $9s^2 = 4$

*Questions:*

a. How did you find the solution/s of each mathematical sentence?

b. What mathematics concepts or principles did you apply to come up with the solution/s?

c. Which mathematical sentence has only one solution? More than one solution? Describe these mathematical sentences.

How did you find the activity? Were you able to find the solution/s of each mathematical sentence? Did you find difficulty in solving each mathematical sentence? If not, then you are ready to proceed to the next activity.

**Activity 2: Which Are Not Quadratic Equations?**

*Directions:* Use the mathematical sentences below to answer the questions that follow.

| | | | |
|---|---|---|---|
| $x^2 + 9x + 20 = 0$ | $2t^2 < 21 - 9t$ | $r^2 + 10r \leq -16$ | $3w^2 + 12w \geq 0$ |
| $2s^2 + 7s + 5 > 0$ | $15 - 6h^2 = 10$ | $4x^2 - 25 = 0$ | $m^2 = 6m - 7$ |

1. Which of the given mathematical sentences are quadratic equations?

2. How do you describe quadratic equations?

3. Which of the given mathematical sentences are not quadratic equations? Why?

4. How would you describe those mathematical sentences which are not quadratic equations?

   How are they different from those equations which are quadratic?



In the activity done, were you able to distinguish mathematical sentences which are quadratic equations and which are not quadratic equations? Were you able to describe mathematical sentences that make use of equality and inequality symbols? In the next activity, you will learn how mathematical sentences involving inequalities are illustrated in real life.



**Activity 3: Let's Do Gardening!**

Directions: Use the situation below to answer the questions that follow.

*Mr. Bayani has a vacant lot in his backyard. He wants to make as many rectangular gardens as possible such that the length of each garden is 2 m longer than its width. He also wants the length of the garden of smallest area to be 3 m.*

1. Illustrate the different rectangular gardens that Mr. Bayani could make.

2. What are the dimensions of the different gardens that Mr. Bayani wants to make?
3. What is the area of each garden in item 2?

4. What is the area of the smallest garden that Mr. Bayani can make? How about the area of the largest garden? Explain your answer.

5. What general mathematical sentence would represent the possible areas of the gardens? Describe the sentence.

6. Using the mathematical sentence formulated, do you think you can find other possible dimensions of the gardens that Mr. Bayani wants to make? If YES, how? If NOT, explain.

7. Suppose the length of each garden that Mr. Bayani wants to make is 3 m longer than its width and the area of the smallest garden is 10 $m^2$. What general mathematical sentence would represent the possible areas of the gardens? How are you going to solve the mathematical sentence formulated? Find at least 3 possible solutions of the mathematical sentence.

8. Draw a graph to represent the solution set of the mathematical sentence formulated in item 7. What does the graph tell you?

9. Are all solutions that can be obtained from the graph true to the given situation? Why?

How did you find the preceding activities? Do you think you are already equipped with those knowledge and skills needed to learn the new lesson? I'm sure you are!!! From the activities done, you were able to find the solutions of different mathematical sentences. You were able to differentiate quadratic equations from those which are not. More important, you were able to perform an activity that will lead you in understanding the new lesson. But how are quadratic inequalities used in solving real-life problems and in making decisions? You will find these out in the activities in the next section. Before doing these activities, read and understand first some important notes on Quadratic Inequalities and the examples presented.



A **quadratic inequality** is an inequality that contains a polynomial of degree 2 and can be written in any of the following forms.

| | |
|---|---|
| $ax^2 + bx + c > 0$ | $ax^2 + bx + c \geq 0$ |
| $ax^2 + bx + c < 0$ | $ax^2 + bx + c \leq 0$ |

where a, b, and c are real numbers and a ≠ 0.

*Examples:* 1. $2x^2 + 5x + 1 > 0$ 3. $-3r^2 + r - 5 \geq 0$

2. $s^2 - 9 < 2s$ 4. $t^2 + 4t \leq 10$

To solve a quadratic inequality, find the roots of its corresponding equality. The points corresponding to the roots of the equality, when plotted on the number line, separates the line into two or three intervals. An interval is part of the solution of the inequality if a number in that interval makes the inequality true.

*Example 1:* Find the solution set of $x^2 + 7x + 12 > 0$.

The corresponding equality of $x^2 + 7x + 12 > 0$ is $x^2 + 7x + 12 = 0$.

Solve $x^2 + 7x + 12 = 0$.

| | |
|---|---|
| $(x+3)(x+4) = 0$ | Why? |
| $x+3 = 0$ and $x+4 = 0$ | Why? |
| $x = -3$ and $x = -4$ | Why? |

Plot the points corresponding to -3 and -4 on the number line.

The three intervals are: $-\infty < x < -4$, $-4 < x < -3$, and $-3 < x < \infty$.

Test a number from each interval against the inequality.

| For $-\infty < x < -4$, let $x$ = -7. | For $-4 < x < -3$, let $x$ = -3.6 | For $-3 < x < \infty$, let $x$ = 0 |
|---|---|---|
| $x^2 + 7x + 12 > 0$<br>$(-7)^2 + 7(-7) + 12 \overset{?}{>} 0$<br>$49 - 49 + 12 \overset{?}{>} 0$<br>$12 > 0$ (True) | $x^2 + 7x + 12 > 0$<br>$(-3.6)^2 + 7(-3.6) + 12 \overset{?}{>} 0$<br>$12.96 - 25.2 + 12 \overset{?}{>} 0$<br>$-0.24 > 0$ (False) | $x^2 + 7x + 12 > 0$<br>$(0)^2 + 7(0) + 12 \overset{?}{>} 0$<br>$0 + 0 + 12 \overset{?}{>} 10$<br>$12 > 10$ (True) |

We also test whether the points $x$ = –3 and $x$ = –4 satisfy the inequality.

| | |
|---|---|
| $x^2 + 7x + 12 > 0$<br>$(-3)^2 + 7(-3) + 12 \overset{?}{>} 0$<br>$9 - 21 + 12 \overset{?}{>} 0$<br>$0 > 0$ (False) | $x^2 + 7x + 12 > 0$<br>$(-4)^2 + 7(-4) + 12 \overset{?}{>} 0$<br>$16 - 28 + 12 \overset{?}{>} 0$<br>$0 > 0$ (False) |

Therefore, the inequality is true for any value of $x$ in the interval $-\infty < x < -4$ or $-3 < x < \infty$, and these intervals *exclude* –3 and –4. The solution set of the inequality is $\{x : x < -4 \text{ or } x > -3\}$, and its graph is shown below.

Note that hollow circles are used in the graph to show that –3 and –4 are not part of the solution set.

Another way of solving the quadratic inequality $x^2 + 7x + 12 > 0$ is by following the procedure in solving quadratic equations. However, there are cases to be considered. Study the procedure in solving the quadratic inequality $x^2 + 7x + 12 > 0$ below. Discuss the reason for each step followed.

Notice that the quadratic expression $x^2 + 7x + 12$ is greater than zero or positive. If we write the expression in factored form, what must be true about its factors?



| | |
|---|---|
| $(x+3)(x+4) > 0$ | Why? |
| Case 1:<br>$(x+3) > 0$ and $(x+4) > 0$ | Why? |
| Case 2:<br>$(x+3) < 0$ and $(x+4) < 0$ | Why? |

| **Case 1**<br>$(x+3) > 0$ and $(x+4) > 0$ | **Case 2**<br>$(x+3) < 0$ and $(x+4) < 0$ | Why? |
|---|---|---|
| $x + 3 > 0$ and $x + 4 > 0$<br>$x > -3$ and $x > -4$ | $x + 3 < 0$ and $x + 4 < 0$<br>$x < -3$ and $x < -4$ | |
| $x > -3$ | $x < -4$ | |

The solution set of the inequality is $\{x : x < -4 \text{ or } x > -3\}$. Why?

To check, consider any number greater than -3 or less than -4. Substitute this number to $x$ in the inequality $x^2 + 7x + 12 > 0$.

Consider -2 and 3 which are both greater than -3.

When $x$ = -2:

$$x^2 + 7x + 12 > 0 \quad \rightarrow \quad (-2)^2 + 7(-2) + 12 \overset{?}{>} 0$$

$$4 - 14 + 12 \overset{?}{>} 0$$

$$2 > 0 \qquad \text{(True)}$$

When $x = 3$:

$$x^2 + 7x + 12 > 0 \quad \rightarrow \quad (3)^2 + 7(3) + 12 \overset{?}{>} 0$$

$$9 + 21 + 12 \overset{?}{>} 0$$

$$42 > 0 \qquad \text{(True)}$$

This shows that $x^2 + 7x + 12 > 0$ is true for values of $x$ greater than -3.

This time, consider -5 and -8 which are both less than -4.

When $x = -5$:

$$x^2 + 7x + 12 > 0 \quad \rightarrow \quad (-5)^2 + 7(-5) + 12 \overset{?}{>} 0$$

$$25 - 35 + 12 \overset{?}{>} 0$$

$$2 > 0 \qquad \text{(True)}$$

When $x = -8$:

$$x^2 + 7x + 12 > 0 \quad \rightarrow \quad (-8)^2 + 7(-8) + 12 \overset{?}{>} 0$$

$$64 - 56 + 12 \overset{?}{>} 0$$

$$20 > 0 \qquad \text{(True)}$$



The inequality $x^2 + 7x + 12 > 0$ is also true for values of $x$ less than -4.

Will the inequality be true for any value of $x$ greater than or equal to -4 but less than or equal to -3?



When $x = -3$:

$$x^2 + 7x + 12 > 0 \quad \rightarrow \quad (-3)^2 + 7(-3) + 12 \overset{?}{>} 0$$

$$9 - 21 + 12 \overset{?}{>} 0$$

$$0 \overset{?}{>} 0 \qquad \text{(Not True)}$$

The inequality is not true for $x = -3$.

When $x = -3.5$:

$$x^2 + 7x + 12 > 0 \quad \rightarrow \quad (-3.5)^2 + 7(-3.5) + 12 \overset{?}{>} 0$$

$$12.25 - 24.5 + 12 \overset{?}{>} 0$$

$$-0.25 \overset{?}{>} 0 \qquad \text{(Not True)}$$

The inequality is not true for $x = -3.5$.

This shows that $x^2 + 7x + 12 > 0$ is not true for values of $x$ greater than or equal to -4 but less than or equal to -3.

*Example 2:* $2x^2 - 5x \le 3$

Rewrite $2x^2 - 5x \le 3$ to $2x^2 - 5x - 3 \le 0$. Why?

Notice that the quadratic expression $2x^2 - 5x - 3$ is less than or equal to zero. If we write the expression in factored form, the product of these factors must be zero or negative to satisfy the inequality. Remember that if the product of two numbers is zero, either one or both factors are zeros. Likewise, if the product of two numbers is negative, then one of these numbers is positive and the other is negative.

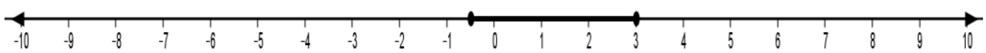
| | |
|---|---|
| $(2x+1)(x-3) \le 0$ | Why? |
| Case 1: $(2x+1) \le 0$ and $(x-3) \ge 0$ | Why? |
| Case 2: $(2x+1) \ge 0$ and $(x-3) \le 0$ | Why? |
| | |



| **Case 1** $(2x+1) \le 0$ and $(x-3) \ge 0$ | **Case 2** $(2x+1) \ge 0$ and $(x-3) \le 0$ | Why? |
|---|---|---|
| $2x + 1 \le 0$ and $x - 3 \ge 0$<br>$2x \le -1$ and $x \ge 3$<br>$x \le -\frac{1}{2}$ and $x \ge 3$ | $2x + 1 \ge 0$ and $x - 3 \le 0$<br>$2x \ge -1$ and $x \le 3$<br>$x \ge -\frac{1}{2}$ and $x \le 3$ | |
| No solution | $-\frac{1}{2} \le x \le 3$ | |

The solution set of the inequality is $\left\{x : -\frac{1}{2} \le x \le 3\right\}$. Why?

The figure below shows the graph of the solution set of the inequality.



Note that $-\frac{1}{2}$ and 3 are represented by points, to indicate that they are part of the solution set.

To check, consider any number greater than or equal to $-\frac{1}{2}$ but less than or equal to 3. Substitute this number to $x$ in the inequality $2x^2 - 5x \le 3$.

When $x$ = 0:

$$2x^2 - 5x \le 3 \quad \rightarrow \quad 2(0)^2 - 5(0) \overset{?}{\le} 3$$
$$0 - 0 \overset{?}{\le} 3$$
$$0 \le 3 \qquad \text{(True)}$$

When $x$ = 2:

$$2x^2 - 5x \le 3 \quad \rightarrow \quad 2(2)^2 - 5(2) \overset{?}{\le} 3$$
$$8 - 10 \overset{?}{\le} 3$$
$$-2 \le 3 \qquad \text{(True)}$$

This shows that $2x^2 - 5x \le 3$ is true for values of $x$ greater than or equal to $-\frac{1}{2}$ but less than or equal to 3.



| Will the inequality be true for any value of $x$ less than $-\frac{1}{2}$ or greater than 3? |
|---|



When $x$ = -2:

$$2x^2 - 5x \le 3 \quad \rightarrow \quad 2(-2)^2 - 5(-2) \overset{?}{\le} 3$$
$$8 + 10 \overset{?}{\le} 3$$
$$18 \overset{?}{\le} 3 \qquad \text{(Not True)}$$

When $x$ = 5:

$$2x^2 - 5x \le 3 \quad \rightarrow \quad 2(5)^2 - 5(5) \overset{?}{\le} 3$$
$$50 - 25 \overset{?}{\le} 3$$
$$25 \overset{?}{\le} 3 \qquad \text{(Not True)}$$

This shows that $2x^2 - 5x \le 3$ is not true for values of $x$ less than $-\frac{1}{2}$ or greater than 3.

There are quadratic inequalities that involve two variables. These inequalities can be written in any of the following forms.

| | |
|---|---|
| $y > ax^2 + bx + c$ | $y \geq ax^2 + bx + c$ |
| $y < ax^2 + bx + c$ | $y \leq ax^2 + bx + c$ |

where a, b, and c are real numbers and a ≠ 0.

*Examples:* 1. $y < x^2 + 3x + 2$ 3. $y + 9 \geq -4x^2$

2. $y > 2x^2 - 5x + 1$ 4. $y - 7x \leq 2x^2$

How are quadratic inequalities in two variables illustrated in real life?

The following is a situation where quadratic inequality in two variables is illustrated.

*The city government is planning to construct a new children's playground. It wants to fence in a rectangular ground using one of the walls of a building. The length of the new playground is 15 m longer than it is wide and its area is greater than the old playground.*



In the given situation, the width of the room can be represented by *w* and the length by *w + 15*. Why?

If we represent the area of the old playground as *A*, then the quadratic inequality that would represent the given situation is $w(w + 15) > A$ or $w^2 + 15w > A$. Why?



*Suppose the area of the old playground is 2,200 m². What could be the area of the new playground? What could be its length and width? Is it possible that the value of w is negative? Why?*

The situation tells us that the area of the new playground is greater than the area of the old playground. This means that the area of the new playground is greater than 2,200. It could be 2,300, 3,500, 4,600, and so on.

One possible pair of dimensions of the new playground is 50 m and 65 m, respectively. With these dimensions, the area of the new playground is $(50\text{m})(65\text{m})$ or 3,250 $\text{m}^2$.

It is not possible for *w* to take a negative value because the situation involves measures of length.

The solution set of quadratic inequalities in two variables can be determined graphically. To do this, write the inequality as an equation, then show the graph. The graph of the resulting parabola will be used to graph the inequality.

*Example 1:* Find the solution set of $y < x^2 + 3x + 2$.

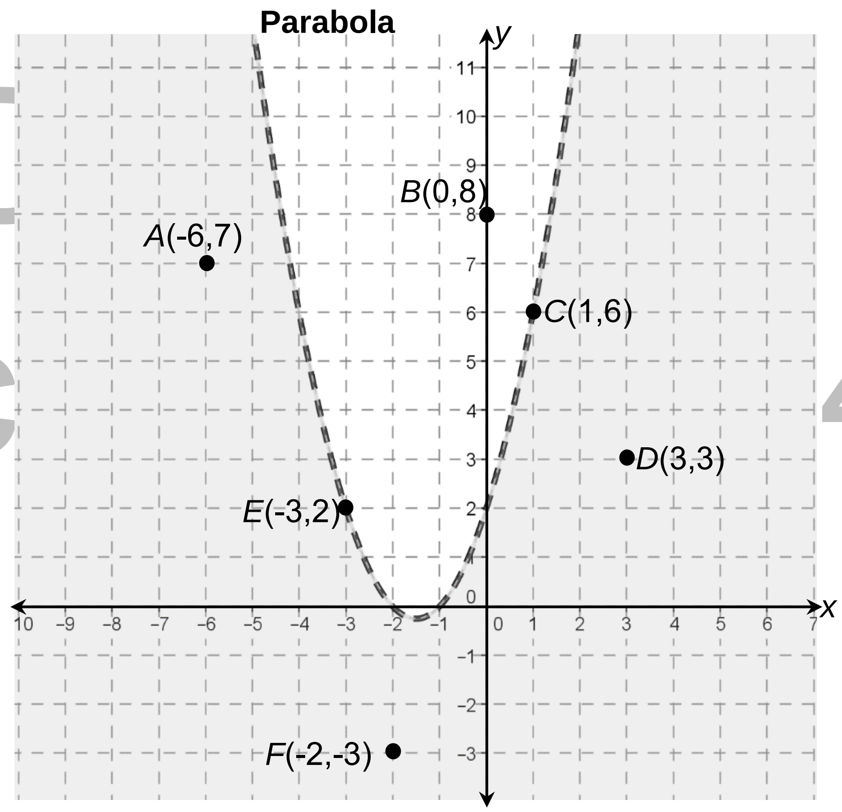
Write the inequality to its corresponding equation.

$y < x^2 + 3x + 2 \quad \rightarrow \quad y = x^2 + 3x + 2$

Construct table of values for *x* and *y*.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| *x* | -5 | -4 | -3 | -2 | -1 | 0 | 1 | 2 |
| *y* | 12 | 6 | 2 | 0 | 0 | 2 | 6 | 12 |

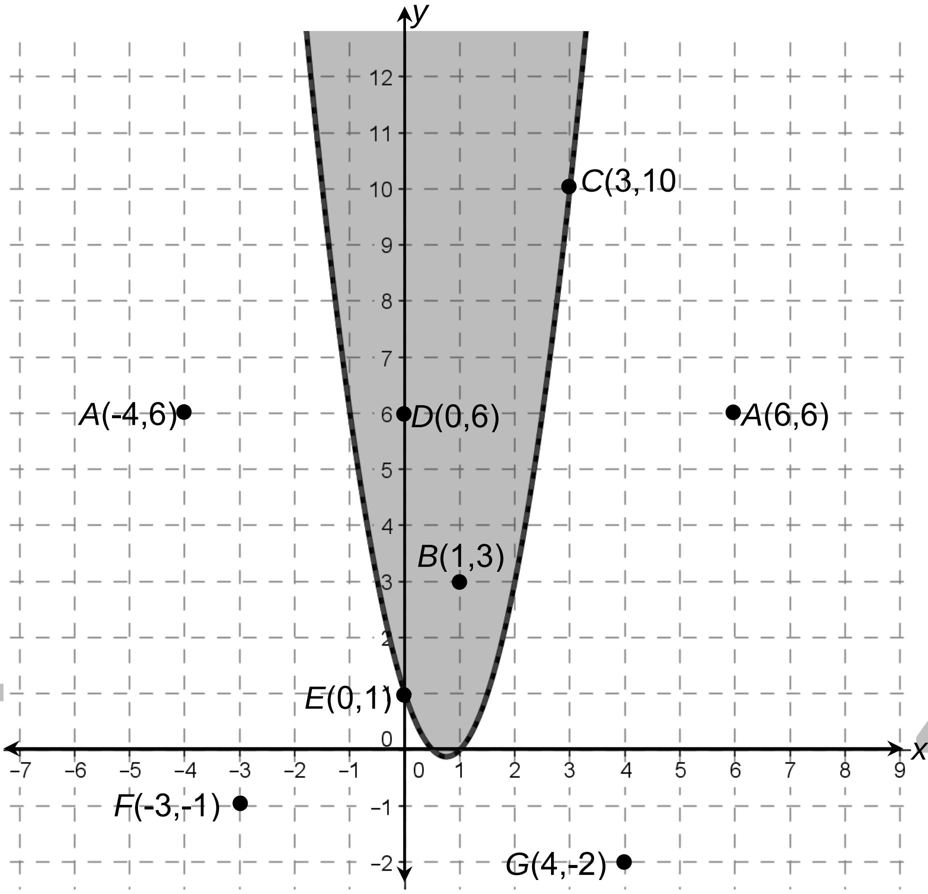
Use these points to graph a parabola. Points *B*(0,8), *C*(1,6), and *E*(-3,2) are points along the parabola. The coordinates of these points do not satisfy the inequality $y < x^2 + 3x + 2$. Therefore, they are *not* part of the solution set of the inequality. We use a broken line to represent the parabola since the points on the parabola do *not* satisfy the given inequality.




The parabola partitions the plane into two regions. Select one point in each region and check whether the given inequality is satisfied. For example, consider the origin (0, 0), and substitute this in the inequality. We obtain 0 < 0 + 0 + 2 or 0 < 2, which is correct. Therefore, the entire region containing (0, 0) represents the solution set and we shade it. On the other hand, the point (0, 8) is on the other region. If we substitute this in the inequality, we obtain 8 < 0 + 0 + 2 or 8 < 2, which is false. Therefore, this

region is *not* part of the solution set and we do not shade this region.
To check, points *A*(-6,7), *D*(3,3), and *F*(-2,-3) are some of the points in the shaded region. If the coordinates of these points are substituted in $y < x^2 + 3x + 2$, the inequality becomes true. Hence, they are part of the solution set.

*Example 2:* Find the solution set of $y \geq 2x^2 - 3x + 1$.






The figure in the preceding page shows the graph of $y \geq 2x^2 - 3x + 1$. All points in the shaded region including those along the solid line (parabola) make up the solution set of the inequality. The coordinates of any point in this region make the inequality true. Points *B*(1,3), *C*(3,10), *D*(0,6), and *E*(0,1) are some of the points on the shaded region and along the parabola. The coordinates of these points satisfy the inequality $y \geq 2x^2 - 3x + 1$.

Consider point *B* whose coordinates are (1,3). If $x$ = 1 and $y$ = 3 are substituted in the inequality, then the mathematical statement becomes true.

$$y \geq 2x^2 - 3x + 1 \quad \rightarrow \quad 3 \overset{?}{\geq} 2(1)^2 - 3(1) + 1$$

$$3 \overset{?}{\geq} 2 - 3 + 1$$

$$3 \geq 0$$

Hence, (1,3) is a solution to the inequality.

Learn more about Quadratic Inequalities through the WEB. You may open the following links.

http://www.mathwarehouse.com/quadratic-inequality/how-to-solve-and-graph-quadratic-inequality.php

http://www.regentsprep.org/regents/math/algtrig/ate6/quadinequal.htm

http://www.mathsisfun.com/algebra/inequality-quadratic-solving.html

http://www.wtamu.edu/academic/anns/mps/math/mathlab/col_algebra/col_alg_tut23_quadineq.htm

http://www.shelovesmath.com/algebra/intermediate-algebra/quadratic-inequalities/

http://www.cliffsnotes.com/math/algebra/algebra-ii/quadratics-in-one-variable/solving-quadratic-inequalities



**What to PROCESS:**

Your goal in this section is to apply the key concepts of quadratic inequalities. Use the mathematical ideas and the examples presented in the preceding section to answer the activities provided.

**Activity 4: Quadratic Inequalities or Not?**

*Directions:* Determine whether each mathematical sentence is a quadratic inequality or not. Answer the questions that follow.

1. $x^2 + 9x + 14 > 0$
2. $3s^2 - 5s = 1$
6. $3m + 20 \geq 0$
7. $(2r - 5)(r + 4) > 0$

3. $4t^2 - 7t + 2 \le 0$

4. $x^2 < 10x - 3$

5. $12 - 5x + x^2 = 0$

8. $x^2 - 1 < x + 1$

9. $(4h^2 - 9) + (2h + 3) \ge 0$

10. $15 - 2x = 3x^2$

*Questions:*

a. How do you describe quadratic inequalities?

b. How are quadratic inequalities different from linear inequalities?

c. Give at least three examples of quadratic inequalities.

| |
|---|
| Were you able to determine which mathematical sentences are quadratic inequalities? In the next activity, you will find and describe the solution sets of quadratic inequalities. |

**Activity 5: Describe My Solutions!**



*Directions:* Find the solution set of each of the following quadratic inequalities then graph. Answer the questions that follow.

1. $x^2 + 9x + 14 > 0$

2. $r^2 - 10r + 16 < 0$

3. $x^2 + 6x \ge -5$

4. $m^2 - 7m \le 10$

5. $x^2 - 5x - 14 > 0$

6. $2t^2 + 11t + 12 < 0$

7. $3x^2 - 8x + 5 \ge 0$

8. $4p^2 \le 1$

9. $2x^2 - 3x - 14 > 0$

10. $3q^2 + 2q \ge 5$

*Questions:*

a. How did you find the solution set of each inequality?

b. What mathematics concepts or principles did you apply to come up with the solution set of each mathematical sentence?

c. How did you graph the solution set of each inequality?

d. How would you describe the graph of the solution set of a quadratic inequality?

e. How many solutions does each inequality have?

f. Are the solution/s of each inequality real numbers? Why?

g. Is it possible for a quadratic inequality not to have a real solution? Justify your answer by giving a particular example.

> Were you able to able to find and describe the solution set of each quadratic inequality? Were you able to show the graph of the solution set of each? In the next activity, you will determine if a point is a solution of a given quadratic inequality in two variables.

**Activity 6: Am I a Solution or Not?**

*Directions:* Determine whether or not each of the following points is a solution of the inequality $y < 2x^2 + 3x - 5$. Justify your answer.

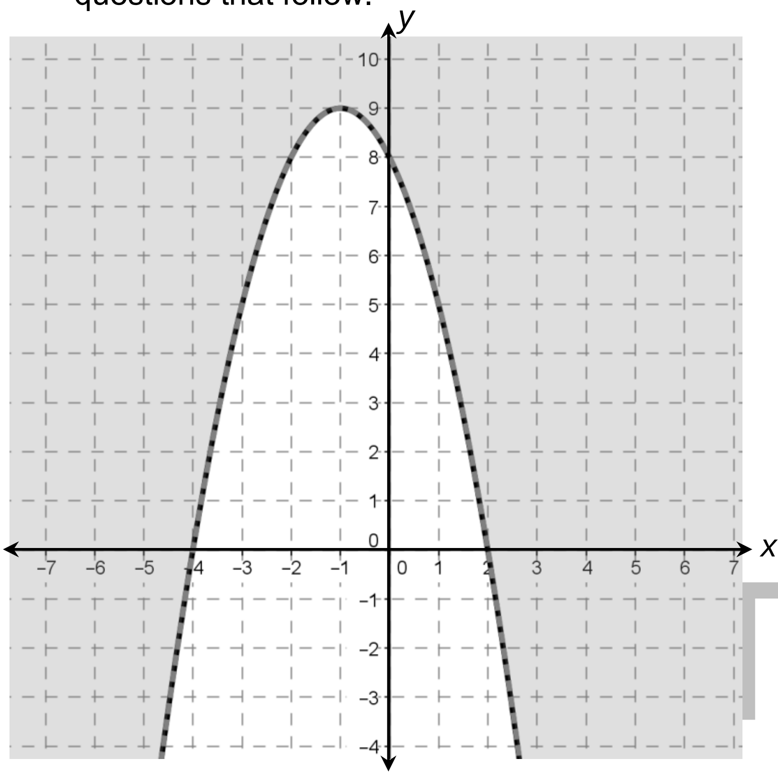
1. $A(-1,6)$
2. $B(1,8)$
3. $C(-5,10)$
4. $D(3,6)$
5. $E(-3,4)$
6. $F(2,9)$
7. $G(-6,-2)$
8. $H(1,-4)$
9. $I(0,0)$
10. $J(-6,7)$

> How did you find the activity? Was it easy for you to determine if a point is a solution of the given inequality? Could you give other points that belong to the solution set of the inequality? I'm sure you could. In the next activity, you will determine the mathematical sentence that is described by a graph.

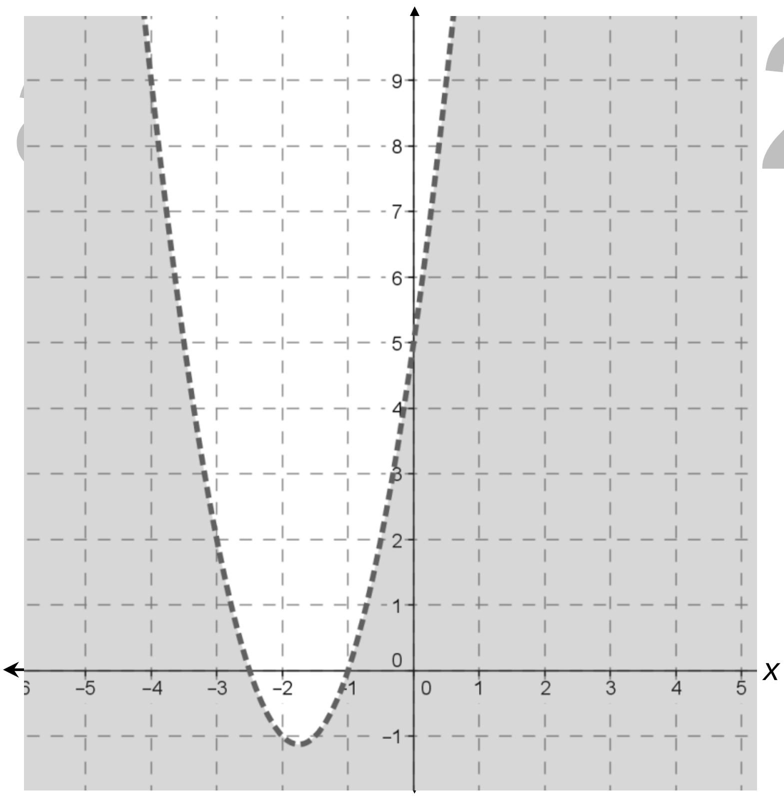
**Activity 7: What Represents Me?**

*Directions:* Select from the list of mathematical sentences on the right side the inequality that is described by each of the following graphs. Answer the questions that follow.

1.




2.




$y > x^2 - 2x + 8$

$y < 2x^2 + 7x + 5$

$y \geq -x^2 - 2x + 8$

$y \geq 2x^2 + 7x + 5$

$y < -x^2 - 2x + 8$

$y > 2x^2 + 7x + 5$

$y \leq -x^2 - 2x + 8$

$y \leq 2x^2 + 7x + 5$

$y > -x^2 - 2x + 8$

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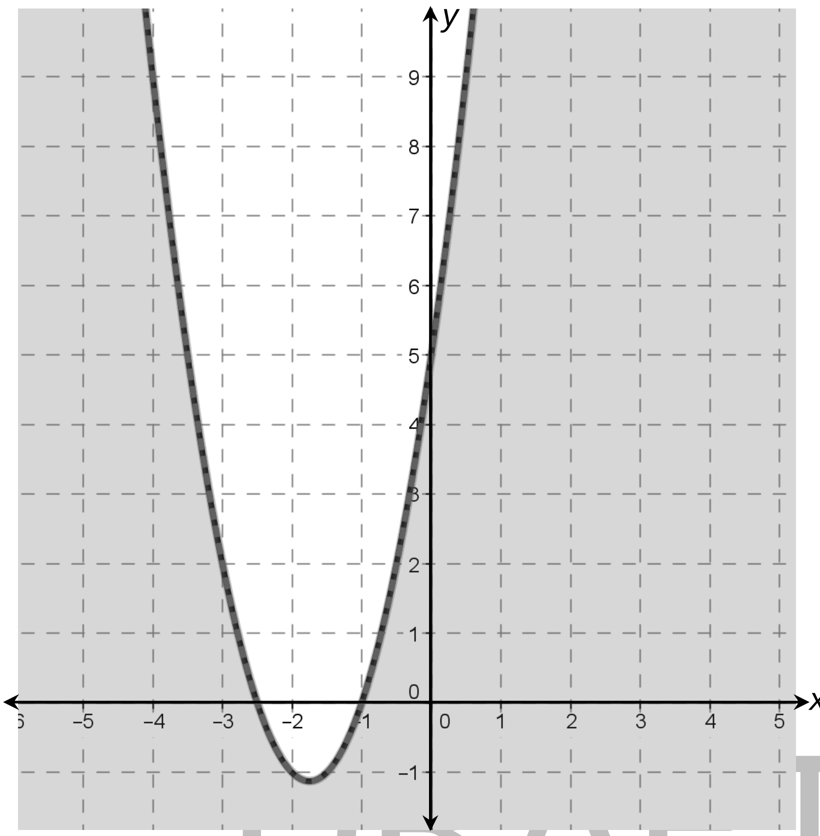


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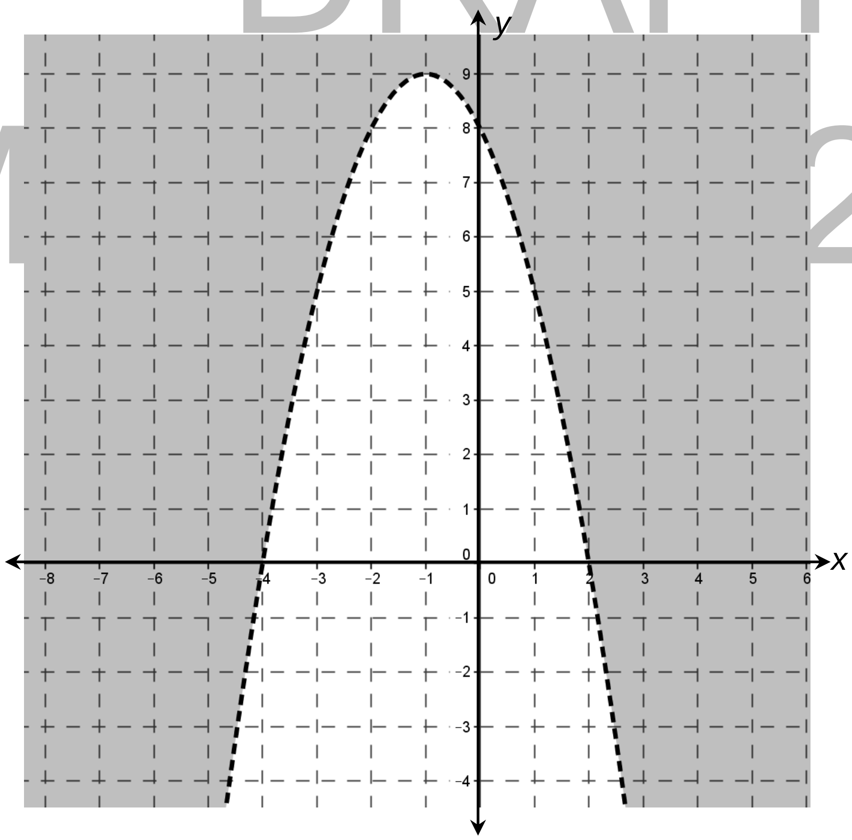



$y > x^2 - 2x + 8$

$y < 2x^2 + 7x + 5$

$y \geq -x^2 - 2x + 8$

$y \geq 2x^2 + 7x + 5$

$y < -x^2 - 2x + 8$

$y > 2x^2 + 7x + 5$

$y \leq -x^2 - 2x + 8$

$y \leq 2x^2 + 7x + 5$

$y > -x^2 - 2x + 8$

*Questions:*

a. How did you determine the quadratic inequality that is described by a given graph?

b. In each graph, what does the shaded region represent?

c. How do the points in the shaded region of each graph facilitate in determining the inequality that defines it?

d. How would you describe the graphs of quadratic inequalities in two variables involving "less than"? "greater than"? "less than or equal to"? "greater than or equal to"?

e. Suppose you are given a quadratic inequality in two variables. How will you graph it?

Were you able to identify the inequality that is described by each graph? Was it easy for you to perform this activity? I'm sure it was! If not, then try to find an easier way of doing this activity. I know you can do it. In the next activity, you will work on a situation involving quadratic inequalities. In this activity, you will further learn how quadratic inequalities are illustrated in real life.



**Activity 8: Make It Real!**

Directions: Read the situation below then answer the questions that follow.

*The floor of a conference hall can be covered completely with tiles. Its length is 36 ft. longer than its width. The area of the floor is less than 2,040 square feet.*

1. How would you represent the width of the floor?

   How about its length?

2. What mathematical sentence would represent the given situation?

3. What are the possible dimensions of the floor?

   How about the possible areas of the floor?

4. Would it be realistic for the floor to have an area of 144 square feet? Explain your answer.

In this section, the discussion was about quadratic inequalities and their solution sets and graphs.

Go back to the previous section and compare your initial ideas with the discussion. How much of your initial ideas are found in the discussion? Which ideas are different and need revision?

Now that you know the important ideas about this topic, let's go deeper by moving on to the next section.

**What to REFLECT ON and FURTHER UNDERSTAND:**

Your goal in this section is to take a closer look at some aspects of the topic. You are going to think deeper and test further your understanding of quadratic inequalities. After doing the following activities, you should be able to answer the following question: *"How are quadratic inequalities used in solving real-life problems and in making decisions?"*



**Activity 9: How Well I Understood…**

*Directions:* Answer the following.

1. How do you describe quadratic inequalities?

2. Give at least three examples of quadratic inequalities.

3. How do you find the solution set of a quadratic inequality in one variable?

   How about quadratic inequalities in two variables?

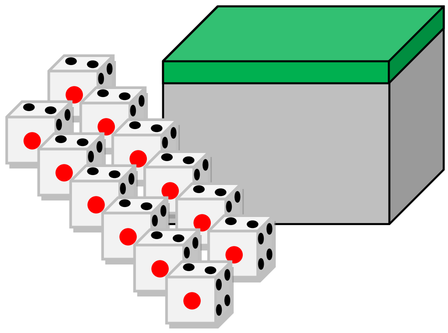
4. How would you describe the solution set of each of the following quadratic inequalities?

   a. $y < x^2 + 9x + 14$
   b. $y > x^2 - 3x - 18$
   c. $y \leq 2x^2 + 11x + 5$
   d. $y \geq 3x^2 + 10x - 8$

5. Do you agree that the solution sets of $y < x^2 + x - 20$ and $y > x^2 + x - 20$ is the set of all points on a plane? Justify your answer by graphing the solution set of each on a coordinate plane.

6. Luisa says that the solutions of $y > 2x^2 - 8x + 7$ are also solutions of $y > x^2 - 4x + 3$. Do you agree with Luisa? Justify your answer.

7. A rectangular box is completely filled with dice. Each die has a volume of 1 $cm^3$. The length of the box is 3 cm greater than its width and its height is 5 cm. Suppose the box holds at most 140 dice. What are the possible dimensions of the box?



8. A company decided to increase the size of the box for the packaging of their canned sardines. The length of the original packaging box was 40 cm longer than its width, the height was 12 cm, and the volume was at most 4,800 $cm^3$.
   a. How would you represent the width of the original packaging box? How about the length of the box?

   b. What expression would represent the volume of the original packaging box?

      How about the mathematical sentence that would represent its volume? Define the variables used.

   c. What could be the greatest possible dimensions of the box if each dimension is in whole centimeters? Explain how you arrived at your answer.

   d. Suppose the length of the new packaging box is still 40 cm longer than its width and the height is 12 cm. What mathematical sentence would represent the volume of the new packaging box? Define the variables used.

   e. What could be the dimensions of the box? Give the possible dimensions of at least three different boxes.



In this section, the discussion was about your understanding of quadratic inequalities and their solution sets and graphs.

What new insights do you have about quadratic inequalities and their solution sets and graphs? How would you connect this to real life? How would you use this in making decisions?

Now that you have a deeper understanding of the topic, you are ready to do the tasks in the next section.

## What to TRANSFER:

Your goal in this section is to apply your learning to real-life situations. You will be given a practical task which will demonstrate your understanding.

### Activity 10: Investigate Me!

*Directions:* Conduct a mathematical investigation for each of the following quadratic inequalities. Prepare a written report of your findings following the format at the right.

1. $ax^2 + bx + c > 0$, where $b^2 - 4ac < 0$

2. $ax^2 + bx + c < 0$, where $b^2 - 4ac < 0$

3. $ax^2 + bx + c \geq 0$, where $b^2 - 4ac < 0$

4. $ax^2 + bx + c \leq 0$, where $b^2 - 4ac < 0$

**Mathematical Investigation Written Report Format**

- Title/Topic For Investigation
- Introduction
- Exploration
- Conjectures
- Testing Of Conjectures
- Justification For Conjectures
- Conclusion



### Activity 11: How Much Would It Cost to Tile a Floor?

*Directions:* Perform the following activity.

1. Find the dimensions of the floors of at least two rooms in your school. Indicate the measures obtained in the table below.

| Rooms | Length | Width |
|---|---|---|
| | | |
| | | |
| | | |
| | | |
| | | |
| | | |

2. Determine the measures and costs of different tiles that are available in the nearest hardware store or advertised in any printed materials or in the internet. Write these in the table below.

| Tiles | Length | Width | Cost |
|---|---|---|---|
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |

3. Formulate quadratic inequalities involving the dimensions of the floor of rooms, and the measures and costs of tiles. Find, then graph the solution sets of these inequalities. Use the rubric provided to rate your work.

Rubric for Real-Life Situations Involving Quadratic Inequalities and Their Solution Sets and Graphs

| 4 | 3 | 2 | 1 |
|---|---|---|---|
| Systematically listed in the table the dimensions of rooms and the measures and costs of tiles, properly formulated and solved quadratic inequalities, and accurately drew the graphs of their solution sets. | Systematically listed in the table the dimensions of rooms and the measures and costs of tiles, and properly formulated and solved quadratic inequalities but unable to draw the graph accurately. | Systematically listed in the table the dimensions of rooms and the measures and costs of tiles, and properly formulated quadratic inequalities but unable to solve these. | Systematically listed in the table the dimensions of rooms and the measures and costs of tiles. |

In this section, your task was to formulate and solve quadratic inequalities based on real-life situations. You were placed in a situation wherein you need to determine the number and total cost of tiles needed to cover the floors of some rooms.

How did you find the performance task? How did the task help you realize the use of the topic in real life?

**Summary/Synthesis/Generalization:**

This lesson was about quadratic inequalities and their solution sets and graphs. The lesson provided you with opportunities to describe quadratic inequalities and their solution sets using practical situations, mathematical expressions, and their graphs. Moreover, you were given the opportunity to draw and describe the graphs of quadratic inequalities and to demonstrate your understanding of the lesson by doing a practical task. Your understanding of this lesson and other previously learned mathematics concepts and principles will facilitate your learning of the next lesson, Quadratic Functions.

**GLOSSARY OF TERMS**

1. Discriminant – This is the value of the expression $b^2 - 4ac$ in the quadratic formula.

2. Extraneous Root or Solution – This is a solution of an equation derived from an original equation. However, it is not a solution of the original equation.

3. Irrational Roots – These are roots of equations which cannot be expressed as quotient of integers.

4. Quadratic Equations in One Variable – These are mathematical sentences of degree 2 that can be written in the form $ax^2 + bx + c = 0$.

5. Quadratic Formula – This is an equation that can be used to find the roots or solutions of the quadratic equation $ax^2 + bx + c = 0$. The quadratic formula is $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

6. Quadratic Inequalities – These are mathematical sentences that can be written in any of the following forms: $ax^2 + bx + c > 0$, $ax^2 + bx + c < 0$, $ax^2 + bx + c \geq 0$, and $ax^2 + bx + c \leq 0$.

7. Rational Algebraic Equations – These are mathematical sentences that contain rational algebraic expressions.

8. Rational Roots – These are roots of equations which can be expressed as quotient of integers.

9. Solutions or Roots of Quadratic Equations – These are the values of the variable/s that make quadratic equations true.

10. Solutions or Roots of Quadratic Inequalities – These are the values of the variable/s that make quadratic inequalities true.

**DEPED INSTRUCTIONAL MATERIALS THAT CAN BE USED AS ADDITIONAL RESOURCES FOR THE LESSON QUADRATIC EQUATIONS AND INEQUALITIES:**

1. EASE Modules Year II Modules 1, 2 and 3

2. BASIC EDUCATION ASSISTANCE FOR MINDANAO (BEAM) Mathematics 8 Module 4 pp. 1-55

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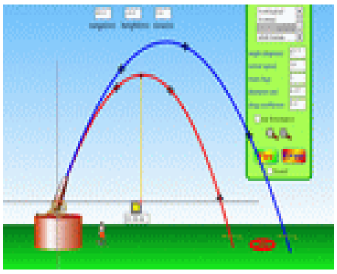
# MODULE 2. QUADRATIC FUNCTIONS

## I. Introduction

Have *you* ever asked *yourself* why PBA star players are good in free throws? How do angry bird expert players hit their targets? Do *you* know the secret key in playing this game? What is the maximum height reached by an object thrown vertically upward given a particular condition?

http://www.popularmechanics.com/cm/popularmechanics/images/y6/free-throw-0312-mdn.jpg


http://sites.davidson.edu/mathmovement/wp content/uploads/2011/11/trajectories.png



http://web.mnstate.edu/lindaas/phys160/lab/Sims/projectileMotion.gif -throw-0312-mdn.jpg

One of the most interesting topics in mathematics is the quadratic function. It has many applications and has played a fundamental role in solving many problems related to human life. In this module, *you* will be able to learn important concepts in quadratic functions which will enable *you* to answer the questions above. Moreover, *you* will also deal with the most common applications of quadratic functions.



## II. Lesson and Coverage

This module consists of five lessons namely:

Lesson 1- Introduction to Quadratic Functions

Lesson 2 - Graphs of Quadratic Functions

Lesson 3 - Finding the Equation of Quadratic Function

Lesson 4 - Applications of Quadratic Functions

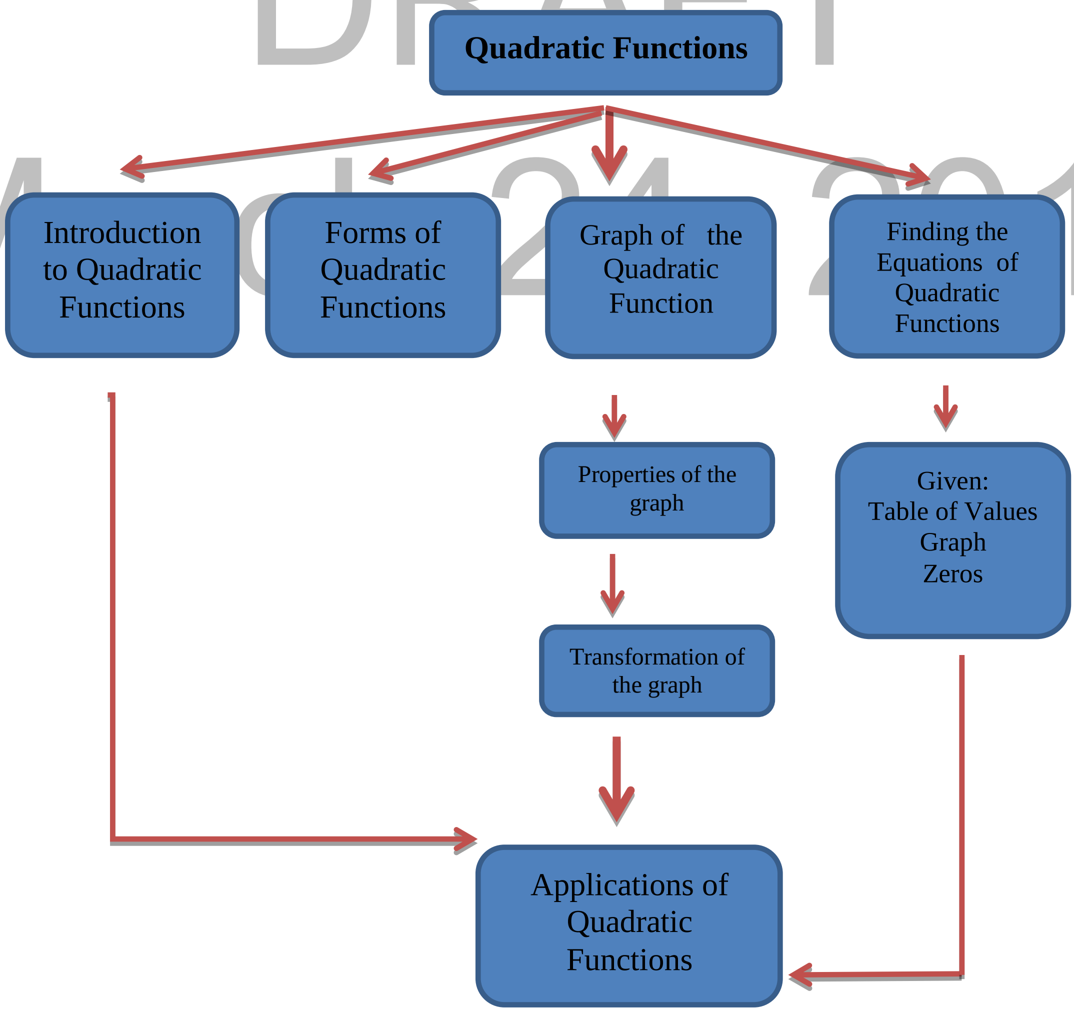
In this module, *you* will learn to:

| | |
|---|---|
| Lesson 1 | • model real life situations using quadratic functions<br>• differentiate quadratic functions from linear or other functions.<br>• represent and identify the quadratic function given<br>- table of values<br>- graphs<br>- equation<br>• transform the quadratic function in general form $y = ax^2 + bx + c$ into standard form (vertex form) $y = a(x - h)^2 + k$ and vice versa. |

| | |
|---|---|
| Lesson 2 | • draw the graph of the quadratic function<br>• given a quadratic function, determine the following: domain, range, intercepts, axis of symmetry and the opening of the parabola.<br>• investigate and analyze the effects of changes in the variables a, h and k in the graph of quadratic functions $y = a(x - h)^2 + k$ and make generalizations.<br>• apply the concepts learned in solving real life problems. |
| Lesson 3 | • determine the zeros of quadratic functions<br>• derive the equation of the quadratic function given<br>- table of values<br>- graphs<br>- zeros<br>• apply the concepts learned in solving real life problems. |
| Lesson 4 | • solve problems involving quadratic function |

**Module Map**





## III. Pre -Assessment

Part I. Find out how much *you* already know about this module. Write the letter that *you* think is the best answer to each question on a sheet of paper. Answer all items. After taking and checking this short test, take note of the items that *you* were not able to answer correctl*y* and look for the right answer as *you* go through this module.

1. Which of the following equations represents a quadratic function?

   a. $y = 3 + 2x^2$ b. $2y^2 + 3 = x$

   c. $y = 3x - 2^2$ d. $y = 2^x - 3$

2. The quadratic function $f(x) = x^2 + 2x - 1$ is expressed in standard form as

   a. $f(x) = (x + 1)^2 + 1$ b. $f(x) = (x + 1)^2 - 2$

   c. $f(x) = (x + 1)^2 + 2$ d. $f(x) = (x + 1)^2 - 1$

3. What is $f(x) = -3(x + 2)^2 + 2$ when written in the form $f(x) = ax^2 + bx + c$?

   a. $f(x) = -3x^2 + 12x - 10$ b. $f(x) = 3x^2 - 12x + 10$

   c. $f(x) = -3x^2 + 12x + 10$ d. $f(x) = -3x^2 - 12x - 10$

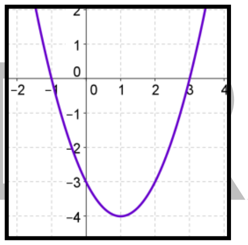
4. The zeros of the quadratic function described by the graph below is

   a. 1, 3

   b. -1, 3

   c. 1, -3

   d. -1, -3




5. The graph of $y = x^2 - 3$ is obtained by sliding the graph of $y = x^2$

   a. 3 units downward b. 3 units upward

   c. 3 units to the right d. 3 units to the left

6. The quadratic function $y = -2x^2 + 4x - 3$ has

   a. real and unequal zeros

   b. real and equal zeros

   c. no real zeros

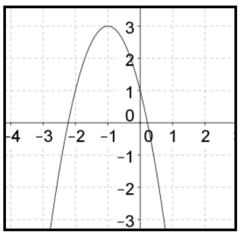
   d. equal and not real

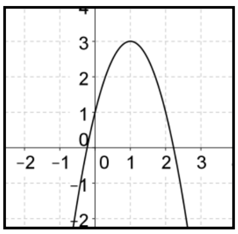
7. What is an equation of a quadratic function whose zeros are twice the zeros of $y = 2x^2 - x - 10$?

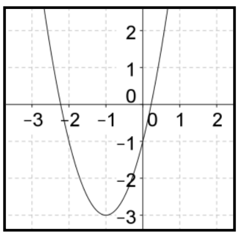
   a. $f(x) = 2x^2 - 20x + 20$ b. $f(x) = x^2 - x - 20$

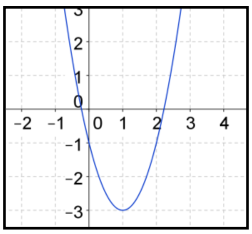
   c. $f(x) = 2x^2 - 2x - 5$ d. $f(x) = 2x^2 - 2x - 10$

8. Which of the following shows the graph of $f(x) = 2(x-1)^2 - 3$




a b. c. d.

9. Richard predicted that the number of mango trees x planted in a farm could yield $y = -20x^2 + 2800x$ mangoes per year. How many trees should be planted to produce the maximum number of mangoes per year?

   a. 60 b.70 c.80 d.90

10. The path of an object when it is thrown can be modeled by $S(t) = -16t^2 + 8t + 4$ where **S** in feet is the height of the object t seconds after it is released. What is the maximum height reached by the object?

   a. 3 ft b. 4 ft c. 5 ft d. 6 ft

11. CJ wrote a function of the path of the stone kicked by Lanlan from the ground. If the equation of the function he wrote is $S(t) = 16t^2 + 8t + 1$, where S is the height of stone in terms of *t* the number of seconds after Lanlan kicks the stone. Which of the statement is true?

   a. CJ's equation is not correct.

   b. CJ's equation described the maximum point reached by the stone.

   c. The equation is possible to the path of the stone.

   d. The equation corresponds to the path of the stone.

12. An object is fired straight up with a velocity of 64 ft/s. Its altitude (height) *h* after *t* seconds is given by $h(t) = -16t^2 + 64t$. When does the projectile hit the ground?

   a. 3 seconds b. 4 seconds c. 5 seconds d. 6 seconds



13. What are the dimensions of the largest rectangular field that can be enclosed with 100m of wire?

   a. 24 m x 26 m b. 25m x 25 m

   c. 50m x 50m d. 50m x 25 m

14. The batter hits the softball and it follows a path in which the height h is given by $h(t) = -2t^2 + 8t + 3$, where t is the time in seconds s elapsed since the ball was pitched. What is the maximum height reached by the softball?

   a. 11 m b. 12m c. 13m d. 14m



**Part II Performance Task**

**Directions:** Apply quadratic functions to solve the problem below. Show your solution.

**Task 1** Being the first grandson, your grandparents decided to give you a rectangular field for your coming wedding. If you are given 200m wires of fencing, what dimensions would you choose to get the maximum area?

a. List all the possible dimensions of the rectangular field.

b. Make a table of values for the possible dimensions.

c. Compute the area for each possible dimension.

d. What is the maximum area you obtained?

e. What are the dimensions of the maximum area you obtained?

**Task 2** You are selling banana bread that costs P 5 each. Each week, you have 50 customers. When you decrease the price by P1, you expect 30 customers to be added. What is the price of the banana bread that yields a maximum profit?

a. Analyze the problem.

b. What is the weekly sale if the cost of the banana bread is P5?

c. If the revenue (R) = number of bread x bread price. Write the equation of the quadratic function given the situation above?

d. What is the price that yields the maximum revenue?

e. Find the maximum revenue.

## IV LEARNING GOALS AND TARGETS

After going through this module, you should be able to demonstrate understanding of the key concepts of quadratic functions and be able to apply these to solve real life problems. You will be able to formulate real life problems involving quadratic functions, and solve them through a variety of techniques with accuracy.



## V INSTRUCTIONAL ACTIVITIES

### Lesson 1. Introduction to Quadratic Function

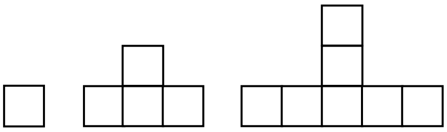
**What to KNOW**

Let us start this lesson by recalling ways of representing a linear function. The knowledge and skills in doing this activity will help you a lot in understanding the quadratic function. In going over this lesson, you will be able to identify quadratic function and represent it in different ways.

**Activity 1. Describe me in many ways!**

**Directions: Perform this activity.**

a. Observe the pattern and draw the 4th and 5th figures.



? ?

1 2 3 4 5

b. Use the table to illustrate the relation of the figure number to the number of blocks.

| Figure Number (x) | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Number of blocks (y) | 1 | 4 | 7 | | |

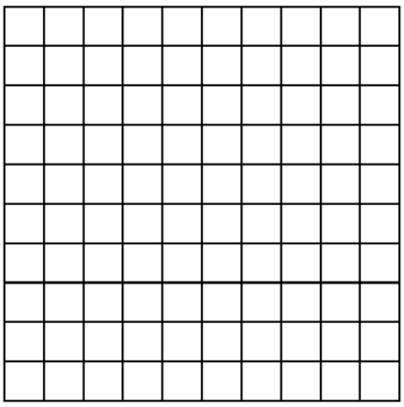
c. Write the pattern observed from the table. _______________

d. List the following:

Set of ordered pairs_______________
Domain _______________ Range _______________

e. What equation describes the pattern? _______________

f. Graph the relation using the Cartesian Plane.



g. What are the independent and dependent variables? _______________

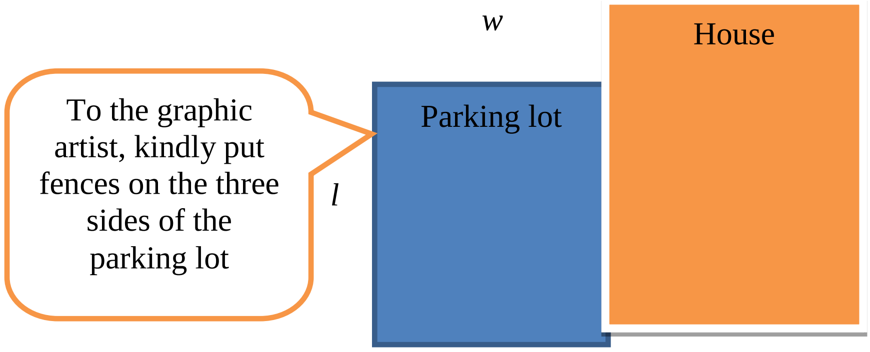
h. What methods are used to describe the relation? _______________

**Activity 2 Parking Lot Problem**

**Directions:** Solve the problem by following the procedure below.

Mr Santos wants to enclose the rectangular parking lot beside his house by putting a wire fence on the three sides as shown in the figure. If the total length of the wire is 80 m, find the dimension of the parking lot that will enclose a maximum area.

Directions. Follow the procedure below:






a. In the figure above, if we let ***w*** be the width and ***l*** be the length, what is the expression for the sum of the measures of the three sides of the parking lot?

b. What is the length of the rectangle in terms of the width?

c. Express the area (*A*) of the parking lot in terms of the width.

d. Fill up the table by having some possible values of *w* and the corresponding areas.

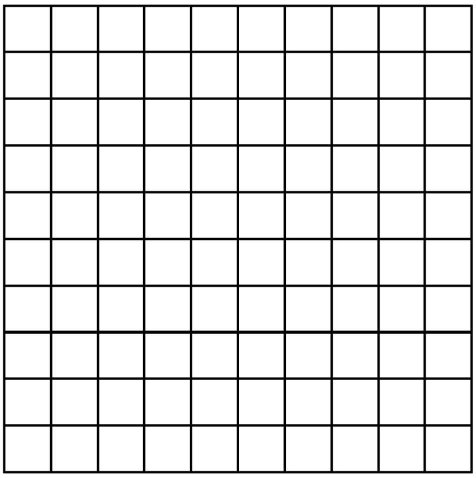
| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Width (*w*) | | | | | | | | |
| Area (*A*) | | | | | | | | |

e. What have you observed about the area (*A*) in relation to the width (*w*)?

f. What is the dependent variable? independent variable?

g. Compare the equation of a linear function with the equation you obtained.

h. From the table of values, plot the points and connect them using a smooth curve.



i. What do you observe about the graph?

j. Does the graph represent a linear function?

How did you find the preceding activity? I hope that you are now ready to learn about quadratic functions. These are functions that can be described by equations of the form **$y = ax^2 + bx + c$,** where, ***a, b*** and ***c*** are real numbers and ***a*** $\neq$ **0.** The highest power of the independent variable x is 2. Thus, the equation of a quadratic function is of degree 2.



**Activity 3 Identify me!**

**Directions:** State whether each of the following equations represents a quadratic function or not. Justify your answer.

| Equations | Yes or No | Justification |
|---|---|---|
| 1. $y = x^2 + 2$ | | |
| 2. $y = 2x - 10$ | | |
| 3. $y = 9 - 2x^2$ | | |
| 4. $y = 2^x + 2$ | | |
| 5. $y = 3x^2 + x^3 + 2$ | | |
| 6. $y = 2^x + 3x + 2$ | | |
| 7. $y = 2x^2$ | | |
| 8. $y = (x - 2)(x + 4)$ | | |
| 9. $0 = (x - 3)(x + 3) + x^2 - y$ | | |
| 10. $3x^3 + y - 2x = 0$ | | |

**Activity 4 Compare Me!** 

**Directions:** Follow the instructions below.

Consider the given functions $f(x) = 2x + 1$ and $g(x) = x^2 + 2x - 1$.

1. What kind of function is $f(x)$? $g(x)$?
2. Complete the following table of values using the indicated function

$f(x) = 2x + 1$

| $x$ | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y$ | | | | | | | |

$g(x) = x^2 + 2x - 1$

| | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y$ | | | | | | | |

3. What are the differences between two adjacent x-values in each table?

4. Find the differences between each adjacent y- values in each table, and write them on the blanks provided.

$f(x) = 2x + 1$

| $x$ | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y$ | | | | | | | |

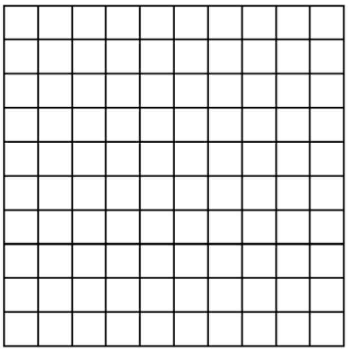
$g(x) = x^2 + 2x - 1$

| $x$ | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y$ | | | | | | | |

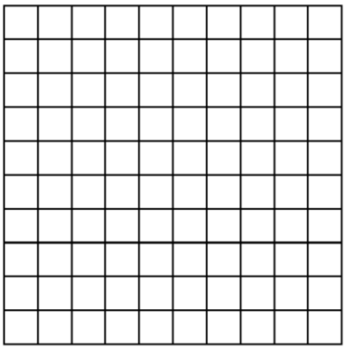
5. What do *y*ou observe?

6. How can you recognize a quadratic function when a table of values is given?

7. Using the table of values, graph the two functions and compare the results.



$y = 2x + 2$



$y = x^2 + 2x - 1$

8. Compare the graph of linear function and quadratic function.

Did you enjoy the activity? You have seen that in a **linear function**, equal differences in x produce equal differences in y. However, in a **quadratic function**, equal differences in x do not lead to equal first differences in y; instead the **second differences** in y are equal. Notice also that the graph of a linear function is a straight line while the graph of a quadratic function is a smooth curve. This smooth curve is ***parabola***.

In a quadratic function, equal differences in the independent variable *x* produce equal second differences in the dependent variable *y*.

**Illustrative example**

Let us consider $y = x^2 - 4$

Differences in *x*: 1 1 1 1 1 1

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| *x* | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
| *y* | 5 | 0 | -3 | -4 | -3 | 0 | 5 |

First Differences in *y*: -5 -3 -1 1 3 5

Second Differences in *y*: 2 2 2 2 2



You have seen in the example above that in the quadratic function **$y = ax^2 + bx + c$**, equal differences in x produce equal second differences in y.

The previous activities familiarized you with the general form **$y = ax^2 + bx + c$** of a quadratic function. In your next activity, the standard form or vertex form **$y = a(x - h)^2 + k$** will be introduced. The standard form will be more convenient to use when working on problems involving the vertex of the graph of a quadratic function.

Study the illustrative examples presented below.

**Example 1**

Express $y = 3x^2 - 4x + 1$ in the form $y = a(x - h)^2 + k$ form and give the values of h and k.

**Solution:**

$y = 3x^2 - 4x + 1$

$y = (3x^2 - 4x) + 1$ Group together the terms containing x

$y = 3(x^2 - \frac{4}{3}x) + 1$ Factor out ***a.*** Here *a* = 3

$y = 3\left[x^2 - \frac{4}{3}x + \left(\frac{2}{3}\right)^2\right] + 1 - 3\left(\frac{2}{3}\right)^2$ Compete the expression in parenthesis to make it a perfect square trinomial by adding the constant $3\left(\frac{\frac{4}{3}}{2}\right)^2 = 3\left(\frac{2}{3}\right)^2 = 3\frac{4}{9} = \frac{4}{3}$ and subtracting the same value from the constant term.

$y = 3\left[x^2 - \frac{4}{3}x + \frac{4}{9}\right] + 1 - \left(\frac{4}{3}\right)$ Simplify and express the perfect square trinomial as square of binomial

$y = 3\left(x - \frac{2}{3}\right)^2 - \frac{1}{3}$



Hence, $y = 3x^2 - 4x + 1$ can be expressed as $y = 3\left(x - \frac{2}{3}\right)^2 - \frac{1}{3}$.

In this case, $h = \frac{2}{3}$ and $k = -\frac{1}{3}$.

***Example 2***

Rewrite $f(x) = ax^2 + bx + c$ in the form $f(x) = a(x - h)^2 + k$.

$y = (ax^2 + bx) + c$ Group together the terms containing x

$y = a\left(x^2 + \frac{b}{a}x\right) + c$ Factor out *a.* Here *a* = 1.

$y = a\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2}\right) + c - \frac{b^2}{4a}$ Compete the expression in the parenthesis to make it a perfect square trinomial by adding a constant $a\left(\frac{b}{2a}\right)^2 = \frac{b^2}{4a}$ and subtracting the same value from the constant term

$y = a\left(x + \frac{b}{2a}\right)^2 + \frac{4ac - b^2}{4a}$ Simplify and express the perfect square trinomial as square of binomial

Hence, the vertex form is y = $a\left(x+\frac{b}{2a}\right)^2+\frac{4ac-b^2}{4a}$. Thus, h = $\frac{-b}{2a}$ and k = $\frac{4ac-b^2}{4a}$.

***Example 3***

Rewrite f (x) = $x^2 - 4x - 10$ in the form f(x) = $a(x - h)^2 + k$.

**Solution 1**

By completing the square:

| | |
|---|---|
| y = $(x^2-4x)-10$ | Group together the terms containing x |
| y = $(x^2-4x)-10$ | Factor out ***a***. Here *a* = 1. |
| y = $(x^2-4x+4)-10-4$ | Compete the expression in parenthesis to make it a perfect square trinomial by adding a constant $\left(\frac{-4}{2}\right)^2$ = 4 and subtracting the same value from the constant term. |
| y = $(x-2)^2-14$ | Simplify and express the perfect square trinomial as square of binomial. |

***Solution 2***

By applying the formula h = $\frac{-b}{2a}$ and k = $\frac{4ac-b^2}{4a}$ :

In the equation y = $x^2 - 4x - 10$, a = 1, b = -4 and c = -10. Thus,

| | |
|---|---|
| h = $\frac{-b}{2a}$ | k = $\frac{4ac-b^2}{4a}$ |
| h = $\frac{-(-4)}{2(1)}$ | k = $\frac{4(1)(-10)-(-4)^2}{4(1)}$ |
| h = $\frac{4}{2}$ | k = $\frac{-40-16}{4}$ |
| h = 2 | k = -14 |

By substituting the solved value of h and k in y = $a(x - h)^2 + k$, we obtain y = $(x - 2)^2 - 14$ .

**Activity 5 Step by step!**

**Directions:** Work in pairs. Transform the given quadratic functions into the form $y = a(x - h)^2 + k$ by following the steps below.

1. $y = x^2 - 4x - 10$
2. $y = 3x^2 - 4x + 1$

| Steps | Task |
|---|---|
| 1. Group the terms containing x | |
| 2. Factor out a | |
| 3. Complete the expression in parenthesis to make it a perfect square trinomial. | |
| 4. Express the perfect square trinomial as square of a binomial | |
| 5. Give the value of h | |
| 6. Give the value of k | |

Did you transform the quadratic function in the form $y = a(x - h)^2 + k$?

To transform a quadratic function from standard form $y = a(x - h)^2 + k$ into general form, consider the examples below.

**Example 4**

Rewrite the equation $y = 3(x - 2)^2 + 4$ in the general form $y = ax^2 + bx + c$.

**Solution:**

$y = 3(x - 2)^2 + 4$ — Expand $(x - 2)^2$.

$y = 3(x^2 - 4x + 4) + 4$ — Multiply the perfect square trinomial by 3.

$y = 3x^2 - 12x + 12 + 4$ — Simplify and add 4.

$y = 3x^2 - 12x + 16$

**Example 5**

Express $f(x) = -2(3x - 1)^2 + 5x$ in the general form $f(x) = ax^2 + bx + c$.

**Solution:**

$f(x) = -2(3x - 1)^2 + 5x$

$f(x) = -2(9x^2 - 6x + 1) + 5x$

$f(x) = -18x^2 + 12x - 2 + 5x$

$f(x) = -18x^2 + 17x - 2$

**Activity 6 Reversing the process**

A. **Directions**: Rewrite $y = 2(x- 1)^2 + 3$ in the form $y = ax^2 + bx + c$ by following the given steps.

| Steps | Task |
|---|---|
| 1. Expand $(x – 1)^2$ | |
| 2. Multiply the perfect square trinomial by 2 | |
| 3. Simplify | |
| 4. Add 3 | |
| 5. Result | |



B. Apply the above steps in transforming the following quadratic functions into the general form.

1. $y = 2(x - 4)^2 + 5$
2. $y = 3\left(x - \frac{1}{2}\right)x + 1$

Did you transform the quadratic function into the form $y = ax^2 + bx + c$?

**What to PROCESS**

Your goal in this section is to master the skills in identifying the quadratic function and transforming it in different forms. Towards the end of this module, you will be encouraged to apply these skills in solving real- life problems.

**Activity 7 Where do *you* belong?**

A. Put the letter of the given equation in the diagram below where *you* think it belongs.

a. $y = x^2 - 1$

b. $y = x$

c. $2x^2 - 2x + 1 = y$

d. $3x - 1 + y = 0$

e. $y = (2x + 3)(x - 1)$

f. $y = x^3 + 1$

g. $2^2 + x = y$

h. $y = 3^x + 2x$

i. $3x + x^2 = y$

j. $2x(x - 3) - y = 0$


How did *you* classify each of the given functions?

What similarities do *you* see in quadratic functions? in linear functions?

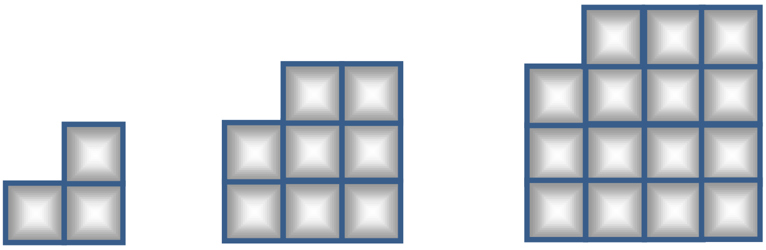
How do a quadratic function and a linear function differ?

What makes a function quadratic?

**Activity 8 Quadratic or Not**

**Directions:** Study the patterns below. Indicate whether the pattern described by the figures is quadratic or not.

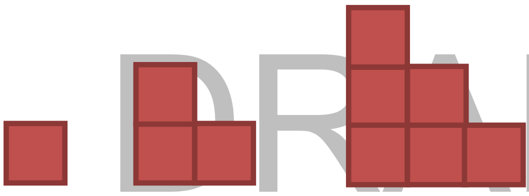
a.



Determine the relationship between the number of blocks in the bottom row and the total number of blocks.

What relationship exists between the two numbers?

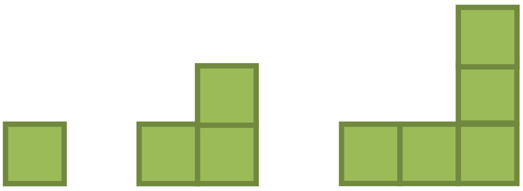
b.





Determine the relationship between the number of blocks in the bottom row and the total number of blocks.

What relationship exists between the two numbers?



c.

Determine the relationship between the number of blocks in the bottom row and the total number of blocks in the figure.

What relationship exists between the two numbers?

**Activity 9 It's your turn**

**Direction**: Match the given quadratic function $y = ax^2 + bx + c$ to its equivalent standard form $y = a(x - h)^2 + k$.

| | |
|---|---|
| $y = x^2 - x + \frac{13}{4}$ | $y = (x - 2)^2 - 3$ |
| $y = \frac{1}{2}x^2 - 3x + 3$ | $y = 2(x-1)^2 + 2$ |
| $y = -2x^2 + 12x - 17$ | $y = -2(x-3)^2 + 1$ |
| $y = x^2 - 4x + 1$ | $y = (x - \frac{1}{2})^2 + 3$ |
| $y = 2x^2 - 4x + 4$ | $y = \frac{1}{2}(x - 3)^2 - \frac{3}{2}$ |



What mathematical concepts did you use in doing the transformation?

Explain how the quadratic function in the form $y = ax^2 + bx + c$ can be transformed into the form $y = a(x - h)^2 + k$.

**Activity 10 The hidden message**

**Directions:** Write the indicated letter of the quadratic function in the form $y = a(x-h)^2 + k$ into the box that corresponds to its equivalent general form $y = ax^2 + bx + c$.

**I** $y = (x - 1)^2 - 4$

**T** $y = (x - 1)^2 - 16$

**S** $y = 2\left(x + \frac{5}{4}\right)^2 - \frac{49}{8}$

**F** $y = (x-3)^2 + 5$

**E** $y = \left(x - \frac{2}{3}\right)^2 + 2$

**M** $y = \left(x - \frac{1}{2}\right)^2 + \frac{3}{2}$

**A** $y = 3(x + 2)^2 - \frac{1}{2}$

**U** $y = -2(x - 3)^2 + 1$

**N** $y = (x - 0)^2 - 36$

**H** $y = 2(x + 1)^2 - 2$



**DIALOG BOX:**

| | |
|---|---|
| | $y = x^2 - x + \frac{7}{4}$ |
| | $y = 3x^2 + 12x + \frac{23}{2}$ |
| | $y = x^2 - 2x - 15$ |
| | $y = 2x^2 + 4x$ |
| | $y = x^2 - 2x - 3$ |
| | $y = 2x^2 + 5x - 3$ |

$y = x^2 - 6x + 14$

$y = -2x^2 + 12x - 17$

$y = x^2 - 36$

How is the square of a binomial obtained without using the long method of multiplication?

Explain how the quadratic function in the form $y = a(x - h)^2 + k$ can be transformed into the form $y = ax^2 + bx + c$.



**Activity 11 Hit or Miss!**

**Direction: Work in Pairs. Solve this problem and show your solution.**

**Problem.** An antenna is 5 m high and 150 meters from the firing place. Suppose the path of the bullet shot from the firing place is determined by the equation $y = -\frac{1}{1500}x^2 + \frac{2}{15}x$, where x is the distance (in meters) of the bullet from the firing place and y is its height. Will the bullet go over the antenna? If yes/no, show your justification.

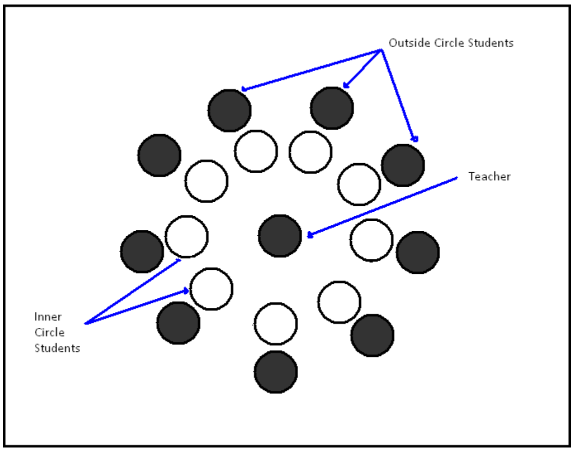
**What to REFLECT or UNDERSTAND**

Your goal in this section is to have a better understanding of the mathematical concepts about quadratic functions. The activities provided for you in this section aim to apply the different concepts that you have learned from the previous activities.

**Activity 12 Inside Outside Circle (Kagan, 1994)**

**Directions:**

1. Form a group of 20 members and arrange yourselves by following the formation below.
2. Listen to your teacher regarding the procedures of the activity.



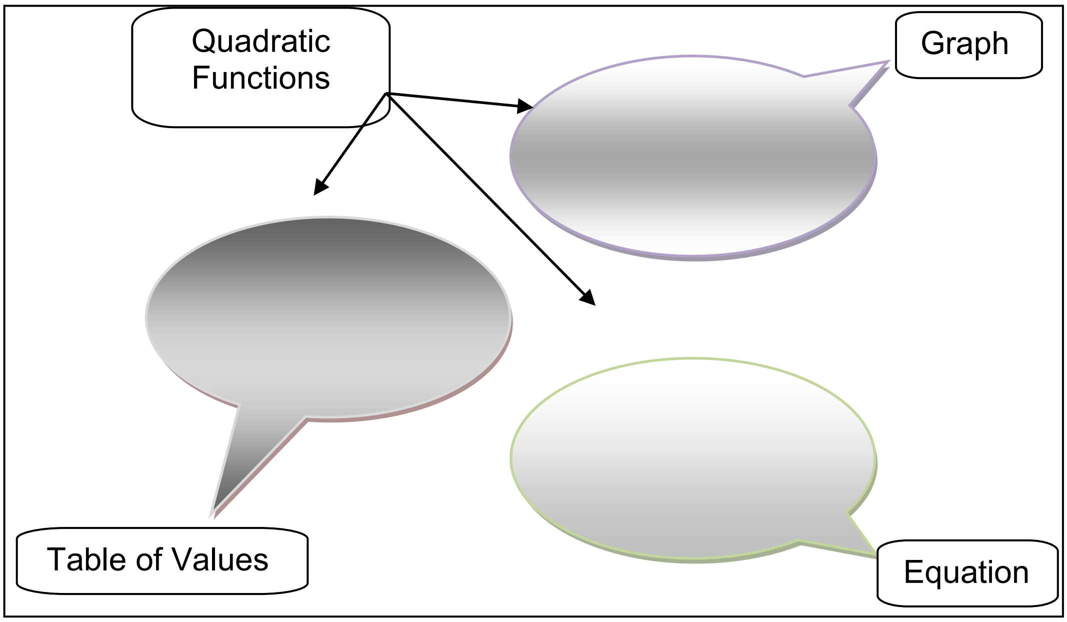

Guide Questions/Topics for the activity.

1. What is a quadratic function?
2. How do *you* differentiate the equation of a quadratic function from that of a linear function?
3. Describe the graph of a linear function and the graph of a quadratic function.
4. Given a table of values, how can *you* determine if the table represents a quadratic function?



**Activity 13. Combination Notes**

A. In the oval callout, describe the ways of recognizing a quadratic function.



B. In the oval callout, make an illustrative example of the indicated mathematical concept.

| Concepts | Illustrative Example |
|---|---|
| Transforming a quadratic function in the form $y = ax^2 + bx + c$ into the form $y = a(x - h)^2 + k$. | |
| Transforming a quadratic function in the form $y = a(x - h)^2 + k$ into the form $y = ax^2 + bx + c$. | |



Based on what you have learned in the preceding activity, you are now ready to apply the concepts that you have learned in other contexts.

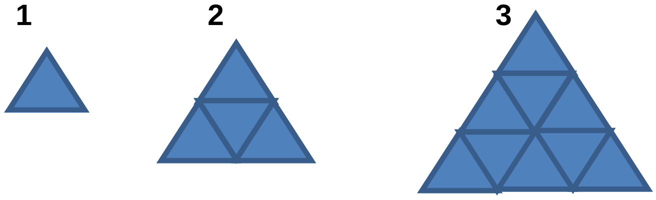
**Activity 14 Find my pattern!**



**Directions**: Group yourselves into 5. Perform the activity below.

Consider the set of figures below. Study the relationship between the term number and the number of unit triangles formed. What is the pattern? Describe the patterns through a table of values, graph and equation. How many triangles are there in the 25th term?

**Term number** **1** **2** **3**



**What to TRANSFER**

The goal of this section is for you to apply what you have learned in a real-life situation. You will be given a task which will demonstrate your understanding of the lesson.

### Activity 15 Investigate!

**Problem.** You are given 50 m of fencing materials. Your task is to make a rectangular garden whose area is maximum. Find the dimensions of such a rectangle. Explain your solution.

### Activity 16 Explore more!

Give at least three parabolic designs that you see in your community. Then create your own design.



### SUMMARY/SYNTHESIS/GENERALIZATION

This lesson introduced quadratic function. The lesson provided you with opportunities to describe a quadratic function in terms of equation, graph, and table of values. You were given a chance to compare and see the difference between quadratic functions and linear functions or other functions.

## Lesson 2. Graphs of Quadratic Functions

### What to KNOW

Let's start this lesson by generating table of values of quadratic functions and plotting the points on the coordinate plane. You will investigate the properties of the graph through guided questions. As you go through this lesson, keep on thinking about this question: **How can the graph of quadratic function be used to solve real life problems?**

### Activity 1 Describe my paths!

**Directions: Follow the procedure in doing the activity.**

a. Given the quadratic functions $y = x^2 - 2x - 3$ and $y = -x^2 + 4x - 1$, transform them into the form $y = a(x - h)^2 + k$.



$y = x^2 - 2x - 3$ $\qquad\qquad$ $y = -x^2 + 4x - 1.$



b. Complete the table of values for x and y.

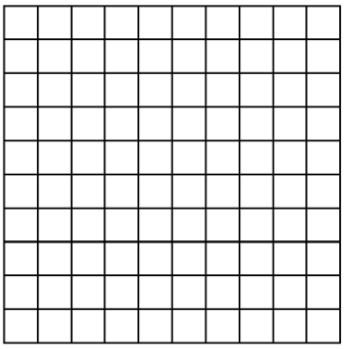
$y = x^2 - 2x - 3$

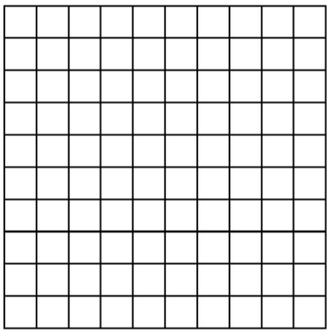
| x | -3 | -2 | -1 | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|---|---|---|
| y | | | | | | | | | |

$y = -x^2 + 4x - 1$

| x | -4 | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|---|
| y | | | | | | | | |

c. Sketch the graph on the cartesian plane.





d. What have you observed about the opening of the curves? Do you have any idea where you can relate the opening of the curves?

e. Which of the 2 quadratic functions has a minimum point? maximum point? Indicate below.

| Quadratic Function | Vertex (Turning Point) | Maximum or Minimum Point |
| --- | --- | --- |
| $y = x^2 - 2x - 3$ | | |
| $y = -x^2 + 4x - 1$ | | |

f. Observe each graph. Can you draw a line that divides the graph in such a way that one part is a reflection of the other part? If there is any, determine the equation of the line?

g. Take a closer look at the minimum point or the maximum point and try to relate it with the values of h and k in the equation $y = a(x - h)^2 + k$ of the function. Write your observations.

h. Can you identify the domain and range of the functions?

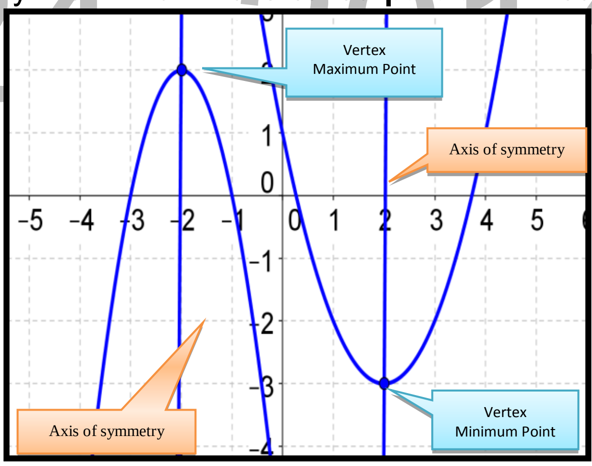
$y = x^2 - 2x - 3$ Domain: ___________ Range: ____________

$y = -x^2 + 4x - 1$ Domain: ___________ Range: ____________



Did you enjoy the activity? To better understand the properties of the graph of quadratic function, study some key concepts below.

The graph of a quadratic function $y = ax^2 + bx + c$ is called **parabola**. You have noticed that the parabola opens **upward or downward**. It has a turning point called vertex which is either the lowest point or the highest point of the graph. If the value of $a > 0$, the parabola opens upward and has a **minimum point.** If $a < 0$, the parabola opens downward and has a **maximum point**. There is a line called the **axis of symmetry** which divides the graph into two parts such that one-half of the graph is a reflection of the other half.




If the quadratic function is expressed in the form $y = a(x - h)^2 + k$, the vertex is the point **(h, k)**. The line **x = h** is the axis of symmetry and **k** is the minimum or maximum value of the function.

The **domain** of a quadratic function is the set of all real numbers. The **range** depends on whether the parabola opens upward or downward. If it opens upward, the range is the set $\{y : y \geq k\}$; if it opens downward, then the range is the set $\{y : y \leq k\}$.

**Activity 2 Draw me!**

**Directions:** Draw the graph of the quadratic function $y = x^2 - 4x + 1$ by following the steps below.

1. Find the vertex and the line of symmetry by expressing the function in the form $y = a(x - h)^2 + k$ or by using the formula **h =** $\frac{-b}{2a}$ **; k =** $\frac{4ac-b^2}{4a}$ if the given quadratic function is in general form.

2. On one side of the line of symmetry, choose at least one value of x and compute the value of y.

Coordinates of points: ___________________________________________

3. Similarly, choose at least one value of x on the other side and compute the value of y.

Coordinates of points: ___________________________________________

4. Plot the points and connect them by smooth curve.

**Activity 3 Play and Learn!**

**Directions:** Work in a group of 5 members. Solve the puzzle and do the activity**.**



**Problem:** Think of a number less than 20. Subtract this number from 20 and multiply the difference by twice the original number. What is the number that will give the largest product?

The first group who gives the largest product wins the game."

a. Record your answer on the table below:

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Number (n) | | | | | | | | | |
| Product (P) | | | | | | | | | |

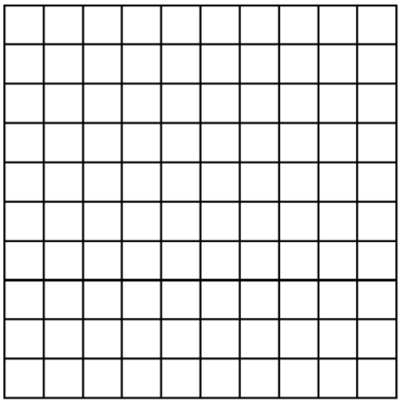
b. Draw the graph.
c. Find the vertex and compare it to your answer in the puzzle.
d. If ***n*** is the number you are thinking, then how can you express the other number, which is the difference of 20 and the number you are thingking?
e. What is the product (***P***) of the two numbers? Formulate the equation.
f. What kind of function is represented by the equation?
g. Express it in standard form.
h. What is the largest product?
i. What is the number that that will give the largest product?
j. Study the graph and try to relate the answer you obtained in the puzzle to the vertex of the graph. Write your observation.

**Activity 4   To the left to the right! Put me up put me down!**

**Directions:** Form groups of 5 members each and perform this activity.

A. Draw the graphs of the following quadratic functions on the same coordinate plane.

1. $y = x^2$
2. $y = 2x^2$
3. $y = 3x^2$
4. $y = \frac{1}{2}x^2$
5. $y = \frac{1}{3}x^2$
6. $y = -x^2$
7. $y = -2x^2$





a. Analyze the graphs.

b. What do you notice  the shape of the graph of the quadratic function $y = ax^2$?

c. What happens to the graph as the value of *a* becomes larger?

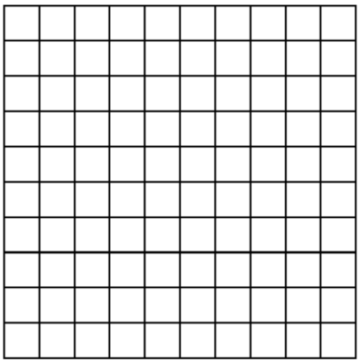
d. What happens when $0 < a < 1$?

e. What happens when $a < 0$ ?  $a > 0$ ?

f. Summarize your observations.

**B. Draw the graphs of the following functions.**

1. $y = x^2$
2. $y = ( x -2 )^2$
3. $y = ( x + 2 )^2$
4. $y = ( x + 1 )^2$
5. $y = ( x - 1 )^2$

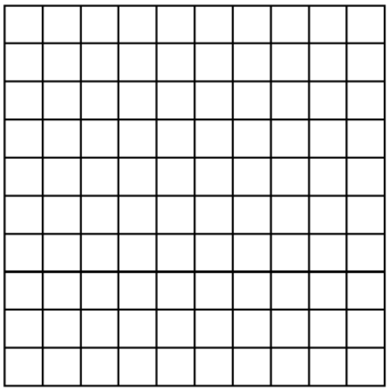


a. Analyze the graphs.

b. What do you notice about the graphs of quadratic functions whose equations are of the form $y = (x - h)^2$?
c. How would you compare the graph of $y = (x - h)^2$ and that of $y = x^2$?
d. Discuss your ideas and observations.

**C. Draw the graphs of the following quadratic functions**

1. $y = x^2$
2. $y = x^2 + 2$
3. $y = x^2 - 2$
4. $y = x^2 - 3$
5. $y = x^2 + 3$





a. Analyze the graphs.

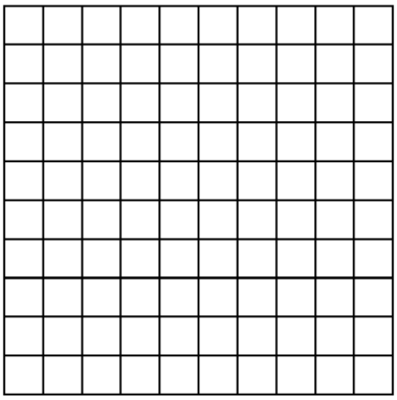
b. What do you notice about the graphs of quadratic functions whose equations are of the form $y = x^2 + k$?

c. How would you compare the graph of $y = x^2 + k$ and that of $y = x^2$ when the vertex is above the origin? below the origin?

d. What conclusion can you give based on your observations?

**D. Draw the graphs of the following quadratic functions:**

1. $y = (x - 2)^2 + 4$
2. $y = (x + 3)^2 - 4$
3. $y = (x - 1)^2 - 3$
4. $y = (x + 4)^2 + 5$
5. $y = (x + 2)^2 - 2$



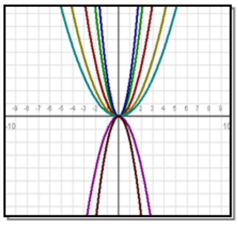
a. Analyze the graphs.

b. What is the effect of the variables **h** and **k** on the graph of $y = (x - h)^2 + k$ as compared to the graph of $y = x^2$?

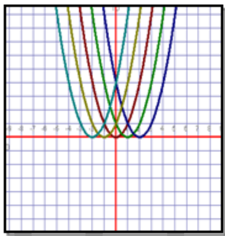
c. Make your generalization on the graph of $y = (x - h)^2 + k$.

Did you enjoy the activity? To better understand the transformation of the graph of quadratic function. Read some key concepts below.

In the graph of $y = ax^2 + bx + c$, the larger the $|a|$ is, the *narrower* is the graph.



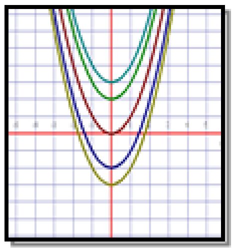




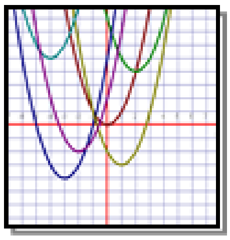
For $a > 0$, the parabola opens upward.

To graph $y = a(x - h)^2$, slide the graph of $y = ax^2$ horizontally h units. If $h > 0$, slide it to the right, if $h < 0$, slide it to the left. The graph has vertex $(h, 0)$ and its axis is the line $x = h$.

To graph $y = ax^2 + k$, slide the graph of $y = ax^2$ vertically **k** units. If **k > 0** slide it upward; if $k < 0$, slide it downward. The graph has vertex $(0, k)$ and its axis of symmetry is the line $x = 0$ (y – axis).



To graph $y = a(x - h)^2 + k$, slide the graph of $y = ax^2$ horizontally h units and vertically k units. The graph has a vertex $(h, k)$ and its axis of symmetry is the line $x = h$.



If $a < 0$, the parabola opens downward. The same procedure can be applied in transforming the graph of a quadratic function.

Vertex of the graph of a quadratic function:

In standard form $f(x) = a(x - h)^2 + k$, the vertex $(h, k)$ can be directly obtained from the values of h and k.

In general form $f(x) = ax^2 + bx + c$, the vertex $(h, k)$ can be obtained using the formulas $h = \dfrac{-b}{2a}$ and $k = \dfrac{4ac - b^2}{4a}$.

### What to PROCESS

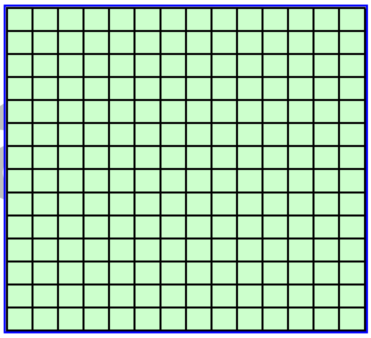
Your goal in this section is to apply the mathematical concepts that you have learned in graphing quadratic functions. Use these mathematical concepts to perform the provided activities in this section.

### Activity 5 Draw and describe me!

**Directions**: Sketch the graph of each quadratic function and identify the vertex, domain, range, and the opening of the graph, State whether the vertex is a minimum or a maximum point, and write the equation of its axis of symmetry.

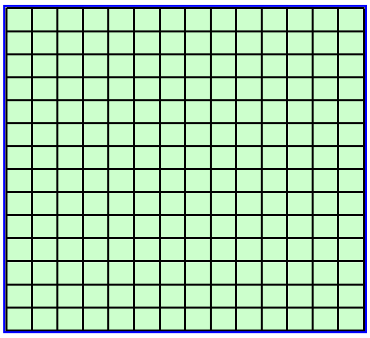


1. $f(x) = x^2$



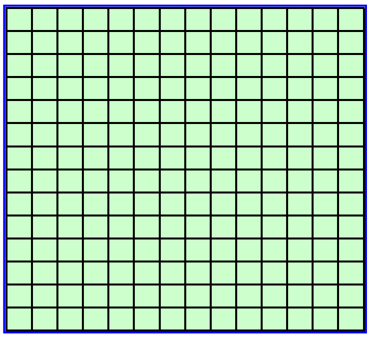
vertex ______________
opening of the graph ______________
Vertex is a ______________ point
Equation of the axis of symmetry _______
Domain: _____ Range: _____

2. $f(x) = 2x^2 + 4x - 3$



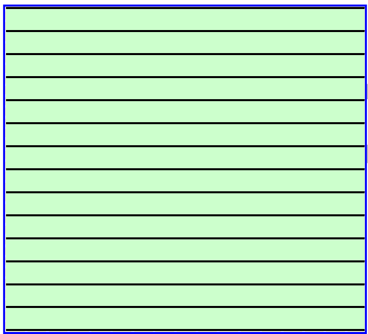
vertex ______________
opening of the graph ______________
Vertex is a ______________ point
Equation of the axis of symmetry ____
Domain: _____ Range: _____

3. $f(x) = \frac{1}{2}x^2 + 2$



vertex ______________
opening of the graph ______________
Vertex is a ______________ point
Equation of the axis of symmetry ____
Domain: _____ Range: _____

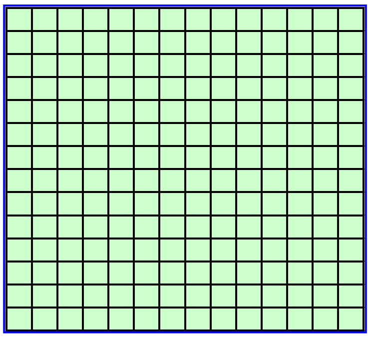
4. $f(x) = -x^2 - 2x - 3$



vertex ______________
opening of the graph ______________
Vertex is a ______________ point
Equation of the axis of symmetry ____
Domain: _____ Range: _____

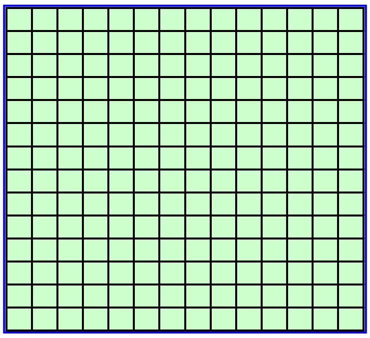


5. $f(x) = (x + 2)^2 + 3$



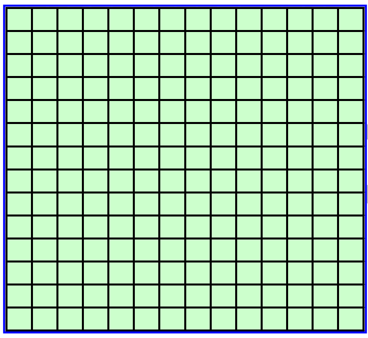
vertex ______________
opening of the graph ______________
Vertex is a ______________ point
Equation of the axis of symmetry ____
Domain: _____ Range: _____

6. $f(x) = 2(x - 2)^2$



vertex ______________
opening of the graph ______________
Vertex is a ______________ point
Equation of the axis of symmetry ____
Domain: _____ Range: _____

7. $f(x) = -2x^2 - 2$



vertex ______________
opening of the graph ______________
Vertex is a ______________ point
Equation of the axis of symmetry ____
Domain: _____ Range: _____

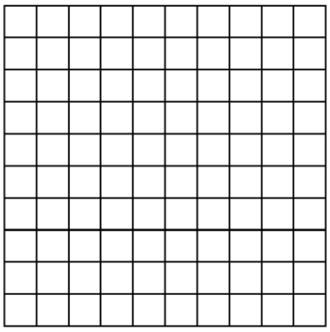


How did you find the activity? Explain the procedure on how to draw the graph of a quadratic function.

**Activity 6 Hit the volleyball**

**Problem**. Carl Allan hit the volleyball at 3 ft above the ground with an initial velocity of 32 ft/sec. The path of the ball is given by the function $S(t) = -16t^2 + 32t + 3$, where S is the height of the ball at *t* seconds. What is the maximum height reached by the ball?

a. What kind of function is used to model the path of the volleyball?
b. Draw the path of the volleyball and observe the curve.



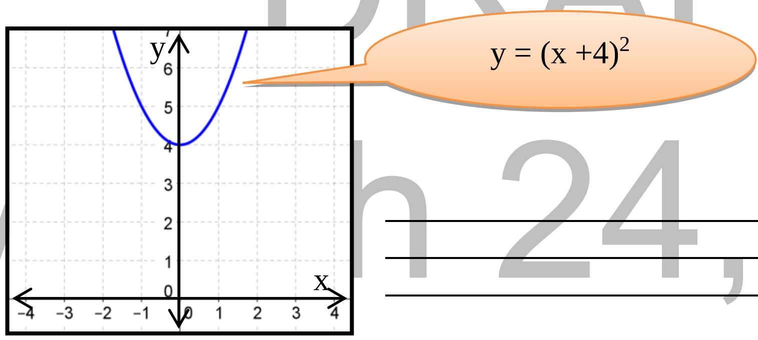
c. What is the maximum height reached by the ball?
d. What is represented by the maximum point of the graph?

**Activity 7 Match or mismatch!**

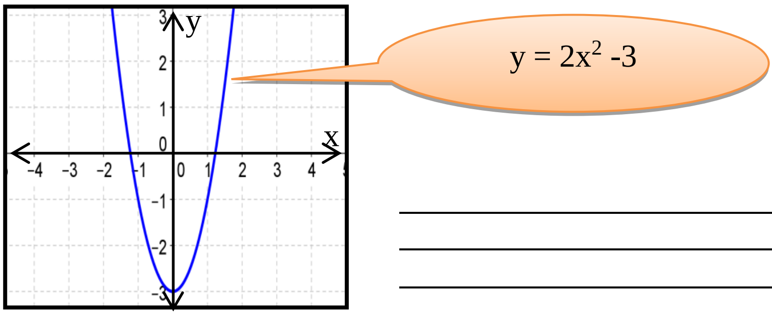
**Directions**: Decide whether the given graph is a match or a mismatch with the indicated equation of quadratic function. Write match if the graph corresponds with the correct equation. Otherwise, indicate the correct equation of the quadratic function.

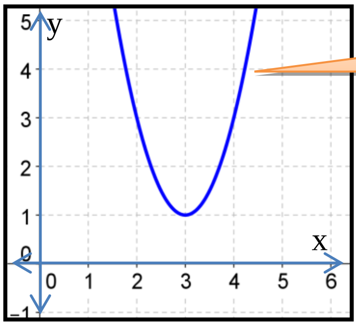


**1.**

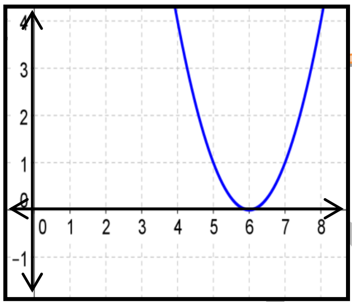



**2.**



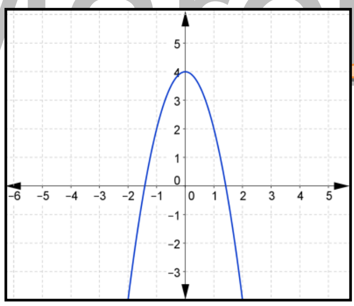
**3.** 

$y = 2x^2 - 12x + 18$

4. 

$y = x^2 - 12x + 36$



5. 

$y = -2x^2 + 4$

Share the technique you used to determine whether the graph and the equation of the quadratic function are match or mismatch.

What characteristics of a quadratic function did you apply in doing the activity?

**Activity 8 Translate Me!**

**Directions:** The graph of $f(x) = 2x^2$ is shown below. Based on this graph, sketch the graphs of the following quadratic functions in the same coordinate system.

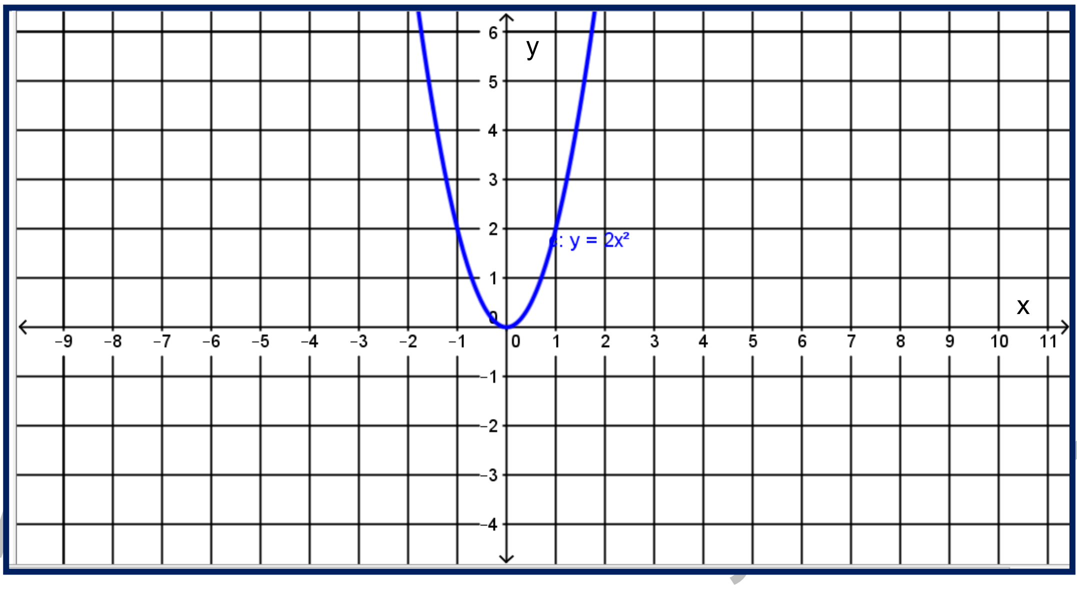
$f(x) = 2x^2 + 3$ f. $f(x) = 2(x + 4)^2$

a. $f(x) = 2(x - 3)^2 - 1$ g. $f(x) = -2x^2 - 1$

b. $f(x) = 2(x + 2)^2$ h. $f(x) = 2(x + 3)^2 + 5$

c. $f(x) = -2x^2 + 3$ i. $f(x) = 2(x - 3)^2 - 2$

d. $f(x) = 2(x + 1)^2 - 2$ j. $f(x) = -2(x - 1)^2$




How did you find the activity?

Describe the movement of the graph for each quadratic function.

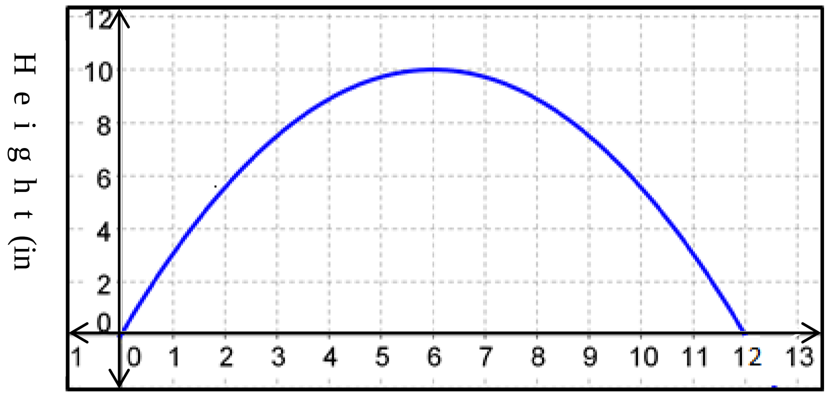
**What to REFLECT or UNDERSTAND**

Your goal in this section is to have a deeper understanding of the graph of quadratic functions. The activities provided for you in this section will be of great help to enable you to apply the concepts in different contexts.

**Activity 9 Let's analyze!**

**Directions**: Analyze the problem and answer the given questions.

**Problem 1**: A ball on the playing ground was kicked by Carl Jasper. The parabolic path of the ball is traced by the graph below. Distance is given in meters.



Horizontal distance (in meters)



**Questions:**

a. How would you describe the graph?
b. What is the initial height of the ball?
c. What is the maximum height reached by the ball?
d. Determine the horizontal distance that corresponds to the maximum distance.
e. Approximate the height of the ball after it has travelled 2 meters horizontally.
f. How far does the ball travel horizontally before it hits the ground?

**Problem 2**: The path when a stone is thrown can be modelled by $y = -16x^2 + 10x + 4$, where ***y*** (in feet) is the height of the stone ***x*** seconds after it is released.

a. Graph the function.
b. Determine the maximum height reached by the stone.
c. How long will it take the stone to reach its maximum height?

## Activity 10 Clock Partner Activity


*Write your name on your clock. Make an appointment with 12 of your classmates , one for each hour on the clock. Be sure you both record the appointment on your clock. Only make an appointment if there is an open slot at that hour on both your clock.*

**Directions**: Use your clock partner to discuss the following questions.

| Time | Topics/Questions to be discussed/answered |
|---|---|
| 2:00 | How can you describe the graph of a quadratic function? |
| 5:00 | Tell something about the axis of symmetry of a parabola. |
| 12:00 | When do we say that the graph has a minimum/maximum value or point? |
| 3:00 | What does the vertex imply? |
| 5:00 | How can you determine the opening of the parabola? |
| 6:00 | How would you compare the graph of $y = a(x - h)^2$ and that of $y = ax^2$? |
| 11:00 | How would you compare the graph of $y = x^2 + k$ with that of $y = x^2$ when the vertex is above the origin? below the origin? |
| 1:00 | How do the values of h and k in $y = a(x-h)^2 + k$ affect the graph of $y = ax^2$? |



## Activity 11 Combination Notes

**Directions:** Tell something about what you have learned.

What are the properties of the graph of a quadratic function? Discuss

Enumerate the steps in graphing a quadratic function.

How can you determine the vertex of a quadratic function?

*Reflections*



**Activity 12 Which of which?**

**Directions:** Answer each of the following questions.

1. Which has the larger area?
   a. A rectangle whose dimensions are 25 m by 20 m
   b. The largest possible area of a rectangle to be enclosed if the perimeter is 50 m

2. Which has the lower vertex?
   a. $y = x^2 + 2x + 3$
   b. $y = x^2 – 4x + 7$

3. Which has the higher vertex?
   a. $y = -x^2 – 6x + 15$
   b. $y = - x^2 + 6$

**Activity 13: ABC in Math!**

**Directions:** Play and Learn with this activity.

In activity 3, you have learned the effects of variables a, h, and k in the graph of $y = a(x - h)^2 + k$ as compared to the graph of $y = ax^2$. Now, try to investigate the effect of variables a, b and c in the graph of quadratic function $y = ax^2 + bx + c$.


Did you enjoy the activities? I hope that you learned a lot in this section and you are now ready to apply the mathematical concepts you gained from all the activities and discussions.



**What to TRANSFER**

In this section, you will be given a task wherein you will apply what you have learned in the previous sections. Your performance and output will show evidence of learning.

**Activity 14 Quadratic design**

**GOAL**: Your task is to design a curtain in a small restaurant that involves a quadratic curve.

**ROLE:** Interior Designer

**AUDIENCE**: Restaurant Owner

**SITUATION:** Mr. Andal, the owner of a restaurant wants to impress some of the visitors, as target clients, in the coming wedding of his friend. As a venue of the reception, Mr Andal wants a new ambience in his restaurant. Mr Andal requested you, as interior designer, to help him to change the interior of the restaurant particularly the design of the curtains. Mr Andal wants you to use parabolic curves in your design. Map out the appearance of the proposed design for the curtains in his 20 by 7 meters restaurant and estimate the approximate budget requirements for the cost of materials based on the height of the design curve.

**PRODUCT:** Proposed plan for the curtain including the proposed budget based on the original garden.

**STANDARDS FOR ASSESSMENT**:

You will be graded based on the rubric designed suitable for your task and performance.

**Activity 15 Webquest Activity. Math is all around.**

**Directions: Make a simple presentation of world famous parabolic arches.**

**Task:**

1. Begin the activity by forming a group of 5 members. Choose someone you can depend on to work diligently and to do his fair share of work.
2. In your free time, start surfing the net for world famous parabolic arches. As you search, keep a record of where you go, and what you find on the site.
3. Complete the project by organizing the data you collected, including the name of the architect and the purpose of creating the design.
4. Once you have completed the data, present it to the class in a creative manner. You can use any of the following but not limited to them.
   - Multimedia presentation
   - Webpages
   - Posters
5. You will be assessed based on the rubric for this activity.



**SUMMARY/SYNTHESIS/GENERALIZATION**

This lesson was about graphs of quadratic functions. The lesson was able to equip you with ample knowledge on the properties of the graph of quadratic functions. You were made to experience graphing quadratic functions and their transformation. You were given opportunities to solve real life problems using graphs of quadratic functions and to create a design out of it.

## Lesson 3. Finding the Equation of a Quadratic Function

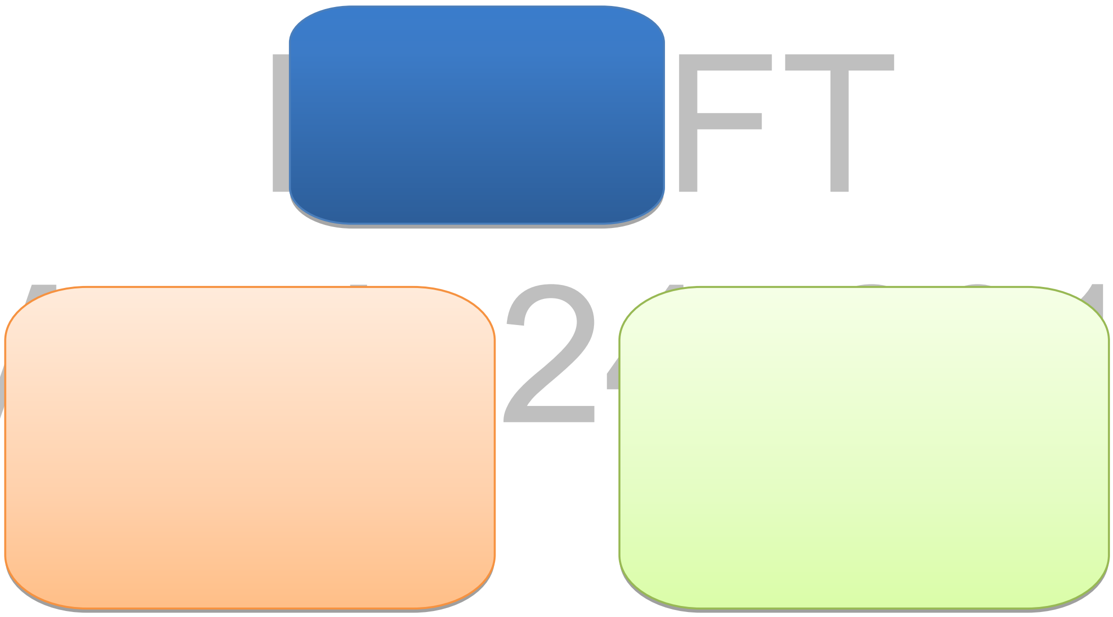
### What to KNOW

Let's begin this lesson by recalling the methods of finding the roots of quadratic equations. Then relate them with the zeros of the quadratic functions. In this lesson, you will be able to formulate patterns and relationship regarding quadratic functions. Furthermore, you will be able to solve real-life problems involving equations of quadratic functions.

### Activity 1. Give me my roots!

Directions: Given a quadratic equation $x^2 - x - 6 = 0$, find the roots in three methods.

*Factoring*



*Quadratic* *Completing the Square*

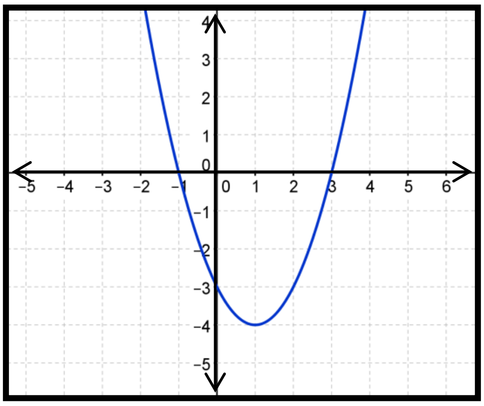
Did you find the roots in 3 different ways? Your skills in finding the roots will also be the methods you will be using in finding the zeros of quadratic functions. To better understand the zeros of quadratic functions and the procedure in finding them, study the mathematical concepts below.

A value of $x$ that satisfies the quadratic equation $ax^2 + bx + c = 0$ is called a ***root*** of the equation.

**Activity 2 What are my zeros?**

**Directions:** Perform this activity and answer the guided questions.

Examine the graph of the quadratic function y = $x^2$ - 2$x$ – 3



a. How would you describe the graph?

b. Give the vertex of the parabola and its axis of symmetry.

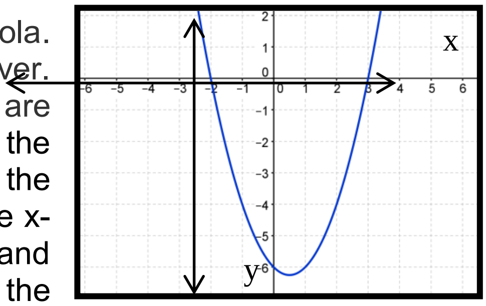
c. At what values of $x$ does the graph intersect the $x$- axis?

d. What do you call these x- coordinates where the curve crosses the $x$- axis?

e. What is the value of $y$ at these values of $x$?



How did you find the activity? To better understand the zeros of a function, study some key concepts below.




The graph of a quadratic function is a parabola. A parabola can cross the $x$-axis once, twice, or never. The x-coordinates of hese points of intersection are called ***x*-intercepts.** Let us consider the graph of the quadratic function y = $x^2$ - $x$ – 6. It shows that the curve crosses the $x$-axis at 3 and -2. These are the x-intercepts of the graph of the function. Similarly, 3 and -2 are the **zeros** of the function since these are the values of $x$ when y equals 0. These zeros of the function can be determined by setting y to 0 and solving the resulting equation through different algebraic methods.

**Example 1.**

Find the zeros of the quadratic function y = $x^2 - 3x + 2$ by factoring method.

Solution:

Set y = 0. Thus,

$$0 = x^2 - 3x + 2$$

$$0 = (x - 2)(x - 1)$$

$$x - 2 = 0 \quad \text{or} \quad x - 1 = 0$$

Then $x = 2$ and $x = 1$

The zeros of y = $x^2 - 3x + 2$ are 2 and 1.

**Example 2.**

Find the zeros of the quadratic function y = $x^2 + 4x - 2$ using the completing the square method.



**Solution:**

Set y = 0. Thus,

$$x^2 + 4x - 2 = 0$$

$$x^2 + 4x = 2$$

$$x^2 + 4x + 4 = 2 + 4$$

$$(x + 2)^2 = 6$$

$$x + 2 = \pm\sqrt{6}$$

$$x = -2 \pm \sqrt{6}$$

The zeros of y = $x^2 + 4x - 2$ are $-2 + \sqrt{6}$ and $-2 - \sqrt{6}$.

**Example 3.**

Find the zeros of the quadratic function f($x$) = $x^2 + x - 12$ using the quadratic formula.

**Solution:**

Set y = 0.

In $0 = x^2 + x - 12$, a = 1, b = 1 and c = -12.

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ Use the quadratic formula

$x = \dfrac{-(1) \pm \sqrt{(1)^2 - 4(1)(-12)}}{2(1)}$ Substitute the values of ***a***, ***b*** and ***c***

$x = \dfrac{-1 \pm \sqrt{1+48}}{2}$ Simplify

$$x = \frac{-1 \pm \sqrt{49}}{2}$$

$$x = \frac{-1 \pm 7}{2}$$

$x = \dfrac{-1+7}{2}$ $x = \dfrac{-1-7}{2}$

$x = \dfrac{6}{2}$ $x = \dfrac{-8}{2}$

$x = 3$ $x = -4$

The zeros of $f(x) = x^2 + x - 12$ are 3 and -4.

**Activity 3 What's my Rule!**

**Directions:** Work in groups of 3 members each. Perform this activity.

The table below corresponds to a quadratic function. Examine it.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| *x* | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
| y | 4 | 0 | -2 | -2 | 0 | 4 | 10 |

**Activity 3. A**

a. Plot the points and study the graph. What have you observed?
b. What are the zeros of the quadratic function? How can you identify them?
c. If the zeros are $r_1$ and $r_2$, express the equation of the quadratic function using $f(x) = a(x - r_1)(x - r_2)$, where a is any non-zero constant.
d. What is the quadratic equation that corresponds to the table?

Can you think of another way of determining the equation of the quadratic function from the table of values above?

What if the table of values does not have its zero/s? How can you derive the equation of the quadratic function?

**Activity 3.B**

The table of values below describes a quadratic function. Find the equation of the quadratic function by following the given procedure.

| $x$ | -3 | -1 | 1 | 2 | 3 |
|---|---|---|---|---|---|
| y | -29 | -5 | 3 | 1 | -5 |

a. Substitute 3 ordered pairs ($x$,y) in y = a$x^2$ + b$x$ + c
b. What are the three equations you came up with?

__________________, __________________, __________________

c. Solve for the values of a, b and c.
d. Write the equation of the quadratic function y = a$x^2$ + b$x$ + c.

How did you obtain the three equations?

What do you call the 3 equations?

How did you solve for the values a, b and c from the three equations?

How can you obtain the equation of a quadratic function from a table of values?

> Did you get the equation of the quadratic function correctly in the activity? You can go over the illustrative examples below to better understand the procedure on how to determine the equation of a quadratic function given the table of values.

Study the illustrative examples below.



**Illustrative example 1**

Find a quadratic function whose zeros are -1 and 4.

Solution: If the zeros are -1 and 4, then

$x$ = -1 or $x$ = 4

It follows that

$x$ + 1 = 0 or $x$ - 4 = 0, then

($x$+1) ($x$ – 4) = 0

$x^2$ – 3$x$ - 4 = 0

The equation of the quadratic function f($x$) = ($x^2$ – 3$x$ – 4) is not unique since there are other quadratic functions whose zeros are -1 and 4 like f(x) = 2$x^2$ -6x -8, f(x) = 3$x^2$ – 9x -12 and many more. These equations of quadratic functions are obtained by multiplying the right-hand side of the equation by a nonzero constant. Thus, the answer is f($x$) = a($x^2$ – 3$x$ – 4) where **a** is any nonzero constant.

**Illustrative example 2**

Determine the equation of the quadratic function represented by the table of values below.

| $x$ | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| y | 24 | 16 | 10 | 6 | 4 | 4 | 6 |

**Solution:**

Notice that you can't find any zeros from the given table of values. In this case, take any three ordered pairs from the table, and use these as the values of $x$ and $y$ in the equation $y = ax^2 + bx + c$. Let's say

using point ( 1, 4 ) $4 = a(1)^2 + b(1) + c$

$4 = a + b + c$ ---------------> equation 1

using point ( -1, 10 ) $10 = a(-1)^2 + b(-1) + c$

$10 = a - b + c$ ----------------> equation 2

using point ( 2, 4 ) $4 = a(2)^2 + b(2) + c$

$4 = 4a + 2b + c$ -------------> equation 3

We obtain a system of 3 equations in $a$, $b$, and $c$.

Add corresponding terms in eq.1 and eq. 2 to eliminate b

eq. 1 + eq. 2 $4 = a + b + c$

$10 = a - b + c$

We have $14 = 2a + 2c$ --------------> equation 4

Multiply the terms in eq. 2 by 2 and add corresponding terms in eq. 3 to eliminate b

2(eq .2 ) + eq. 3 $20 = 2a - 2b + 2c$

$4 = 4a + 2b + c$

We have $24 = 6a + 3c$-------> equation 5

Notice that equation 4 and equation 5 constitute a system of linear equations in two variables. To solve for c, multiply the terms in equation 4 by 3 and subtract corresponding terms in equation 5.

3 (eq. 4) – equation 5 $42 = 6a + 6c$

$-\quad 24 = 6a + 3c$

$18 = 3c$

$c = 6$

Substitute the value of ***c*** in equation 4 and solve for ***a***.

$14 = 2a + 2(6)$

$14 = 2a + 12$

$2a = 14 - 12$

$a = 1$

Substitute the value of ***c*** and ***a*** in equation 1 and solve for b.

$4 = a + b + c$

$4 = 1 + b + 6$

$4 = 7 + b$

$b = 4 - 7$

$b = -3$

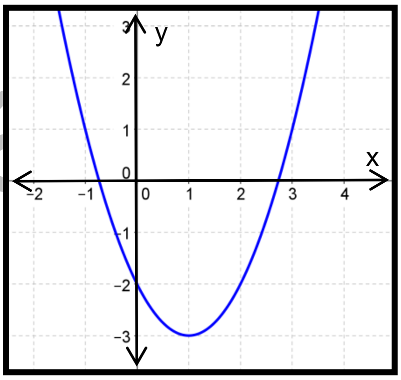
Thus, a = 1, b = -3 and c = 6. Substitute these in $f(x) = ax^2 + bx + c$; the quadratic function is $f(x) = x^2 - 3x + 6$.

**Activity 4 Pattern from curve!**

**Directions:** Work in pairs**.** Determine the equation of the quadratic function given the graph by following the steps below.



Study the graph of a quadratic function below.



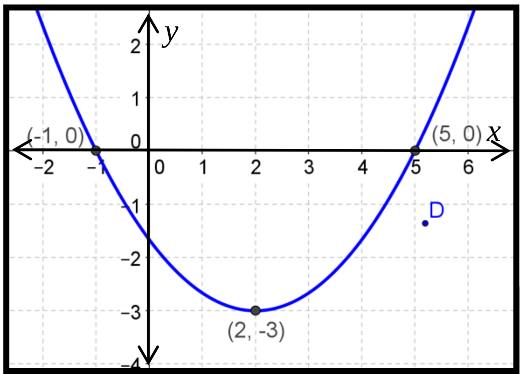

1. What is the opening of the parabola? What does it imply regarding the value of *a*?
2. Identify the coordinates of the vertex.
3. Identify coordinates of any point on the parabola.
4. In the form of quadratic function $y = a(x - h)^2 + k$, substitute the coordinates of the point you have taken in the variables *x* and y and the x- coordinates and y-coordinate of the vertex in place of h and k, respectively.
5. Solve for the value of a.
6. Get the equation of a quadratic in the quadratic form $y = a(x - h)^2 + k$. function by substituting the obtained value of ***a*** and the x and y coordinates of the vertex in ***h*** and ***k*** respectively.

How did you find the activity? Study the mathematical concepts below to have a clear understanding of deriving the quadratic equation from the graph.

When the vertex and any point on the parabola are clearly seen, the equation of the quadratic function can easily be determined by using the form of a quadratic function y = a($x$ – h)$^2$ + k.

**Illustrative example 1.**

Find the equation of the quadratic function determined from the graph below.






**Solution:**

The vertex of the graph of the quadratic function is (2, -3). The graph passes through the point (5, 0). By replacing $x$ and y with 5 and 0, respectively,and h and k with 2 and -3, respectively, we have

$$y = a(x - h)^2 + k$$
$$0 = a(5 - 2)^2 - 3$$
$$0 = a(3)^2 - 3$$
$$3 = 9a$$
$$a = \frac{1}{3}$$

Thus, the quadratic equation is y = $\frac{1}{3}(x-2)^2$ - 3 or y = $\frac{1}{3}x^2 - \frac{4}{3}x - \frac{5}{3}$.

Aside from the method presented above, you can also determine the equation of a quadratic function by getting the coordinates of any 3 points lying on the graph. You can follow the steps in finding the equation of a quadratic function using this method by following the illustrative example presented previously in this section.

**Activity 5 Give my Equation!**

**Directions**: Perform the activity.

A. Study the example below in finding the zeros of the quadratic function and try to reverse the process to find the solution of the problem indicated in the table on the right.

| Find the zeros of $f(x) = 6x^2 - 7x - 3$ using factoring.<br><br>Solution:<br>$f(x) = 6x^2 - 7x - 3$<br><br>$0 = 6x^2 - 7x - 3$<br><br>$0 = (2x - 3)(3x + 1)$<br><br>$2x - 3 = 0$ or $3x + 1 = 0$<br><br>Then $x = \frac{3}{2}$ and $x = -\frac{1}{3}$<br><br>The zeros are $\frac{3}{2}$ and $-\frac{1}{3}$. | If the zeros of the quadratic function are 1 and 2, find the equation.<br>Note: $f(x) = a(x - r_1)(x - r_2)$ where a is any nonzero constant.<br><br>Solution:<br>____________________________<br>____________________________<br>____________________________<br>____________________________<br>____________________________<br>____________________________ |
|---|---|



How did you find the activity?

Explain the procedure you have done to determine the equation of the quadratic function.

B. Find the equation of the quadratic function whose zeros are $2 \pm \sqrt{3}$.
   a. Were you able get the equation of the quadratic function?
   b. If no, what difficulties did you encounter?
   c. If yes, how did you manipulate the rational expression to obtain the quadratic function? Explain.
   d. What is the equation of the quadratic function?

> Study the mathematical concepts below to have a clearer picture on how to get the equation of a quadratic function from its zeros.

If $r_1$ and $r_2$ are the zeros of a quadratic function then $f(x) = a(x - r_1)(x - r_2)$ where a is a nonzero constant that can be determined from other point on the graph.

Also, you can use the sum and product of the zeros to find the equation of the quadratic function. (See the illustrative example in Module 1, lesson 4)

**Example 1**

Find an equation of a quadratic function whose zeros are -3 and 2.

**Solution**

Since the zeros are $r_1$ = -3 and $r_2$ = 2, then

$f(x) = a(x - r_1)(x - r_2)$

$f(x) = a[x - (-3)](x - 2)$ ]

$f(x) = a(x + 3)(x - 2)$

$f(x) = a(x^2 + x - 6)$ where ***a*** is any nonzero constant.

**Example 2**

Find an equation of a quadratic function with zeros $\frac{3 \pm \sqrt{2}}{3}$.

**Solution**

A quadratic expression with irrational roots cannot be written as a product of linear factors with rational coefficients. In this case, we can use another method.

Since the zeros are $\frac{3 \pm \sqrt{2}}{3}$ then,

$x = \frac{3 \pm \sqrt{2}}{3}$

$3x = 3 \pm \sqrt{2}$

$3x - 3 = \pm \sqrt{2}$

Square both sides of the equation and simplify. We get

$9x^2 - 18x + 9 = 2$

$9x^2 - 18x + 7 = 0$

Thus, an equation of a quadratic function is $f(x) = 9x^2 - 18x + 7$.



You learned from the previous activities the methods of finding the zeros of quadratic function. You also have an initial knowledge of deriving the equation of a quadratic function from tables of values, graphs,or zeros of the function. The mathematical concepts that you learned in this section will help you perform the activities in the next section.

**What to PROCESS**

Your goal in this section is to apply the concepts you have learned in finding the zeros of the quadratic function and deriving the equation of aquadratic function. You will be dealing with some activities and problems to have mastery of skills needed to perform some tasks ahead.

## Activity 6 Match the zeros!

**Directions:** Matching Type. Each quadratic function has a corresponding letter. Similarly, each box with the zeros of the quadratic function inside has a corresponding blank below. Write the indicated letter of the quadratic function on the corresponding blank below the box containing the zeros of the function to get the hidden message.

| | |
|---|---|
| **Y** f($x$) = $4x^2 - 25$ | **R** f($x$) = $x^2 - 9$ |
| **V** f($x$) = $9x^2 - 16$ | **E** f($x$) = $x^2 - 5x - 36$ |
| **G** f($x$) = $x^2 + 6x + 9$ | **L** f($x$) = $x^2 - x - 20$ |
| **U** f($x$) = $x^2 - 4x - 21$ | **D** f($x$) = $2x^2 + x - 3$ |
| **S** f($x$) = $6x^2 + 5x - 4$ | **O** f($x$) = $6x^2 - 7x + 2$ |

| $\{-3,-3\}$ | $\left\{\frac{2}{3},\frac{1}{2}\right\}$ | $\left\{-\frac{3}{2},1\right\}$ | $\{5,-4\}$ | $\left\{\frac{2}{3},\frac{1}{2}\right\}$ | $\left\{\frac{4}{3},-\frac{4}{3}\right\}$ | $\{9,-4\}$ | $\left\{-\frac{4}{3},\frac{1}{2}\right\}$ |
|---|---|---|---|---|---|---|---|
| ____ | ____ | ____ | ____ | ____ | ____ | ____ | ____ |

| $\left\{\frac{5}{2},-\frac{5}{2}\right\}$ | $\left\{\frac{2}{3},\frac{1}{2}\right\}$ | $\{7,-3\}$ |
|---|---|---|
| ____ | ____ | ____ |



## Activity 7 Derive my equation!

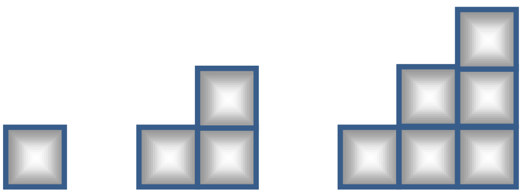
**Directions**: Work in pairs.

A. Determine the equation of the quadratic function represented by the table of values below.

| $x$ | -4 | -3 | -2 | -1 | 0 | 1 |
|---|---|---|---|---|---|---|
| y | -20 | -13 | -8 | -5 | -4 | -5 |

B. The vertex of the parabola is ( -3, 5) and it is the minimum point of the graph. If the graph passes though the point (-2, 7), what is the equation of the quadratic function?

C. Observe the pattern below and draw the 4th and 5th figures.



Make a table of values for the number of squares at the bottom and the total number of unit squares.

What is the resulting equation of the function?

What method did you use to obtain the equation of the quadratic function in A? Explain how you obtained your answer.

In B, explain the procedure you used to arrive at your answer. What mathematical concepts did you apply?

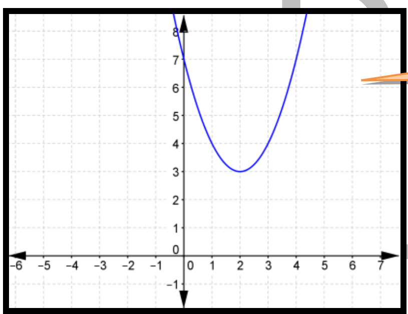
Consider C, did you find the correct equation? Explain the method that you used to get the answer.

**Activity 8 Rule me out!**

**Directions**: Work in pairs to perform this activity. Write the equation in the oval and write your explanation on the blank provided.

A. Derive the equation of the quadratic function presented by each of the following graphs below.

1.



________________________________________

________________________________________

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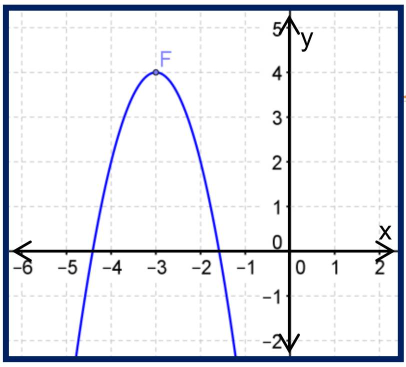
2.

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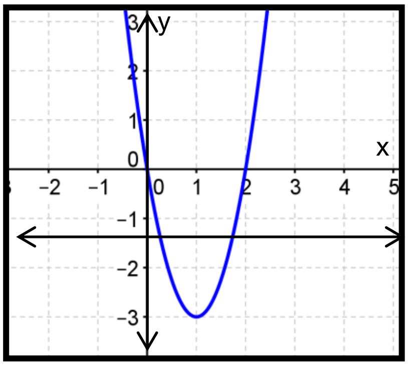
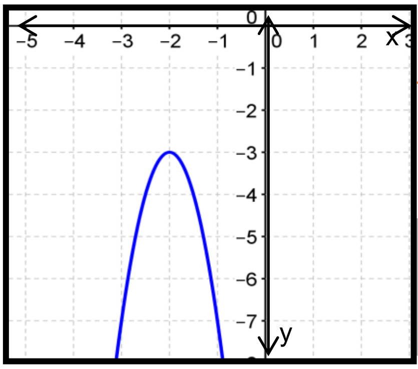
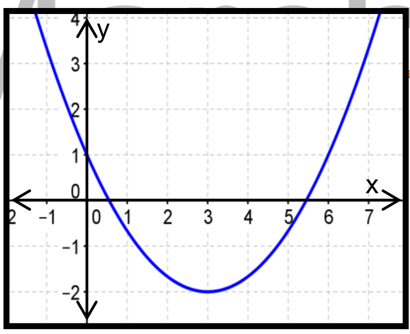
**3.**



**4.**



**5.**





Explain briefly the method that you applied in getting the equation

**Activity 9 Name the translation!**

**Directions:** Give the equation of a quadratic function whose graphs are described below. Write your answers on the blanks provided.

1. The graph of f($x$) = 3$x^2$ shifted 4 units downward ____________________

2. The graph of f($x$) = $4x^2$ shifted 2 units to the left ______________________

3. The graph of f($x$) = $3x^2$ shifted 5 units upward and 2 units to the right ______________________

4. The graph of f($x$) = $-10x^2$ shifted 2 units downward and 6 units to the left ______________________

5. The graph of f($x$) = $7x^2$ shifted half unit upward and half unit to the left ______________________

Describe the method you used to formulate the equations of the quadratic functions above.

**Activity 10. Rule my zeros!**

**Directions:** Find one equation for each of the quadratic function given its zeros.

1. 3, 2 ______________________

2. -2, 5/2 ______________________

3. $1 + \sqrt{3}$ , $1 - \sqrt{3}$ ______________________

4. $\frac{1+\sqrt{2}}{3}$, $\frac{1-\sqrt{2}}{3}$ ______________________

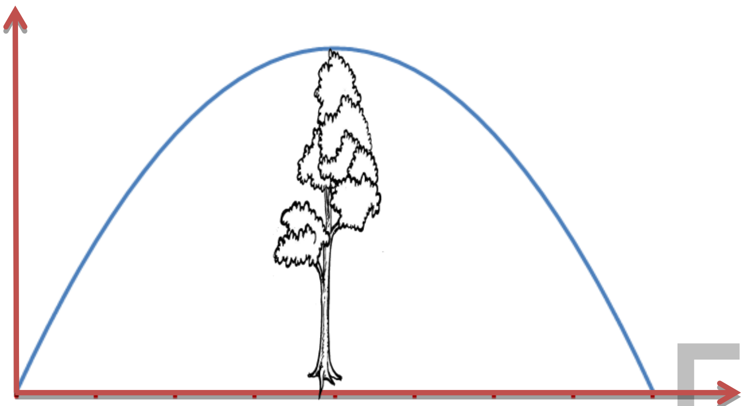
5. $\frac{11}{3}$, $-\frac{11}{3}$ ______________________

In your reflection note, explain briefly the procedure used to get the equation of the quadratic function given its zero/s.

**Activity 11 Dare to hit me!**

**Directions:** Work in pairs. Solve the problem below:

**Problem.** The path of the golf ball follows a trajectory. It hits the ground 400 meters away from the starting position. It just overshoots a tree which is 20 m high and is 300m away from the starting point.



From the given information, find the equation determined by the path of the golf ball.



Did you enjoy the activities in this section?

Go back to the previous section and compare your initial ideas with the discussion. How much of your initial ideas are found in the discussion? Which ideas are different and need to be clarified more?

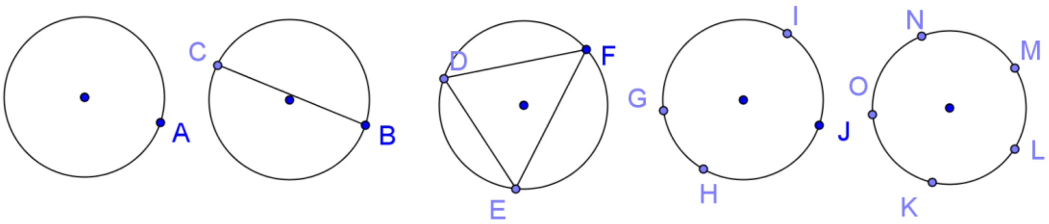
**What to REFLECT or UNDERSTAND**

Your goal in this section is to have a deeper understanding on how to derive the equation of the quadratic functions. You can apply the skills you have learned from previous sections to perform the tasks ahead. The activities provided for you in this section will be of great help to deepen your understanding for further application of the concepts.

### Activity 12 Connect and Relate!

**Directions:** Work in groups of 5 members. Perform this mathematical investigation.

Joining any two points on a circle determine a chord. A chord is a line segment that joins any two points on a circle. Investigate the relationship between the number of points ***n*** on the circle and the maximum number of chords ***C*** that can be drawn. Make a table of values and find the equation of the function representing the relationship. How many chords are there if there are 50 points on a circle?




What kind of function is represented by the relationship?

Given the number of points, how can you easily determine the maximum number of chords that can be drawn?

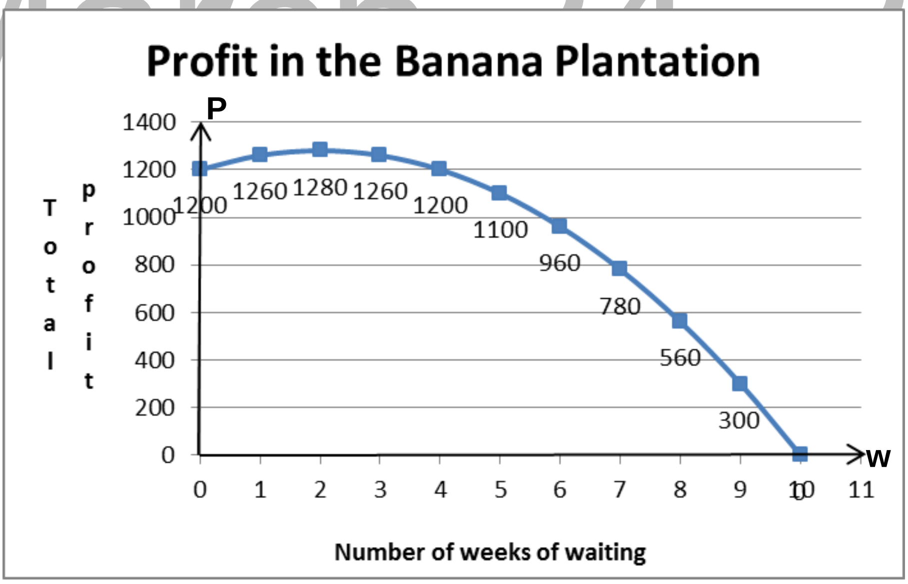
How did you get the pattern or the relationship?



### Activity 13 Profit or Loss!

**Directions:** Work in pairs. Analyze the graph below and answer the questions that follow.






a. Describe the graph.
b. What is the vertex of the graph? What does the vertex represent?
c. How many weeks should the owner of the banana plantation wait before harvesting the bananas to get the maximum profitmmmmmmmmm ?
d. What is the equation of the function?

## Activity 14 What if questions!

**Directions:** Work in groups with 3 members each.

1. Given the zeros $r_1$ and $r_2$ of the following quadratic function, the equation of the quadratic function is f(x) = a(x - $r_1$)(x - $r_2$) where a is any nonzero constant. Consider a = 1 in any of the situations in this activity.

   a. -2 and 3.

   b. $3 \pm \sqrt{3}$

A. What is the equation of the quadratic function?

   a. ____________________

   b. ____________________

B. If we double the zeros, what is the new equation of the quadratic function?

   a. _____________________

   b. _____________________

Find a pattern on how to determine easily the equation of a quadratic function given this kind of condition.

C. What if the zeros are reciprocal of the zeros of the given function? What is the new equation of the quadratic function?

   a. _____________________

   b. _____________________

Find the pattern.

D. What if you square the zeros? What is the new equation of the quadratic function?

   a. _____________________

   b. _____________________

Find the pattern.

2. Find the equation of the quadratic function whose zeros are

   a. the squares of zeros of f($x$) = $x^2 - 3x$ -5.

   b. the reciprocal of the zeros of f($x$) = $x^2 - x$ -6

   c. twice the zeros of f($x$) = $3x^2 - 4x$ -5

How did you find the activity?

**Activity 15  Principle Pattern Organizer!**

**Directions:** Make a summary of what you have learned.

Deriving quadratic function from:

- *Graphs*
- *Zero*
- *Table of Values*



Did you enjoy the activities? I hope that you learned a lot in this section and you are now ready to apply the mathematical concepts you learned in all the activities and discussions from the previous sections.



## What to TRANSFER

In this section, you will be given a task wherein you will apply what you have learned in the previous sections. Your performance and output will show evidence of your learning.

## Activity 16 Mathematics in Parabolic Bridges!

**GOAL**: Look for world famous parabolic bridges and determine the equation of the quadratic functions .

**ROLE:** Researchers

**AUDIENCE**: Head of the Mathematics Department and math teachers

**SITUATION:** For the Mathematics monthly culminating activity, your section is tasked to present a simple research paper in Mathematics. Your group is assigned to make a simple research on the world's famous parabolic bridges and the mathematical equations/functions described by each bridge.

Make a simple research on parabolic bridges and use the data to formulate the equations of quadratic functions pertaining to each bridge.

http://64.19.142.13/australia.gov.au/sites/default/files/agencies/culture/library/images/site_images/bridgesunset-web.jpg

**PRODUCT:** Simple research paper on world famous parabolic bridge.

**STANDARDS FOR ASSESSMENT**:

You will be graded based on the rubric designed suitable for your task and performance.

## Lesson 4. Applications of Quadratic Functions

### What to KNOW

The application of quadratic function can be seen in many different fields like physics, industry, business and in variety of mathematical problems. In this section, you will explore situations that can be modeled by quadratic functions.

Let us begin this lesson by recalling the properties of quadratic functions and how we can apply them to solve real life problems.

### Activity 1

**Directions**: Consider this problem.

1. If the perimeter of the rectangle is 100 m, find its dimensions if its area is maximum.

   a. Complete the table below for the possible dimensions of the rectangle and their corresponding areas. The first column has been completed for you.

| Width (*w*) | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 | 50 |
|---|---|---|---|---|---|---|---|---|---|---|
| Length (l) | 45 | | | | | | | | | |
| Area (*A*) | 225 | | | | | | | | | |

   b. What is the largest area that you obtained?
   c. What are the dimensions of a rectangle with the largest area?
   d. The perimeter **P** of the given rectangle is 100. Make a mathematical statement for the perimeter of the rectangle.
   e. Simplify the obtained equation and solve for the length **l** of the rectangle in terms of its width **w**.
   f. Express the area *A* of a rectangle as a function of width **w**.
   g. What kind of equation is the result?
   h. Express the function in standard form. What is the vertex?
   i. Graph the data from the table in **a** showing the relationship between the width and the area.
   j. What have you observed about the vertex of the graph in relation to the dimensions and the largest area?

How did you find the activity? Can you still recall the properties of quadratic function? Did you use them in solving the given problem?

To better understand how the concepts of the quadratic function can be applied to solve geometry problems, study the illustrative example presented below.

**Example 1.**

What are the dimensions of the largest rectangular field that can be enclosed by 80 m of fencing wire?

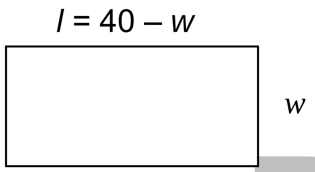
**Solution:**

Let *l and w* be the length and width of a rectangle. Then, the perimeter P of a rectangle is $P = 2l + 2w$. Since $P = 80$ m, thus,

$$2l + 2w = 80$$

$l + w = 40$ It follows that

$l = 40 - w$ expressing the length as a function of $\boldsymbol{w}$




Substituting in the formula for the area ***A*** of a rectangle

$A(w) = wl$

$A(w) = \boldsymbol{w}\,( 40 - \boldsymbol{w} )$

$A(w) = -\boldsymbol{w}^2 + 40\boldsymbol{w}$

By completing the square,

$$\boldsymbol{A(w)} = -( \boldsymbol{w} - 20 )^2 + 400$$

The vertex the graph of the function A(w) is (20, 400). This point indicates the a maximum value of 400 for A(w) that occurs when w = 20. Thus, the maximum area is 400 $m^2$ when the width is 20. If the width is 20 m, then the length is (40 – 20) m or 20 m also. The field with maximum area is a square.



**Activity 2 Catch me when I fall!**

**Directions:** Work in groups with 3 members each. Do the following activity.

**Problem:** The height (H) of the ball thrown into the air with an initial velocity of 9.8 m/s from a height of 2 m above the ground is given by the equation $H(t) = -4.9t^2 + 9.8t + 2$, where *t* is the time in seconds that the ball has been in the air.

a. What maximum height did the object reach?
b. How long will it take the ball to reach the maximum height?
c. After how many seconds is the ball at a height of 4 m?

Guide questions:

1. What kind of function is the equation $H(t) = -4.9t^2 + 9.8t + 2$?
2. Transform the equation into the standard form.
3. What is the vertex?
4. What is the maximum height reached by the ball?

5. How long will it take the ball to reach the maximum height?
6. If the height of the ball is 4 *m*, what is the resulting equation?
7. Find the value of *t* to determine the time it takes the ball to reach 4 m .

How did you find the preceding activity? The previous activity allowed you to recall your understanding of the properties of quadratic function and gave you an opportunity to solve real-life related problems that deal with quadratic function

The illustrative example below is intended for you to better understand the key ideas necessary to solve real-life problems involving quadratic function.

Free falling objects can be modeled by a quadratic function $h(t) = -4.9t^2 + V_0t + h_0$, where h(t) is the height of an object at *t* seconds, when it is thrown with an initial velocity of $V_0$ m/s and an initial height of $h_0$ meters. If units are in feet, then the function is $h(t) = -16t^2 + V_0t + h_0$.

**Illustrative example 1**



From a 96-foot building, an object is thrown straight up into the air then follows a trajectory. The height S(t) of the ball above the building after t seconds is given by the function $S(t) = 80t - 16t^2$.

1. What maximum height will the object reach?
2. How long will it take the object to reach the maximum height?
3. Find the time at which the object is on the ground.

***Solution***

1. The maximum height reached by the object is the ordinate of vertex of the parabola of the function $S(t) = 80t - 16t^2$. By transforming this equation in completed square form we have,

$$S(t) = 80t - 16t^2$$

$$S(t) = -16t^2 + 80t$$

$$S(t) = -16(t^2 - 5t)$$

$$S(t) = -16(t^2 - 5t + \frac{25}{4}) + 100$$

$$S(t) = -16(t - \frac{5}{2})^2 + 100$$

The vertex is $(\frac{5}{2}, 100)$ Thus the maximum height reached by the object is 100 ft from the top of the building. This is 196 ft from the ground.

2. The time for an object to reach the maximum height is the abscissa of the vertex of the parabola or the value of h.

$S(t) = 80t - 16t^2$

$S(t) = -16(t - \frac{5}{2})^2 + 100$

Since the value of h is $\frac{5}{2}$ or 2.5, then the object is at its maximum height after 2.5 seconds.

3. To find the time it will take the object to hit the ground, let $S(t) = -96$ , since the height of the building is 96 ft. The problem requires us to solve for t.

$h(t) = 80t - 16t^2$

$-96 = 80t - 16t^2$

$16t^2 - 80t - 96 = 0$

$t^2 - 5t - 6 = 0$

$(t - 6)(t + 1) = 0$

$t = 6$ or $t = -1$

Thus, it will take 6 seconds before the object hits the ground.



**Activity 3 Harvesting Time!**

**Directions:** Solve the problem by following the given steps.

Problem: Marvin has a mango plantation. If he picks the mangoes now, he will get 40 small crates and make a profit of P 100 per crate. For every week that he delays picking, his harvest increases by 5 crates. But the selling price decreases by P10 per crate. When should Marvin harvest his mangoes for him to have the maximum profit?



a. Complete the following table of values.

| No. of weeks of waiting (w) | 0 | 1 | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| No. of crates | 40 | | | | | | | | |
| Profit per crates (P) | 100 | | | | | | | | |
| Total profit (T) | | | | | | | | | |

b. Plot the points and draw the graph of the function.
c. How did you determine the total profit?
d. Express the profit P as a function of the number of weeks of waiting.
e. Based on the table of values and graph, how many weeks should Marvin wait before picking the mangoes to get the maximum profit?

*This problem is adapted from PASMEP Teaching Resource Materials, Volume II.*

How did you find the activity? A quadratic function can be applied in business/industry to determine the maximum profit, the break-even situation and the like.

Suppose $x$ denotes the number of units a company plans to produce or sell. The revenue function $R(x)$ is defined as $R(x)$= (price per unit) x (number of units produced or sold).

Study the example below.

**Illustrative example**

Problem. A garments store sells about 40 t-shits per week at a price of Php100 each. For each P10 decrease in price, the sales lady found out that 5 more t-shits per week were sold.

a. Write a quadratic function in standard form that models the revenue from t-shirt sales.
b. What price produces the maximum revenue?

Solution:

You know that Revenue $R(x)$ = (price per unit) x (number of units produced or sold).



*Therefore, Revenue R(x) = (Number of t-shirts sold) (Price per t-shirt)*

Revenue $R(x) = (40 + 5x)(100-10x)$



$R(x) = -50x^2 + 100x + 4000$

If we transform the function into the form $y = a(x- h)^2 + k$

$R(x) = -50(x -1)^2 + 4050$

The vertex is (1, 4050).

Thus, the maximum revenue is Php 4050

The price of the t-shirt to produce maximum revenue can be determined by

$P(x) = 100 – 10x$

$P(x) = 100 – 10 (1) = 90$

Thus, Php 90 is the price of the t-shirt that produces maximum revenue.

## What to PROCESS

Your goal in this section is to extend your understanding and skill in the use of quadratic function to solve real-life problems.

**Activity 4 Hit the mark!**

**Directions**: Analyze and solve this problem.

**Problem A**.

A company of cellular phones can sell 200 units per month at P 2 000 each. Then they found out that they can sell 50 more cell phone units every month for each P 100 decrease in price.

a. How much is the sales amount if cell phone units are priced at P2000 each?
b. How much would be their sales if they sell each cell phone unit at P 1600?
c. Write an equation for the revenue function?.
d. What price per cell phone unit gives them the maximum monthly sales?
e. How much is the maximum sale?



**Problem B**

The ticket to a film showing costs P 20. At this price, the organizer found out that all the 300 seats are filled. The organizer estimates that if the price is increased, the number of viewers will fall by 50 for every P 5 increase.

a. What ticket price results in the greatest revenue?
b. At this price, what is the maximum revenue?



What properties of a quadratic function did you use to come up with the correct solution to the problem A? problem B?

**Activity 5. Equal Border!**

**Directions:** Work in pairs and perform this activity.

A photograph is 16 inches wide and 9 inches long and is surrounded by a frame of uniform width **x**. If the area of the frame is 84 square inches, find the uniform width of the frame.

a. Make an illustration of the described photograph.
b. What is the area of the picture?
c. If the width of the frame is $x$ inches, what is the length and width of the photograph and frame?
d. What is the area of the photograph and frame?
e. Given the area of the frame which is 84 square inches, formulate the relationship among three areas and simplify.
f. What kind of equation is formed?
g. How can you solve the value of $x$?
h. How did you find the activity? What characteristics of quadratic functions did you apply to solve the previous problem?

**Activity 6 Try this!**

**Directions:** With your partner, solve this problem. Show your solution.

A. An object is thrown vertically upward with a velocity of 96 m/sec. The distance S(*t*) above the ground after t seconds is given by the formula $S(t) = 96t - 5t^2$.

   a. How high will it be at the end of 3 seconds?
   b. How much time will it take the object to be 172 m above the ground?
   c. How long will it take the object to reach the ground?

B. Suppose there are 20 persons in a birthday party. How many handshakes are there altogether if everyone shakes hands with each other?

   a. Make a table of values for the number of persons and the number of handshakes.
   b. What is the equation of the function?
   c. How did you get the equation?
   d. If there are 100 persons, how many handshakes are there given the same condition?

In problem A, what mathematical concepts did you apply to solve the problem?

If the object reaches the ground, what does it imply?

In problem B, what are the steps you follow to arrive at your final answer?

Did you find any pattern to answer the question d?



Did you enjoy the activities in this section? Go back to the previous section and compare your initial ideas with the discussion. How much of your initial ideas are found in the discussion? Which ideas are different and need revision?

**What to REFLECT or UNDERSTAND**

Your goal in this section is to have a deeper understanding on how to solve problems involving quadratic functions. The activities provided for you in this section will be of great help to practice the key ideas developed throughout the lesson and to stimulate your synthesis of the key principles and techniques in solving problems on quadratic functions.

## Activity 7 Geometry and Number!

**Directions:** Solve the problems. Show your solution.

1. What are the dimensions of the largest rectangular field that can be enclosed with 60 m of wire?
2. Find the maximum rectangular area that can be enclosed by a fence that is 364 meters long.
3. Find two numbers whose sum is 36 and whose product is a maximum.
4. The sum of two numbers is 28. Find the two numbers such that the sum of their squares is a minimum?
5. Marlon wants to fence a rectangular area that has one side bordered by an irrigation. If he has 80 m of fencing materials, what are the dimensions and the maximum area he can enclose?
6. The length of a rectangular field is 8 m longer than its width If the area is $2900m^2$, find the dimensions of the lot.
7. The sum of two numbers is 24. Find the numbers if their product is to be a maximum.

## Activity 8 It's high time!



**Directions:** Work in group with 5 members each. Solve the problems. Show your solution.

1. A ball is launched upward at 14 m/s from a platform that is 30 m high.
   a. Find the maximum height the ball reaches.
   b. How long it will take the ball to reach the maximum height?
   c. How long will it take the ball to reach the ground?

2. On top of a hill, a rocket is launched from a distance 80 feet above a lake. The rocket will fall into the lake after its engine burns out. The rocket's height *h*, in feet above the surface of the lake is given by the equation $h = -16t^2 + 64t + 80$, where t is time in seconds. What is the maximum height reached by the rocket?

3. A ball is launched upward at 48 ft/s from a platform that is 100 ft. high. Find the maximum height the ball reaches and how long it will take to get there.

4. An object is fired vertically from the top of a tower. The tower is 60.96 ft high. The height of the object above the ground t seconds after firing is given by the formula $h(t) = -16t^2 + 80t + 200$. What is the maximum height reached by the object? How long after firing does it reach the maximum height?

5. The height in meters of a projectile after t seconds is given by $h(t) = 160t - 80t^2$. Find the maximum height that can be reached by the projectile.

6. Suppose a basketball is thrown 8 ft from the ground. If the ball reaches the 10m basket at 2.5 seconds, what is the initial velocity of the basketball?

**Activity 9	Reach the target!**

**Directions**: Work in pairs to solve the problems below. Show your solution.

1. A store sells lecture notes, and the monthly revenue R of this store can be modelled by the function $R(x) = 3000 + 500x - 100x^2$, where $x$ is the peso increase over Php 4. What is the maximum revenue?

2. A convention hall has a seating capacity of 2000. When the ticket price in the concert is Php160, attendance is 500. For each Php 20 decrease in price, attendance increases by 100.

   a. Write the revenue R of the theater as a function of concert ticket price $x$.
   b. What ticket price will yield maximum revenue?
   c. What is the maximum revenue?

3. A smart company has 500 customers paying P 600 each month. If each Php 30 decrease in price attracts 120 additional customers, find the approximate price that yields maximum revenue?

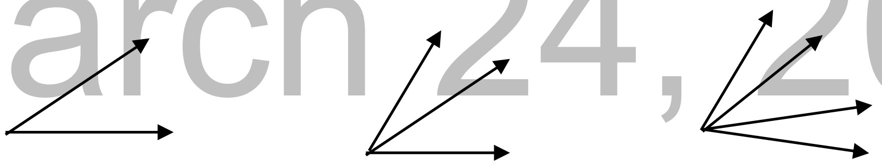
**Activity 10	Angles Count!**



**Directions:** Work in groups of 5 members each. Perform this mathematical Investigation.

Problem. An angle is the union of two noncollinear rays. If there are 100 rays in the figure, how many angles are there?





a. What kind of function is represented by the relationship?

b. Given the number of rays, how can you determine the number of angles?

c. How did you get the pattern or the relationship?

In working on problems and exploration in this section, you studied the key ideas and principles to solve problems involving quadratic functions. These concepts will be used in the next activity which will require you to illustrate a real-life application of a quadratic function.

**What to TRANSFER**

In this section, you will be given a task wherein you will apply what you have learned in the previous sections. Your performance and output in the activity must show evidence of your learning.

### Activity 11 Fund Raising Project!

**GOAL**: Apply quadratic concepts to plan and organize a fund raising activity

**ROLE**: Organizers of the Event

**SITUATION:** The Mathematics Club plan to sponsor a film viewing on the last Friday of the Mathematics month. The primary goal for this film viewing is to raise funds for their Math Park Project and of course to enhance the interest of the students in Mathematics.

To ensure that the film viewing activity will not lose money, careful planning is needed to guarantee a profit for the project. As officers of the club, your group is tasked to make a plan for the event. Ms. De Guzman advised you to consider the following variables in making the plan.

a) Factors affecting the number of tickets sold
b) Expenses that will reduce profit from ticket sales such as:
   - promoting expenses
   - operating expenses
c) How will the expenses depend on the number of people who buy tickets and attend?
d) Predicted income and ticket price
e) Maximum income and ticket price
f) Maximum participation regardless of the profit
g) What is the ticket price for which the income is equal to the expenses?



Make a proposed plan for the fund raising activity showing the relationship of the related variables and the predicted income, price, maximum profit, maximum participation, and also the break-even point.



**AUDIENCE:** Math Club Advisers, Department Head-Mathematics, Mathematics teachers

**PRODUCT**: Proposed plan for the fund raising activity (Film showing)

**STANDARD:** Product/Performance will be assessed using a rubric.

#### Summary/Synthesis/Feedback

This module was about concepts of quadratic functions. In this module, you were encouraged to discover by yourself the properties and characteristics of quadratic functions. The knowledge and skills gained in this module help you solve real-life problems involving quadratic functions which would lead you to perform practical tasks. Moreover, concepts you learned in this module allow you to formulate real-life problems and solve them in a variety of ways.

**Glossary of Terms**

**axis of symmetry** – the vertical line through the vertex that divides the parabola into two equal parts

**direction of opening of a parabola –** can be determined from the value of a in $f(x) = ax^2 + bx + c$. If $a > 0$, the parabola opens upward; if $a < 0$, the parabola opens downward.

**domain of a quadratic function** – the set of all possible values of x. Thus, the domain is the set of all real numbers.

**maximum value** – the maximum value of $f(x) = ax^2 + bx + c$ where $a < 0$, is the y-coordinate of the vertex.

**minimum value** – the minimum value of $f(x) = ax^2 + bx + c$ where $a > 0$, is the y-coordinate of the vertex.

**parabola** – the graph of a quadratic function.

**quadratic function**- a second-degree function of the form $f(x) = ax^2 + bx + c$, where a, b, and c are real numbers and $a \neq 0$. This is a function which describes a polynomial of degree 2.

R**ange of quadratic function** - consists of all y greater than or equal to the y-coordinate of the vertex if the parabola opens upward.

- consists of all y less than or equal to the y-coordinate of the vertex if the parabola opens downward

**vertex** – the turning point of the parabola or the lowest or highest point of the parabola. If the quadratic function is expressed in standard form $y = a(x – h)^2 + k$, the vertex is the point (h, k).

**zeros of a quadratic function** – the x-intercepts of the parabola.

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## WEBLINKS

### Website links for Learning Activities

1. http://www.analyzemath.com/quadraticg/Problems1.html
2. http://www.learner.org/workshops/algebra/workshop4/
3. http://www.cehd.umn.edu/ci/rationalnumberproject/01_1.html
4. http://www.montereyinstitute.org/courses/Algebra1/COURSE_TEXT_RESOURCE/U10_L2_T1_text_container.html
5. http://answers.yahoo.com/question/index?qid=20090531221500AAoXyOi
6. http://algebra2trigplt.wikispaces.com/file/view/cb_sb_math_miu_L6_U3_embedded_assessment3-1_SE_v2.pdf
7. http://www.youtube.com/watch?v=BYMd-7Ng9Y8
8. http://www.utdanacenter.org/mathtoolkit/downloads/scope/OLDalg2scope/ovadawatta.pdf
9. http://www.westfargo.k12.nd.us/district/academic/images/MathProblemSolvingRubric.pdf
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13. http://www.montereyinstitute.org/courses/Algebra1/COURSE_TEXT_RESOURCE/U10_L2_T1_text_container.html
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15. http://www.youtube.com/watch?v=5bKch8vitu0
16. http://ts2.mm.bing.net/th?id=H.4577714805345361&pid=15.1
17. http://www.mathopenref.com/quadraticexplorer.html
18. http://fit-ed.org/intelteachelements/assessment/index.php

### Website links for Images

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