1

SPECIAL PRODUCTS AND FACTORS

I. INTRODUCTION AND FOCUS QUESTIONS

http://dmciresidences.com/home/2011/01/cedar-crest-condominiums/

http://frontiernerds.com/metal-box

http://mazharalticonstruction.blogspot.com/2010/04/architectural-drawing.html

Have you at a certain time asked yourself how a basketball court was painted using the least number of paint? Or how the architect was able to maximize the space of a building and was able to place all amenities the owners want? Or how a carpenter was able to create a utility box using minimal materials? Or how some students were able to multiply polynomial expressions in a least number of time?

This module will help you recognize patterns and techniques in finding products, and factors, and mathematical as well as real-life problems.

After finishing the module, you should be able to answer the following questions:

a. How can polynomials be used to solve geometric problems?
b. How are products obtained through patterns?
c. How are factors related to products?

II. LESSONS AND COVERAGE

In this module, you will examine the aforementioned questions when you study the following lessons:

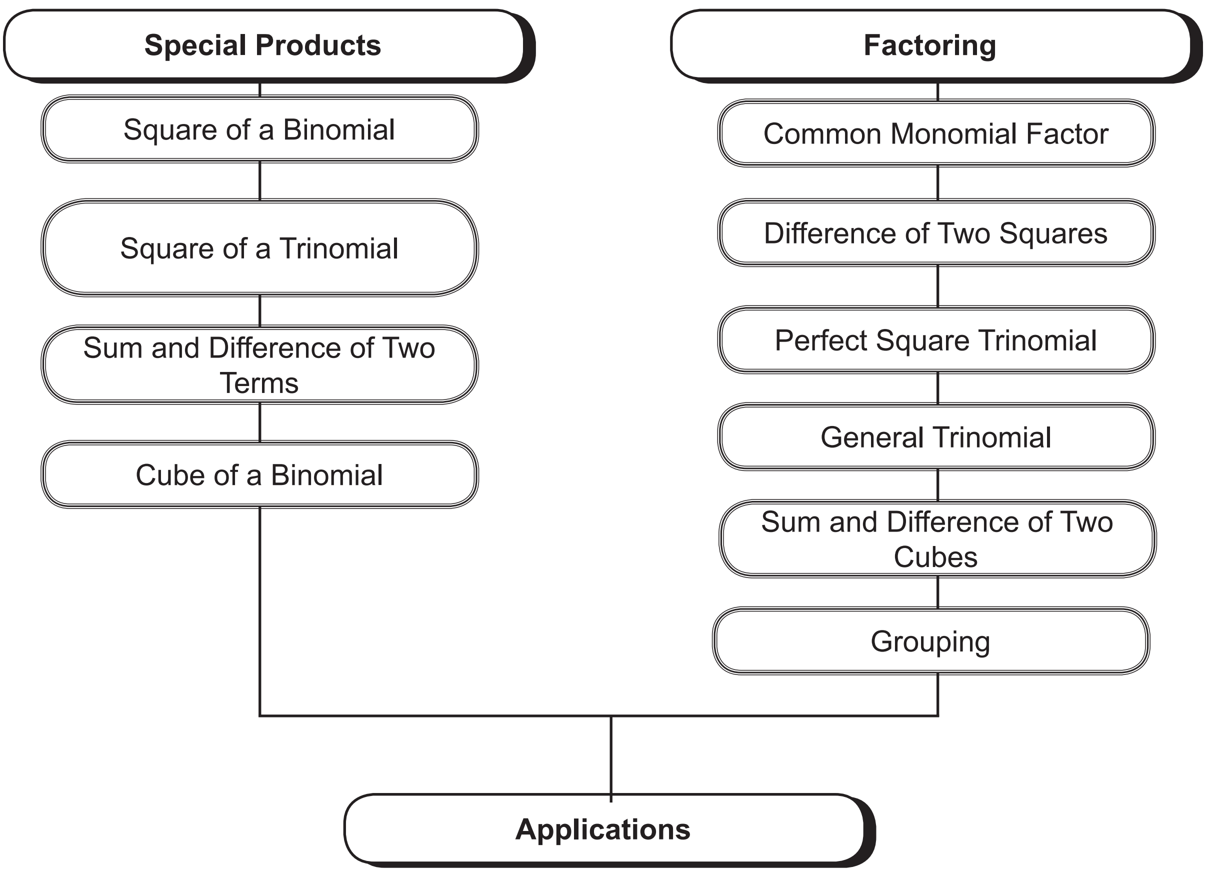
Lesson 1 – Special Products
Lesson 2 – Factoring

In these lessons, you will learn to:

Lesson 1 *Special Products*	• identify polynomials which are special products through pattern recognition • find special products of certain polynomials • apply special products in solving geometric problems • solve problems involving polynomials and their products
Lesson 2 *Factoring*	• factor completely different types of polynomials • find factors of products of polynomials • solve problems involving polynomials and their factors.

Module Map

Here is a simple map of the lessons that will be covered in this module:



Find out how much you already know about this module. Write the letter that corresponds to the best answer on your answer sheet.

III. PRE-ASSESSMENT

1. Which mathematical statement is correct?

   a. $6x^2 - 5xy + y^2$
   b. $16x^2 - 40x + 25$
   c. $6x^2 + 13x - 28$
   d. $4x^2 + 20x + 25$

2. Which of the following DOES NOT belong to the group?

   a. $\frac{1}{4}x^4 - 1$
   b. $x^2 - 0.0001y^4$
   c. $8(x - 1)^3 - 27$
   d. $(x + 1)^4 - 4x^6$

3. Which of the following gives a product of $x^2 + 5x + 4$?

   a. $(x + 1)(x + 4)$
   b. $(x + 2)(x + 2)$
   c. $(x + 5)(x - 1)$
   d. $(x + 2)^2$

4. A polynomial expression is evaluated for the x- and y-values shown in the table below. Which expression gives the values shown in the third column?

X	Y	Value of the Expression
0	0	0
-1	-1	0
1	1	0
1	-1	4

   a. $x^2 - y^2$
   b. $x^2 + 2xy + y^2$
   c. $x^2 - 2xy + y^2$
   d. $x^3 - y^3$

5. Find the missing terms: $(x + \_\_\_)(3x + \_\_\_) = 3x^2 + 27x + 24$

   a. 6, 4
   b. 4, 6
   c. 8, 3
   d. 12, 2

6. The length of a box is five meters less than twice the width. The height is 4 meters more than three times the width. The box has a volume of 520 cubic meters. Which of the following equations can be used to find the height of the box?

a. W(2L – 5) (3H + 4) = 520
b. W(2W + 5) (3W – 4) = 520
c. W(2W – 5) (3W – 4) = 520
d. W(2W – 5) (3W + 4) = 520

7. One of the factors of $2a^2 + 5a - 12$ is $a + 4$. What is the other factor?

a. $2a - 3$
b. $2a + 3$
c. $2a - 8$
d. $2a + 8$

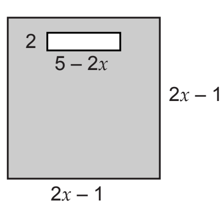
8. The area of a square is $4x^2 + 12x + 9$ square units. Which expression represents the length of the side?

a. $(3x + 2)$ units
b. $(2x + 3)$ units
c. $(4x + 9)$ units
d. $(4x + 3)$ units

9. The side of a square is x cm long. The length of a rectangle is 5 cm longer than the side of the square and the width is 5 cm shorter. Which statement is true?

a. The area of the square is greater than the area of the rectangle.
b. The area of the square is less than the area of the rectangle.
c. The area of the square is equal to the area of the rectangle.
d. The relationship cannot be determined from the given information.

10. A square piece of land was rewarded by a landlord to his tenant. They agreed that a portion of it represented by the rectangle inside should be used to construct a grotto. How large is the area of the land that is available for the other purposes?

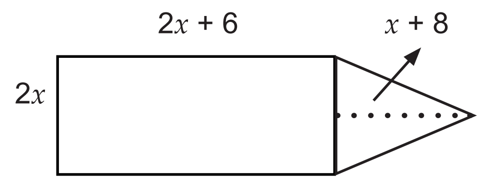


a. $4x^2 - 9$
b. $4x^2 + x + 9$
c. $4x^2 - 8x - 9$
d. $4x^2 + 9$

11. Which value for x will make the largest area of the square with a side of $3x + 2$?

a. $\frac{3}{4}$
b. 0.4
c. $\frac{1}{3}$
d. 0.15

12. Which procedure could not be used to solve for the area of the figure below?



a. $A = 2x\,(2x + 6) + \frac{1}{2}\,(2x)(x + 8)$

$A = 4x^2 + 12x + x^2 + 8x$

$A = 5x^2 + 20x$

b. $A = 2x(3x + 14) - 2(\frac{1}{2})(x)(x + 8)$

$A = 6x^2 + 28x - x^2 - 8x$

$A = 5x^2 + 20x$

c. $A = [2x(2x + 6) + (x + 8)(2x)] - 2(\frac{1}{2})(x)(x + 8)$

$A = [4x^2 + 12x) + (2x^2 + 16x) - (x^2 + 8x)$

$A = 6x^2 + 28x - x^2 - 8x$

$A = 5x^2 + 20x$

d. $A = 2x(2x + 6) + (\frac{1}{2})(2 + x)(x + 8)$

$A = 4x^2 + 12x + x^2 + 8x$

$A = 5x^2 + 20x$

13. Your classmate was asked to square $(2x - 3)$, he answered $4x^2 - 9$. Is his answer correct?

a. Yes, because squaring a binomial always produces a binomial product.
b. Yes, because the product rule is correctly applied.
c. No, because squaring a binomial always produces a trinomial product.
d. No, because the answer must be $4x^2 + 9$

.

14. Let A: $4x^2 - 81$, and let B: $(2x - 9)(2x + 9)$. If $x = 2$, which statement is true about A and B?

a. A > B b. A < B c. A = B d. A ≠ B

15. Your sister plans to remodel her closet. She hired a carpenter to do the task. What should your sister do so that the carpenter can accomplish the task according to what she wants?

    a. Show a replica of a closet.
    b. Download a picture from the internet.
    c. Leave everything to the carpenter.
    d. Provide the layout drawn to scale.

16. Which of the following standards would best apply in checking the carpenter's work in item number 15?

    a. accuracy of measurements and wise utilization of materials
    b. accuracy of measurements and workmanship
    c. workmanship and artistic design
    d. workmanship and wise utilization of materials

17. The city mayor asked you to prepare a floor plan of the proposed day care center in your *barangay*. The center must have a small recreational corner. As head of the city engineering office, what will you consider in preparing the plan?

    a. Feasibility and budget
    b. Design and budget
    c. Design and Feasibility
    d. Budget and lot area

18. Suppose there is a harvest shortage in your farm because of malnourished soil. What will you do to ensure a bountiful harvest in your farmland?

    a. Hire number of workers to spread fertilizers in the farmland.
    b. Buy several sacks of fertilizers and use them in your farmland.
    c. Find the area of the farmland and buy proportionate number of sacks of fertilizers.
    d. Solve for the number of sacks of fertilizers proportionate to the number of workers.

19. The *Punong Barangay* in your place noticed that garbage is not properly disposed because the garbage bins available are too small. As the chairman of the health committee, you were tasked to prepare garbage bins which can hold 24 ft^3 of garbage. However, the location where the garbage bins will be placed is limited. H ow will you maximize the area?

    a. Find the dimensions of the planned bin according to the capacity given.
    b. Make trial and error bins until the desired volume is achieved.
    c. Solve for the volume and use it in creating bins.
    d. Find the area of the location of the bins.

20. As head of the marketing department of a certain construction firm, you are tasked to create a new packaging box for the soap products. What criteria will you consider in creating the box?

    a. Appropriateness and the resources used
    b. Resources used and uniqueness
    c. Appropriateness and uniqueness
    d. Appropriateness and capacity

How was your performance in the pre–test? Were you able to answer all the problems? Did you find difficulties in answering them? Are there questions familiar to you?

IV. LEARNING GOALS AND TARGETS

In this module, you will have the following targets:

- Demonstrate understanding of the key concepts of special products and factors of polynomials.
- Formulate real-life problems involving special products and factors and solve these with utmost accuracy using a variety of strategies.

Lesson 1 Special Products

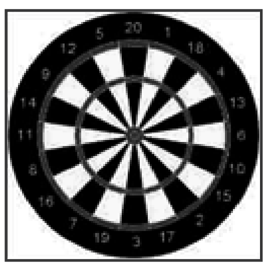
What to Know

Let us start our study of this module by reviewing first the concepts on multiplying polynomials, which is one of the skills needed in the study of this module. Discuss the questions below with a partner.

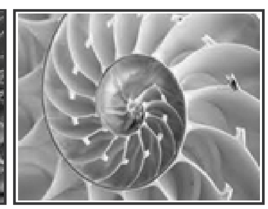
PATTERNS WHERE ARE YOU?

Have you ever looked around and recognized different patterns? Have you asked yourself what the world's environment would look like if there were no patterns? Why do you think there are patterns around us?

Identify the different patterns in each picture. Discuss your observations with a partner.



http://meganvanderpoel.blogspot.com/2012/09/pattern-precedents.html



http://gointothestory.blcklst.com/2012/02/doodling-in-math-spirals-fibonacci-and-being-a-plant-1-of-3.html

Have you ever used patterns in simplifying mathematical expressions? What advantages have you gained in doing such? Let us see how patterns are used to simplify mathematical expressions by doing the activity below. Try to multiply the following numerical expressions. Can you solve the following numerical expressions mentally?

$$97 \times 103 =$$
$$25 \times 25 =$$
$$99 \times 99 \times 99 =$$

Now, answer the following questions:

1. What do you notice about the given expressions?
2. Did you solve them easily? Did you notice some patterns in finding their answers?
3. What technique/s did you use? What difficulties did you encounter?

The indicated products can be solved easily using different patterns.

Are your solutions different from your classmates? What did you use in order to find the products easily?

> *The problems you have answered are examples of the many situations where we can apply knowledge of special products. In this lesson, you will do varied activities which will help you answer the question, "**How can unknown quantities in geometric problems be solved**?"*

Let's begin by answering the "I" portion of the IRF Worksheet shown below. Fill it up by writing your initial answer to the topical focus question:

Activity 1 IRF WORKSHEET

Description: Below is the IRF worksheet which will determine your prior knowledge about the topical question.

Direction: Answer the topical questions: **(1) What makes a product special**? and **(2) What patterns are involved in multiplying algebraic expressions**? Write your answer in the initial part of the IRF worksheet.

IRF Worksheet
Initial Answer
Revised Answer
Final Answer

Activity 2 COMPLETE ME!

Description: This activity will help you review multiplication of polynomials, the pre-requisite skill to complete this module.

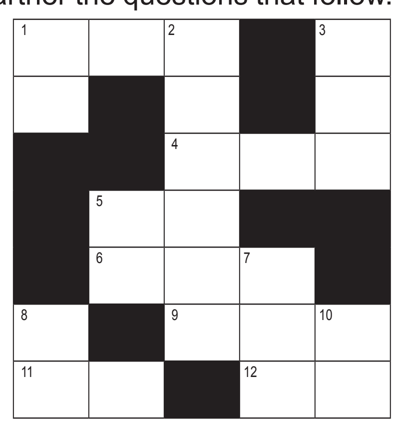
Directions: Complete the crossword polynomial by finding the indicated products below. After completing the puzzle, discuss with a partner the questions that follow.

Across

1. $(a + 3)(a + 3)$
4. $(b + 4a)^2$
5. $2a(-8a + 3a)$
6. $(b - 2)(b - 4)$
9. $-2a(b + 3 - 2a)$
11. $(5b^2 + 7a^2)(-5b^2 + 7a^2)$
12. $(a - 6b)(a + 6b)$

Down

1. $(a + 9)(a - 9)$
2. $(3 + a + b)^2$
3. $(3b - 4a)(3b - 4a)$
5. $(-4a + b)(4a + b)$
7. $(2 - a)(4 - a)$
8. $(4a^3 - 5b^2)(4a^3 + 5b^2)$
10. $(2a + 6b)(2a - 6b)$



QUESTIONS ?

1. How did you find each indicated product?
2. Did you encounter any difficulty in finding the products? Why?
3. What concept did you apply in finding the product?

Activity 3 GALLERY WALK

Description: This activity will enable you to review multiplication of polynomials.

Direction: Find the indicated product of the expressions that will be handed to your group. Post your answers on your group station. Your teacher will give you time to walk around the classroom and observe the answers of the other groups. Answer the questions that follow.

Remember:

To multiply polynomials:
- $a(b + c)$
  $= ab + ac$
- $(a + b)(c + d)$
  $= ac + ad + bc + bd$

CASE 1:

$(x + 5)(x – 5) =$
$(a – b)(a + b) =$
$(x + y)(x – y) =$
$(x – 8)(x + 8) =$
$(2x + 5)(2x – 5) =$

CASE 2:

$(x + 5)(x + 5) =$
$(a – b)^2 =$
$(x + y)(x + y) =$
$(x – 8)^2 =$
$(2x + 5)(2x + 5) =$

CASE 3:

$(x + 5)^3 =$
$(a – b)(a – b)(a – b) =$
$(x + y)^3 =$
$(x + 4)(x + 4)(x + 4) =$
$(x + 2y)^3 =$

CASE 4:

$(a + b + c)(a + b + c) =$
$(x + y + z)(x + y + z) =$
$(m + 2n – 3f)^2 =$

QUESTIONS ?

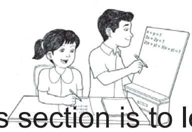
1. How many terms do the products contain?
2. Compare the product with its factors. What is the relationship between the factors and the terms of their product?
3. Do you see any pattern in the product?
4. How did this pattern help you in finding the product?

You just tried finding the indicated products through the use of patterns. Are the techniques applicable to all multiplication problems? When is it applicable and when is it not?

Let us now find the answers by going over the following section. What you will learn in the next sections will enable you to do the final project. This involves making a packaging box using the concepts of special products and factoring.

Let us start by doing the next activity.

What to Process



Your goal in this section is to learn and understand key concepts related to finding special products. There are special forms of algebraic expressions whose products are readily seen. These are called **special products**. There are certain conditions which would make a polynomial special. Discovering these conditions will help you find the product of algebraic expressions easily. Let us start in squaring a binomial

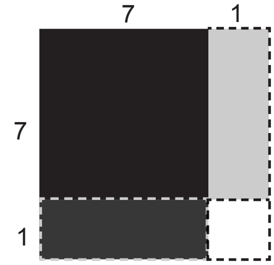
The square of a binomial is expressed as $(x + y)^2$ or $(x + y)(x + y)$ and $(x - y)^2$ or $(x - y)(x - y)$. In your previous grade, you did this by applying the FOIL method, which is sometimes tedious to do. There is an easier way in finding the desired product and that is what we will consider here.

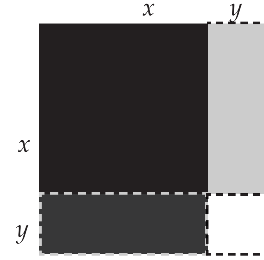
Activity 4 FOLD TO SQUARE

Description: In this activity, you will model the square of a binomial through paper folding. Investigate the pattern that can be produced in this activity. This pattern will help you find the square of a binomial easily. You can do this individually or with a partner.

Directions: Get a square paper measuring 8” × 8”.

1. Fold the square paper 1” with from an edge and make a crease.
2. Fold the upper right corner by 1” and make a crease.
3. Unfold the paper.
4. Continue the activity by creating another model for squaring a binomial by changing the measures of the folds to 2 in. and 3 in. Then answer the questions below.





Remember:

- Area of square = s^2
- Area of rectangle = lw

QUESTIONS

1. How many different regions are formed? What geometric figures are formed? Give the dimensions of each region?
2. What is the area of each region?
3. What will be the area if the longer part is replaced by x? by x *and* 1?
4. What is the sum of the areas? Write the sum of areas in the box below.
5. If 1 is replaced by y, what will be the area?

	FIRST TERM	SECOND TERM	LAST TERM
$(x + 1)^2$			
$(x + 2)^2$			
$(x + 3)^2$			
$(x + y)^2$			

Did you find any pattern? What pattern is it?

1. How is the first term of the product related to the first term of the given binomial?
2. How is the last term of the product related to the last term of the given binomial?
3. What observation do you have about the middle term of the product and the product of the first and last terms of the binomial?

Observe the following examples:

a. $$\begin{aligned}(x - 3)^2 &= (x)^2 - 3x - 3x + (3)^2\\ &= x^2 - 2(3x) + 9\\ &= x^2 - 6x + 9\end{aligned}$$

b. $$\begin{aligned}(x + 5)^2 &= (x)^2 + 5x + 5x + (5)^2\\ &= x^2 + 2(5x) + 25\\ &= x^2 + 10x + 25\end{aligned}$$

c. $$\begin{aligned}(3x + 4y)^2 &= (3x)^2 + 12xy + 12xy + (4y)^2\\ &= 9x^2 + 2(12xy) + 16y^2\\ &= 9x^2 + 24xy + 16y^2\end{aligned}$$

Remember:

- Product rule
  $(a^m)(a^n) = a^{m+n}$
- Raising a power to a power
  $(a^m)^n = a^{mn}$

The square of a binomial consists of:

a. *the square of the first term;*
b. *twice the product of the first and last terms; and*
c. *the square of the last term.*

Remember that the square of a binomial is called a ***perfect square trinomial****.*

LET'S PRACTICE!

Square the following binomials using the pattern you have just learned.

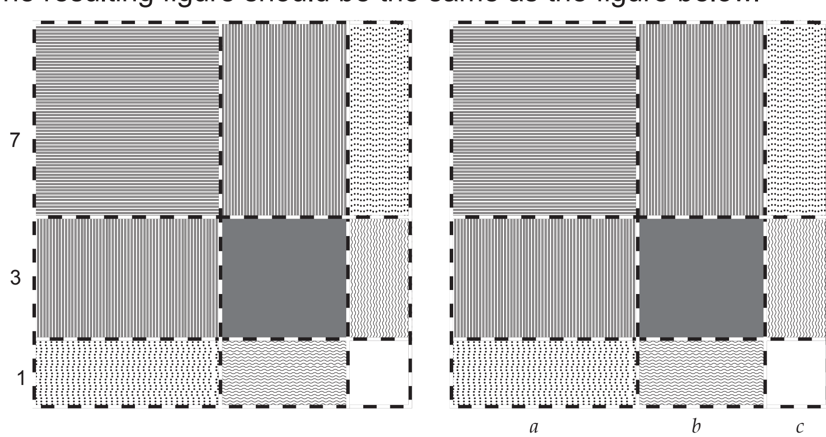
1. $(s + 4)^2$
2. $(w - 5)^2$
3. $(e - 7)^2$
4. $(2q - 4)^2$
5. $(3z + 2k)^2$
6. $(5d - 7d^2t)^2$
7. $(7q^2w^2 - 4w^2)^2$
8. $(\frac{2}{2}e - 6)^2$
9. $(\frac{4}{5}kj - 6)^2$
10. $[(x + 3) - 5]^2$

The square of a binomial is just one example of special products. Do the next activity to discover another type of special product, that is squaring a trinomial.

Activity 5 DISCOVER ME AFTER! (PAPER FOLDING AND CUTTING)

Description: In this activity you will model and discover the pattern on how a trinomial is squared, that is $(a + b + c)^2$. Investigate and observe the figure that will be formed.

Directions: Get a 10" × 10" square paper. Fold the sides 7", 3" and 1" vertically and make creases. Using the same measures, fold horizontally and make creases. The resulting figure should be the same as the figure below.



QUESTIONS ?

1. How many regions are formed? What are the dimensions of each region in the figure?
2. What are the areas of each region?
3. If the side of the biggest square is replaced by a, how will you represent its area?
4. If one of the dimensions of the biggest rectangle is replaced by b, how will you represent its area?
5. If the side of the smaller square is replaced by c, how will you represent its area?
6. What is the sum of the areas of all regions? Do you observe any pattern in the sum of their areas?

Observe the following examples and take note of your observation.

a. $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2xz$
b. $(m + n - d)^2 = m^2 + n^2 + d^2 + 2mn - 2md - 2nd$
c. $(3d + 2e + f)^2 = 9d^2 + 4e^2 + f^2 + 12de + 6df + 4ef$

The square of a trinomial consists of:
a. the sum of the squares of the first, second and last terms;
b. twice the product of the first and the second terms;
c. twice the product of the first and the last terms; and
d. twice the product of the second and the last terms.

LET'S PRACTICE!

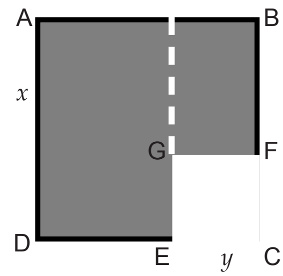
Square the following trinomials using the pattern you have learned.

1. $(r - t + n)^2$
2. $(e + 2a + q)^2$
3. $(m + a - y)(m + a - y)$
4. $(2s + o - 4n)^2$
5. $(2i^2 + 3a - 5n)^2$
6. $(15a - 4n - 6)^2$
7. $(4a + 4b + 4c)^2$
8. $(9a^2 + 4b^2 - 3c^2)^2$
9. $(1.5a^2 - 2.3b + 1)^2$
10. $(\frac{3x}{4} + \frac{4y}{3} - 6)^2$

Activity 6 TRANSFORMERS

Description: This activity will help us model the product of the sum and difference of two terms $(x - y)(x + y)$ and observe patterns to solve it easily.

Directions: Prepare a square of any measure; label its side as x. Cut a small square of side y from any of its corner (as shown below). Answer the questions that follow.



In terms of x and y, answer the following:

1. What is the area of the original big square (**ABCD**)?
2. What is the area of the small square (**GFCE**)?
3. How are you going to represent the area of the new figure?

Cut along the broken line as shown and rearrange the pieces to form a rectangle.

1. What are the dimensions of the rectangle formed?
2. How will you get the area of the rectangle?
3. Represent the area of the rectangle that was formed. Do you see any pattern in the product of the sum and difference of two terms?

Study the relationship that exists between the product of the sum and difference of two terms and the factors. Take note of the pattern formed.

a. $(x + y)(x - y) = x^2 - y^2$
b. $(a - b)(a + b) = a^2 - b^2$
c. $(m + 3)(m - 3) = m^2 - 9$
d. $(w - 5)(w + 5) = w^2 - 25$
e. $(2x - 5)(2x + 5) = 4x^2 - 25$

> *The product of the sum and difference of two terms is the **difference of the squares of the terms**. In symbols, $\mathbf{(x + y)(x - y) = x^2 - y^2}$. Notice that the product is always a binomial.*

LET'S PRACTICE!

Multiply the following binomials using the patterns you have learned.

1. $(w - 6)(w + 6)$
2. $(a + 4c)(a - 4c)$
3. $(4y - 5d)(4y + 5d)$
4. $(3sd + 4f)(4f - 3sd)$

5. $(12x - 3)(12x + 3)$

6. $(3s^2r^2 + 7q)(3s^2r^2 - 7q)$

7. $(l^3o^4v^5 - 6e^3)(l^3o^4v^5 + 6e^3)$

8. $(\frac{5}{6}g^2a^2 - \frac{2}{3}d^2)(\frac{5}{6}g^2a^2 + \frac{2}{3}d^2)$

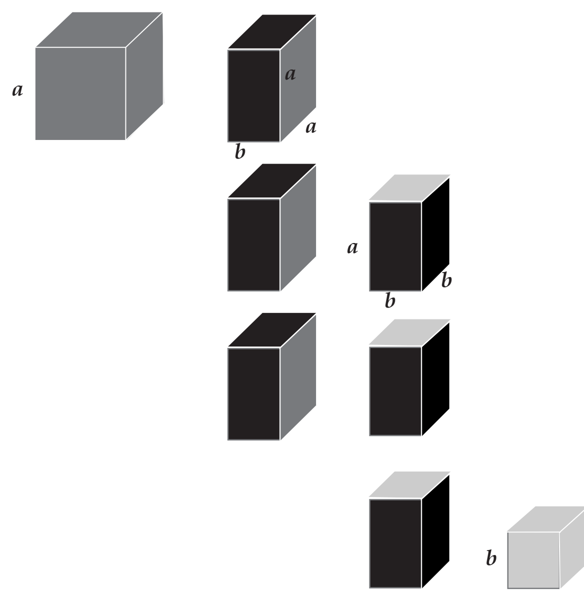
9. $(2s^nq^m + 3d^{3k})(2s^nq^m - 3d^{3k})$

10. $[(s + 2) - 4][(s + 2) + 4]$

The previous activity taught you how to find the product of the sum and difference of two terms using patterns. Perform the next activity to discover another pattern in simplifying expressions of polynomials.

Activity 7 CUBRA CUBE

Description: A cubra cube is a set of cubes and prisms connected by nylon. The task is to form a bigger cube using all the figures provided. Your teacher will help you how to form a cubra cube. After performing the activity, answer the questions that follow.



Remember:

- Volume of a cube = s^3
- Volume of a rectangular prism = *lwh*

QUESTIONS ?

1. How many big cubes did you use? Small cubes?
2. How many different prisms do you have?
3. How many prisms are contained in the new cube?
4. What is the total volume of the new cube formed?
5. If the side of the big cube is marked as a and the smaller cube is marked as b, what is the volume of each figure?
6. What will be the total volume of the new cube?
7. What are the dimensions of the new cube?

This time let us go back to the Gallery Walk activity and focus on case 3, which is an example of a cube of binomial $(x + y)^3$ or $\mathbf{(x + y)(x + y)(x + y)}$ **and** $(x - y)^3$ or $\mathbf{(x - y)(x - y)(x - y)}$.

To find the cube of a binomial of the form $(x + y)^3$:

a. Find the cube of each term to get the first and the last terms.
$\mathbf{(x)^3, (y)^3}$
b. The second term is three times the product of the square of the first term and the second term. $\mathbf{3(x)^2(y)}$
c. The third term is three times the product of the first term and the square of the second term. $\mathbf{3(x)(y)^2}$

Hence, $\mathbf{(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3}$

To find the cube of a binomial of the form $(x - y)^3$:

a. Find the cube of each term to get the first and the last terms.
$\mathbf{(x)^3, (-y)^3}$
b. The second term is three times the product of the square of the first term and the second term. $\mathbf{3(x)^2(-y)}$
c. The third term is three times the product of the first term and the square of the second term. $\mathbf{3(x)(-y)^2}$

Hence, $\mathbf{(x - y)^3 = x^3 - 3x^2y + 3xy^2 - y^3}$

Activity 8 IRF WORKSHEET

Description: Using the "R" portion of the IRF Worksheet, answer the following topical focus questions: **What makes a product special? What patterns are involved in multiplying algebraic expression?**

IRF Worksheet
Initial Answer
Revised Answer
Final Answer

HTTP://www.

You can visit these websites for more games.

http://math123xyz.com/Nav/Algebra/Polynomials_Products_Practice.php
http://worksheets.tutorvista.com/special-products-of-polynomials-worksheet.html#

WEB – BASED ACTIVITY: DRAG AND DROP

Description: Now that you have learned the various special products, you will now do an interactive activity which will allow you to drag sets of factors and drop them beside special products. The activity is available in this website:
http://www.media.pearson.com.au/schools/cw/au_sch_bull_gm12_1/dnd/2_spec.html.

QUESTIONS:

1. What special products did you use in the activity?
2. Name some techniques which you used to make the work easier.
3. What generalizations can you draw from the examples shown?
4. Given the time constraint, how could you do the task quickly and accurately?

Activity 9 3-2-1 CHART

Description: In this activity, you will be asked to complete the 3-2-1 Chart regarding the special products that you have discovered.

3-2-1 Chart
Three things I found out:
1. ____________________ 2. ____________________ 3. ____________________
Two interesting things:
1. ____________________ 2. ____________________
One question I still have:
1. ____________________

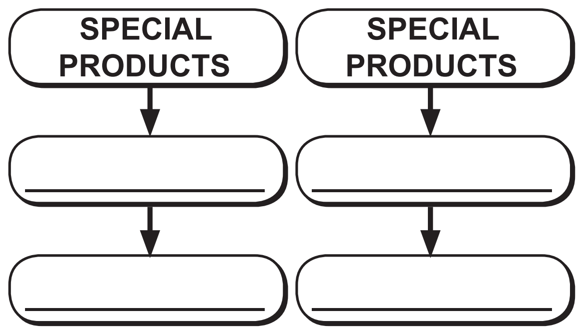
Activity 10 WHAT'S THE WAY? THAT'S THE WAY!

Description: This activity will test if you have understood the lesson by giving the steps in simplifying expressions containing special products in your own words.

Directions: Give the different types of special products and write the steps/process of simplifying them. You may add boxes if necessary.

Video Watching:
You can visit the following websites to watch different discussions and activities on special products.

1. http://www.youtube.com/watch?v=bFtjG45-Udk (Square of binomial)
2. http://www.youtube.com/watch?v=OWu0tH5RC2M (Sum and difference of binomials)
3. http://www.youtube.com/watch?v=PcwXRHHnV8Y (Cube of a binomial)



Now that you know the important ideas about how patterns on special products were used to find the product of a algebraic expressions, let's go deeper by moving on to the next section.

REFLECTION

What I have learned so far...

What to Understand

You have already learned and identified the different polynomials and their special products. You will now take a closer look at some aspects of the topic and check if you still have misconceptions about special products.

Activity 11 DECISION, DECISION, DECISION!

Directions: Help each person decide what to do by applying your knowledge on special products on each situation.

1. Jem Boy wants to make his 8-meter square pool into a rectangular one by increasing its length by 2 m and decreasing its width by 2 m. Jem Boy asked your expertise to help him decide on certain matters.

    a. What will be the new dimensions of Jem Boy's pool?
    b. What will be the new area of Jem Boy's pool? What special product will be used?
    c. If the sides of the square pool is unknown, how will you represent its area?
    d. If Jem Boy does not want the area of his pool to decrease, will he pursue his plan? Explain your answer.

2. Emmanuel wants to tile his rectangular floor. He has two kinds of tiles to choose from, one of which is larger than the other. Emmanuel hired your services to help him decide which tile to use.

    a. What area will be covered by the 8" x 8" tile? 16" x 16" tile?
    b. If the rectangular floor has dimensions of 74" x 128", how many small square tiles are needed to cover it?
    c. How many big square tiles are needed to cover the rectangular floor?
    d. If each small tile costs Php 15.00 and each big tile costs Php 60.00, which tile should Emmanuel use to economize in tiling his floor? Explain why.

Activity 12 AM I STILL IN DISTRESS?

Description: This activity will help you reflect about things that may still confuse you in this lesson.

Directions: Complete the phrase below and write it on your journal.

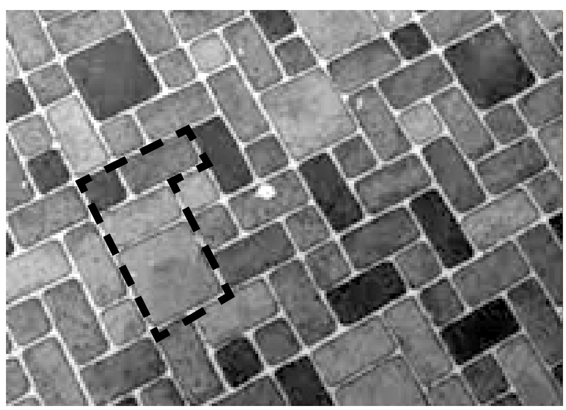
The part of the lesson that I still find confusing is ___________________ because ____.

Let us see if your problem will be solved doing the next activity.

Activity 13 BEAUTY IN MY TILE!

Description: See different tile patterns on the flooring of a building and calculate the area of the region bounded by the broken lines, then answer the questions below.

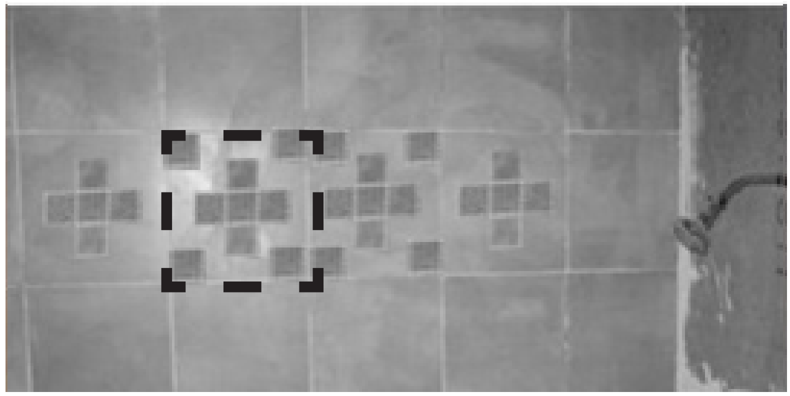
1.



http://www.apartmenttherapy.com/tile-vault-midcentury-rec-room-39808

a. What is the area represented by the big square? small square? rectangles?
b. What is the total area bounded by the region?
c. What special product is present in this tile design?
d. Why do you think the designer of this room designed it as such?

2.



a. What is the area represented by the big square? Small square?
b. What is the sum of all areas of small squares?
c. If the small squares were to be removed, how are you going to represent the area that will be left?

QUESTIONS ?

1. What difficulties did you experience in doing the activity?
2. How did you use special products in this activity?
3. What new insights did you gain?
4. How can unknown quantities in geometric problems be solved?

Activity 14 WHERE IS THE PATTERN?

Descriptions: Take a picture/sketch of a figure that makes use of special products. Paste it on a piece of paper.

QUESTIONS ?

1. Did you find difficulty in looking for patterns where the concept of special products was applied?
2. What special products were applied in your illustration?
3. What realization do you have in this activity?

Activity 15 LET'S DEBATE!

Description: Form a team of four members and debate on the two questions below. The team that can convince the other teams wins the game.

- "Which is better to use in finding products, patterns or long multiplication?"
- "Which will give us more benefit in life, taking the shortcuts or going the long way?

Activity 16 IRF WORKSHEET

Description: Now that you have learned the different special products, using the "F" portion of the IRF Worksheet, answer the topical focus question: **What makes a product special? What patterns are involved in multiplying algebraic expressions?**

Initial Answer
Revised Answer
Final Answer

Now that you have a deeper understanding of the topic, you are ready to do the tasks in the next section.

What to Transfer

Let us now apply your learning to real–life situations. You will be given a practical task which will demonstrate your understanding.

Activity 17 MAKE A WISH

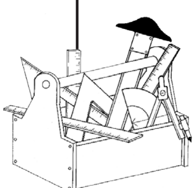
Description: The concept of squaring binomials is used in the field of Genetics through PUNNETT squares. PUNNETT SQUARES are used in genetics to model the possible combinations of parents' genes in offspring. In this activity you will discover how it will be used.

Direction: Investigate how squaring trinomials are applied in PUNNETT squares and answer the following questions.

One cat carries heterozygous, long-haired traits (Ss), and its mate carries heterozygous, long-haired traits (Ss). To determine the chances of one of their offsprings having short hair, we can use PUNNETT squares.

	S	s
S	SS	Ss
s	Ss	ss

QUESTIONS ?



1. What are the chances that the offspring is a long–haired cat? A short–haired cat?
2. What are the different possible offsprings of the mates?
3. How many homozygous offsprings will they have? Heterozygous?
4. How is the concept of squaring binomials used in this process?
5. Do you think it is possible to use the process of squaring trinomials in the field of genetics?
6. Create another model of PUNNET square using a human genetic component. Explain the possible distribution of offsprings and how squaring trinomials help you in looking for its solution.
7. Create your own PUNNET square using the concept of squaring trinomials, using your dream genes.

Now that you have seen the different patterns that can be used in simplifying polynomial expressions, you are now ready to move to the next lesson which is factoring. Observe the different patterns in factoring that are related to special products so that you can do your final project, the making of a packaging box.

Punnett square is named after **Reginald C. Punnett**, who devised the approach. It is used by biologists to determine the chances of an offspring having a particular genotype. The Punnett square is a tabular summary of every possible combination of one maternal allele with one paternal allele for each gene being studied in the cross.

REFLECTION

In this lesson, I have understood that ____.

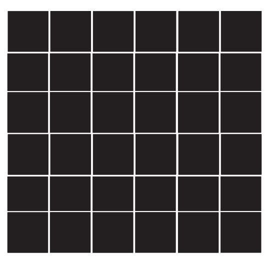
Lesson 2 Factoring

What to Know

Your goal in this section is to see the association of products to factors by doing the activities that follow.

Before you start doing the activities in this lesson, do this challenge first.

The figure below is a square made up of 36 tiles. Rearrange the tiles to create a rectangle, having the same area as the original square. How many such rectangles can you create? What do you consider in looking for the other dimensions? What mathematical concepts would you consider in forming different dimensions? Why? Suppose the length of one side is increased by unknown quantities (e.g. x) how could you possibly represent the dimensions?



This module will help you break an expression into different factors and answer the topical questions, "**What algebraic expressions can be factored**? **How are patterns used in finding the factors of algebraic expressions**? **How can unknown quantities in geometric problems be solved**?"

To start with this lesson, perform the activities that follow:

Activity 1 LIKE! UNLIKE!

Description: This activity will help gauge how ready you are for this lesson through your responses.

Directions: Answer all the questions below honestly by pasting the *like* or *unlike* thumb that your teacher will provide you. *Like* means that you are the one being referred to and *unlike* thumb means that you have no or little idea about what is being asked.

SKILLS ACQUIRED	RESPONSES
1. Can factor numerical expressions easily	
2. Can divide polynomials	
3. Can apply the quotient rule of exponents	
4. Can add and subtract polynomials	
5. Can work with special products	
6. Can multiply polynomials	

Before you proceed to the next topic, answer first the IRF form to determine how much you know in this topic and see your progress.

Activity 2 IRF WORKSHEETS

Description: Complete the table by filling up first the initial column of the chart with your answer to each item. This activity will determine how much you know about this topic and your progress.

	Initial	Revise	Final
Express the following as product of factors. 1. $4x^2 - 12x$ = ________ 2. $9m^2 - 16n^2$ = ________ 3. $4a^2 + 12a + 9$ = ________ 4. $2x^2 + 9x - 5$ = ________ 5. $27x^3 - 8y^3$ = ________ 6. $a^3 + 125b^3$ = ________ 7. $xm + hm - xn - hn$ = ________			

Activity 3 MESSAGE FROM THE KING (Product – Factor Association)

Description: This activity will give you an idea on how factors are associated with products. You will match the factors in column A with the products in column B to decode the secret message.

COLUMN A		COLUMN B	
1.	$4x(3x-5)$	A.	$6x^2y^2+3xy^3-3xy^2$
2.	$3xy^2(2x+y-1)$	F.	x^3-27
3.	$(x+y)(x-y)$	G.	$4x^2-9$
4.	$(2x+3)(2x-3)$	R.	$4x^2+12x+9$
5.	$(x-5y)(x+5y)$	U.	$12x^2-20x$
6.	$(x+y)^2$	E.	$6x^2+x-2$
7.	$(2x+3)^2$	T.	$ac-ad+bc-bd$
8.	$(x-5y)^2$	S.	$mr-nr+ms-ns$
9.	$(x+4)(x-3)$	C.	x^2-y^2
10.	$(2x-1)(3x+2)$	I.	$2x^2-x-10$
11.	$(x+2)(2x-5)$	O.	$x^2-10xy+25y^2$
12.	$(x-3)(x^2+3x+9)$	N.	x^2+x-12
13.	$(x+3)(x^2-3x+9)$	H.	x^3-27
14.	$(a+b)(c-d)$	M.	$x^2+2xy+y^2$
15.	$(m-n)(r+s)$	L.	x^2-25y^2
16.	$(3x+4)(3x-4)$	P.	$9x^2-16$
17.	$(3x-4)^2$	V.	$9x^2-24x+16$

___ 12 ___ 2 ___ 3 ___ 14 ___ 8 ___ 7 ___ 11 ___ 9 ___ 4   ___ 11 ___ 15   ___ 14 ___ 13 ___ 10

___ 7 ___ 10 ___ 17 ___ 10 ___ 7 ___ 15 ___ 10   ___ 8 ___ 12

___ 6 ___ 1 ___ 5 ___ 14 ___ 11 ___ 16 ___ 5 ___ 11 ___ 3 ___ 2 ___ 14 ___ 11 ___ 8 ___ 9

QUESTIONS ?

1. What are your observations on the expressions in column A? Compare them with those in column B.
2. Do you see any pattern?
3. Are the two expressions related?
4. Why is it important to know the reverse process of multiplication?

What did you discover between the relationship of products and its factors? You have just tried finding out the relationship between factors and their product. You can use this idea to do the next activities.

What you will learn in the next session will also enable you to do the final project which involves model and lay–out making of a packaging box.

What to Process

The activity that you did in the previous section will help you understand the different lessons and activities you will encounter here.

The process of finding the factors of an expression is called **factoring**, which is the reverse process of multiplication. A **prime number** is a number greater than 1 which has only two positive factors: 1 and itself. Can you give examples of prime numbers? Is it possible to have a prime that is a polynomial? If so, give examples.

The first type of factoring that you will encounter is **Factoring the Greatest Common Monomial Factor**. To understand this let us do some picture analysis.

Activity 4 FINDING COMMON

Description: Your task in this activity is to identify common things that are present in the three pictures.

http://blog.ningin.com/2011/09/04/10-idols-and-groups-pigging-out/

http://k-pop-love.tumblr.com/post/31067024715/eating-sushi

QUESTIONS ?

1. What are the things common to these pictures?
2. Are there things that make them different?
3. Can you spot things that are found on one picture but not on the other two?
4. What are the things common to two pictures but are not found on the other?

The previous activity gave us the idea about the **Greatest Common Monomial Factor** that appears in every term of the polynomial. Study the illustrative examples on how factoring the Greatest Common Monomial Factor is being done.

Factor $12x^3y^5 - 20x^5y^2z$

a. Find the greatest common factor of the numerical coefficients.

The GCF of 12 and 20 is 4.

b. Find the variable with the least exponent that appears in each term of the polynomial.

*x and y are both common to all terms and 3 is the smallest exponent for x and 2 is the smallest exponent of y, thus, x^3y^2 is the **GCF** of the variables.*

c. The product of the **greatest common factor** in **(a)** and **(b)** is the GCF of the polynomial.

Hence, $\mathbf{4x^3y^2}$ is the GCF of $12x^3y^5 - 20x^5y^2z$.

d. To completely factor the given polynomial, divide the polynomial by its GCF, the resulting quotient is the other factor.

Thus, the factored form of $12x^3y^5 - 20x^5y^2z$ is $4x^3y^2(3y^3 - 5x^2z)$

Below are other examples of Factoring the Greatest Monomial Factor.

a. $8x^2 + 16x \Rightarrow$ $\mathbf{8x}$ is the **greatest monomial factor**. Divide the polynomial by $8x$ to get the other factor.

$\mathbf{8x(x + 2)}$ is the factored form of $8x^2 + 16x$.

b. $12x^5y^4 - 16x^3y^4 + 28x^6 \Rightarrow 4x^3$ is the greatest monomial factor. Divide the given expression by the greatest monomial factor to get the other factor.

Thus, $\mathbf{4x^3\ (3x^2y^4 - 4y^4 + 7x^3)}$ is the factored form of the given expression.

Complete the table to practice this type of factoring.

Polynomial	Greatest Common Monomial Factor (CMF)	Quotient of Polynomial and CMF	Factored Form
$6m + 8$	2	$3m + 4$	$2\ (3m + 4)$
	$4mo^2$		$4mo^2\ (3m + o)$
$27d^4o^5t^3a^6 - 18d^2o^3t^6 - 15d^6o^4$		$9d^2o^2t^3a^6 - 6t^6 - 5d^4$	
4(12) + 4(8)	4		
$12WI^3N^5 - 16WIN + 20WINNER$			

Now that you have learned how to factor polynomials using their greatest common factor we can move to the next type of factoring, which is the difference of two squares. ***Why do you think it was given such name?*** *To model it, let's try doing the activity that follows.*

Activity 5 INVESTIGATION IN THE CLASSROOM

Description: This activity will help you understand the concepts of difference of two squares and how this pattern is used to solve numerical expressions. Investigate the number pattern by comparing the products, then write your generalizations afterwards.

NUMBER PATTERN:

a. (11)(9) = (10 + 1)(10 – 1) = 100 – 1 =
b. (5)(3) = (4 + 1)(4 – 1) = 16 – 1 =
c. (101)(99) = (100 + 1)(100 – 1) = 10000 – 1 =
d. (95)(85) = (90 + 5)(90 – 5) = 8100 – 25 =
e. $(n - 5)(n + 5)$ =

How do you think the products are obtained? What are the different techniques used to solve for the products?

What is the relationship of the product to its factor? Have you seen any pattern in this activity?

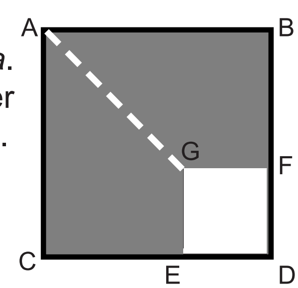
For you to have a clearer view of this type of factoring, let us have a paper folding activity again.

Activity 6 INVESTIGATION IN PAPER FOLDING

Description: This activity will help you visualize the pattern of difference of two squares.

Directions:

1. Get a square paper and label the sides as *a*.
2. Cut – out a small square in any of its corner and label the side of the small square as *b*.
3. Cut the remaining figure in half.
4. Form a rectangle



QUESTIONS ?

1. What is the area of square ABDC?
2. What is the area of the cutout square GFDE?
3. What is the area of the new figure formed?
4. What is the dimension of the new figure formed?
5. What pattern can you create in the given activity?

For you to have a better understanding about this lesson, observe how the expressions below are factored. Observe how each term relates with each other.

a. $x^2 - y^2 = (x + y)(x - y)$

b. $4x^2 - 36 = (2x + 6)(2x - 6)$

c. $a^2b^4 - 81 = (ab^2 - 9)(ab^2 + 9)$

d. $16a^6 - 25b^2 = (4a^3 - 5b)(4a^3 + 5b)$

e. $(\frac{9}{16}r^4 - \frac{1}{25}t^2 n^6) = (\frac{3}{4}r^2 + \frac{1}{5}tn^3)(\frac{3}{4}r^2 - \frac{1}{5}tn^3)$

QUESTIONS ?

1. What is the first term of each polynomial?
2. What is the last term of each polynomial?
3. What is the middle sign of the polynomial?
4. How was the polynomial factored?
5. What pattern is seen in the factors of the difference of two terms?
6. Can all expressions be factored using difference of two squares? Why or why not?
7. When can you factor expressions using difference of two squares?

Remember the factored form of a polynomial that is a difference of two squares is the sum and difference of the square roots of the first and last terms.

- *$4x^2 - 36y^2$ ➔ the square root of $4x^2$ is $2x$ and the square root of $36y^2$ is $6y$. To write their factors, write the product of the sum and difference of the square roots of $4x^2 - 36y^2$, that is $(2x + 6y)(2x - 6y)$.*

Activity 7 PAIR MO KO NYAN!

Description: This game will help you develop your factoring skills by formulating your problem based on the given expressions. You can integrate other factoring techniques in creating expressions. Create as many factors as you can.

Directions: Form difference of two squares problems by pairing two squared quantities, then find their factors. (Hint: You can create expressions that may require the use of the greatest common monomial factor.)

	x^2y^3	$16s^2$	25	$81m^4$	k^2	$4m^6$
$24p^2$	$100n^8$	w^6d^{18}	k^6u^{12}	$36h^{10}$	9	
	$20a^4$	$25a^2b^3$	144	$121c^4$	$88m^4$	
$49x^2y^8$	1	$(x + 3)$	$\frac{16}{64}$	$36z^4x^{16}$	$121h^{18}$	
	169	$225v^{22}$	$(x - 7)^2$	$30o^4p^6$	$196d^{18}$	

You have learned from the previous activity how factoring the difference of two squares is done and what expression is considered as the difference of two squares. You are now ready to find the factors of the sum or difference of two cubes. To answer this question, find the indicated product and observe what pattern is evident.

a. $(a + b)(a^2 - ab + b^2)$
b. $(a - b)(a^2 + ab + b^2)$

What are the resulting products? How are the terms of the products related to the terms of the factors? What if the process was reversed and you were asked to find the factors of the products? How are you going to get the factor? Do you see any common pattern?

Activity 8 ROAD MAP TO FACTOR

Answer the following problems by using the map as your guide.

Is the given expression a sum or a difference of two cubes?

If No → Use other factoring technique/method

If Yes → Are the binomials sums or differences of two cubes?

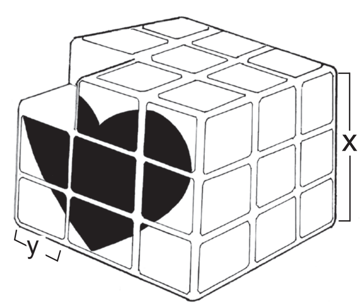
If Difference →

1. What are the cube roots of the first and last terms?
2. Write their **difference** as the first factor. $(x - y)$.
3. For the second factor, get the trinomial factor by:
   a. Squaring the first term of the first factor;
   b. Adding the product of the first and second terms of the first factor;
   c. Squaring the last term of the first factor.
4. Write them in factored form.
   $\mathbf{(x - y)(x^2 + xy + y^2)}$

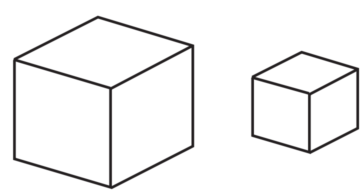
If Sum →

1. What are the cube roots of the first and last terms?
2. Write their sum as the first factor. $\mathbf{(x + y)}$.
3. For the second factor, get the trinomial factor by:
   a. Squaring the first term of the first factor;
   b. Subtracting the product of the first and second terms of the first factor;
   c. Squaring the last term of the first factor.
4. Write them in factored form.
   $\mathbf{(x + y)(x^2 - xy + y^2)}$

1. Represent the volume of this figure. What is the factored form of the volume of a given figure?



2. What are the volumes of the cubes? If the cubes are to be joined to create a platform for a statue, what will be the volume of the platform? What are the factors of the volume of the platform?



Activity 9 Let's tile it up!

Directions: Prepare the following:

1. 4 big squares measuring 4" × 4" represent each square...
2. 8 rectangular tiles measuring 4" × 1" represent each square...
3. 16 small squares measuring is 1" × 1" represent each square...

Form squares using:

- 1 big square tile, 2 rectangular tiles, and 1 small square.
- 1 big square tile, 4 rectangular tiles, and 4 small squares.
- 1 big square tile, 6 rectangular tiles, and 9 small squares.
- 4 big square tiles, 4 rectangular tiles, and 1 small square.
- 4 big square tiles, 8 rectangular tiles, and 4 small squares.

QUESTIONS ?

1. How will you represent the total area of each figure?
2. Using the sides of the tiles, write all the dimensions of the squares.
3. What did you notice about the dimensions of the squares?
4. Did you find any pattern in their dimensions? If yes, what are those?
5. How can unknown quantities in geometric problems be solved?

The polynomials formed are called ***perfect square trinomials.***

A perfect square trinomial *is the result of squaring a binomial. A perfect square trinomial has first and last terms which are perfect squares and a middle term which is twice the product of the square root of the first and last terms.*

Activity 10 PERFECT HUNT

Directions: Look for the different perfect square trinomials found in the box. Answers might be written diagonally, horizontally, or vertically.

$25x^2$	$10x$	81	$18x$	x^2	4
$15x$	$16x^2$	$-24x$	9	$10x$	$28x$
$4x^2$	$-16x$	16	$15x$	25	$49x^2$
$16x^2$	49	$8x$	16	$24x^2$	9
25	$14x$	$8x$	$40x$	$30x$	$10x$
$7x$	x^2	$12x$	$25x^2$	40	$12x^2$

To factor perfect square trinomials:

1. Get the square root of the first and last terms.
2. List down the square root as sum/difference of two terms as the case may be.

You can use the following relationships to factor perfect square trinomials:

(First term)² + 2(First term)(Last term) + (Last term)² = (First term + Last term)²
(First term)² – 2(First term)(Last term) + (Last term)² = (First term – Last term)²

Remember to factor out first the greatest common monomial factor before factoring the perfect square trinomial.

Ex. 1. Factor $n^2 + 16n + 64$

Solution:

a. Since $n^2 = (n)^2$ and $64 = (8)^2$, then both the first and last terms are perfect squares. And $2(n)(8) = 16n$, then the given expression is a perfect square polynomial.

b. The square root of the first term is n and the square root of the last term is 8. The polynomial is factored as $(n + 8)^2$.

Ex. 2. Factor $\mathbf{4r^2 - 12r + 9}$

Solution:

a. Since $4r^2 = (2r)^2$ and $9 = (3)^2$, and since $(-12r) = (-2)(2r)(3)$ then it follows the given expression is a perfect square trinomial.

b. The square root of the first term is $2r$ and the square root of the last term is 3 so that its factored form is $(2r - 3)^2$.

Ex. 3. Factor $\mathbf{75t^3 + 30t^2 + 3t}$

Solution:

a. Notice that $3t$ is common to all terms, thus, factoring it out first we have:

$3t\,(25t^2 + 10t + 1)$

b. Notice that $25t^2 = (5t)^2$ and $1 = (1)^2$, and $10t = 2(5t)(1)$, then $25t^2 + 10t + 1$ is a perfect square trinomial.

c. Factoring $25t^2 + 10t + 1$ is $(5t + 1)^2$, thus, the factors of the given expression are $3t\,(5t + 1)^2$.

Explain why in **Example 3**, $(5t + 1)^2$ is not the only factor. What is the effect of removing $3t$?

Exercises

Supply the missing term to make a true statement.

a.	$m^2 + 12m + 36$	=	$(m + \_\_\_\_)^2$
b.	$16d^2 - 24d + 9$	=	$(4d - \_\_\_\_)^2$
c.	$a^4b^2 - 6abc + 9c^2$	=	$(a^2b\ \_\_\_\_\ \_\_\_\_)^2$
d.	$9n^2 + 30nd + 25d^2$	=	$(\_\_\_\_\ \_\_\_\ 5d)^2$
e.	$49g^2 - 84g + 36$	=	$(\_\_\_\_\ \_\_\_\ \_\_\_\_)^2$
f.	$121c^4 + 66c^2 + 9$	=	$(\_\_\_\_\ \_\_\_\ \_\_\_\_)^2$
g.	$25r^2 + 40rn + 16n^2$	=	$(\_\_\_\_\ \_\_\_\ \_\_\_\_)^2$
h.	$\frac{1}{16}x^2 + \frac{1}{3}x + \frac{4}{9}$	=	$(\_\_\_\ \_\_\_\ \_\_\_)^2$
i.	$18h^2 + 12h + 2$	=	$2\,(\_\_\_\_\ \_\_\_\ \_\_\_\_)^2$
j.	$20f^4 - 60f^3 + 45f^2$	=	$\_\_\_\_\,(2f\ \_\_\_\ \_\_\_\_)^2$

Is $q^2 + q - 12$ a perfect square trinomial? Why?

Are all trinomials perfect squares? How do we factor trinomials that are not perfect squares?

In the next activity, you will see how trinomials that are not perfect squares are factored.

Activity 11 TILE ONCE MORE!!

Description: You will arrange the tiles according to the instructions given to form a polygon and find its dimensions afterwards.

Directions:
1. Cutout 4 pieces of 3 in by 3 in card board and label each as x^2 representing its area.
2. Cutout 8 pieces of rectangular cardboard with dimensions of 3 in by 1 in and label each as x representing its area.
3. Cutout another square measuring 1 in by 1 in and label each as 1 to represent its area.

Form rectangles using the algebra tiles that you prepared. Use only tiles that are required in each item below.

a. 1 big square tile, 5 rectangular tiles, and 6 small square tiles
b. 1 big square tile, 6 rectangular tiles, and 8 small square tiles
c. 2 big square tiles, 7 rectangular tiles, and 5 small square tiles
d. 3 big square tiles, 7 rectangular tiles, and 4 small square tiles
e. 4 big square tiles, 7 rectangular tiles, and 3 small square tiles

QUESTIONS ?

1. What is the total area of each figure?
2. Using the sides of the tiles, write all the dimensions of the rectangles.
3. How did you get the dimensions of the rectangles?
4. Did you find difficulty in getting the dimensions?

Based on the previous activity, **how can the unknown quantities in geometric problems be solved?**

If you have noticed, there were two trinomials formed in the preceding activity. The term with the highest degree has a numerical coefficient greater than 1 or equal to 1 in these trinomials.

Let us study first how trinomials whose leading coefficient is 1 are being factored.

Ex. Factor $p^2 + 5p + 6$
Solution: a. List all the possible factors of 6.

Factors of 6	
2	3
6	1
-2	-3
-6	-1

b. Find factors of 6 whose sum is 5.

- **2 + 3 = 5**
- 6 + 1 = 7
- (-2) + (-3) = -5
- (-6) + (-1) = -7

c. Thus, the factor of $p^2 + 5p + 6 =$ **$(p + 2)(p + 3)$.**

Ex. Factor $\boldsymbol{v^2 + 4v - 21}$

Solution: a. List all the factors of **– 21**

Factors of - 21	
-3	7
-7	3
-21	1
-1	21

b. Find factors of -21 whose sum is 4.

- **-3 + 7 = 4**
- -7 + 3 = -4
- -21 + 1 = -20
- -1 + 20 = 19

c. Hence, the factors of $v^2 + 4v - 21 =$ **$(v - 3)(v + 7)$.**

Factor $\boldsymbol{2q^3 - 6q^2 - 36q}$. Since there is a common monomial factor, begin by factoring out $2q$ first. Rewriting it, you have $2q\,(q^2 - 3q - 18)$.

a. Listing all the factors of – 18.

Factors of -18	
-1	18
-2	9
-3	6
-18	1
-9	2
-6	3

b. Since – 6 and 3 are the factors of 18 whose sum is –3, then the binomial factors of $q^2 - 3q - 18$ are **$(q - 6)(q + 3)$**.

c. Therefore, the factors of $2q^3 - 6q - 36q$ are $2q(q - 6)(q + 3)$.

Remember:

To factor trinomials with 1 as the numerical coefficient of the leading term:

a. factor the leading term of the trinomial and write these factors as the leading terms of the factors;
b. list down all the factors of the last term;
c. identify which factor pair sums up to the middle term; then
d. write each factor in the pairs as the last term of the binomial factors.

NOTE: ***Always perform factoring using greatest common monomial factor first before applying any type of factoring.***

Activity 12 FACTOR BINGO GAME!

Description: Bingo game is an activity to practice your factoring skills with speed and accuracy.

Instruction: On a clean sheet of paper, draw a 3 by 3 square grid and mark the center as **FACTOR**. Pick **8** different factors from the table below and write them in the grid. As your teacher reads the trinomial, you will locate its factors and mark them x. The first one who makes the x pattern wins.

$(n + 4)(n - 5)$	$(n + 2)(n + 9)$	$(n - 8)(n - 9)$	$(n + 2)(n + 3)$
$(n + 9)(n + 8)$	$(n + 1)(n + 8)$	$(n - 8)(n + 4)$	$(n - 7)(n - 5)$
$(n + 6)(n + 4)$	$(n - 7)(n + 6)$	$(n - 12)(n + 4)$	$(n - 8)(n + 6)$
$(n + 3)(n + 6)$	$(n - 2)(n + 16)$	$(n + 3)(n + 8)$	

QUESTIONS ?

1. How did you factor the trinomials?
2. What did you do to factor the trinomials easily?
3. Did you find any difficulty in factoring the trinomials? Why?
4. What are your difficulties? How will you address those difficulties?

What if the numerical coefficient of the leading term of the trinomial is not 1, can you still factor it? Are trinomials of that form factorable? Why?

Trinomials of this form are written on the form $ax^2 + bx + c$, where a and b are the numerical coefficients of the variables and c is the constant term. There are many ways of factoring these types of polynomials, one of which is by inspection.

Trial and error being utilized in factoring this type of trinomials. Here is an example:

Factor $\mathbf{6z^2 - 5z - 6}$ through trial and error:
Give all the factors of $6z^2$ and -6

Factors of:	
$6z^2$	-6
$(3z)(2z)$	(3)(-2)
$(6z)(z)$	(-3)(2)
	(1)(-6)
	(-1)(6)

Write all possible factors using the values above and determine the middle term by multiplying the factors.

Possible Factors	**Sum of the product of the outer terms and the product of the inner terms**
$(3z - 2)(2z + 3)$	$9z - 4z = 5z$
$(3z + 3)(2z - 2)$	$-6z + 6z = 0$
$(3z - 3)(2z + 2)$	$6z - 6z = 0$
$\mathbf{(3z + 2)(2z - 3)}$	$\mathbf{-9z + 4z = -5z}$
$(3z + 1)(2z - 6)$	$-18z + 2z = -16z$
$(3z - 6)(2z + 1)$	$3z - 12z = -9z$
$(6z + 3)(z - 2)$	$-12z + 3z = -9z$
$(6z - 2)(z +3)$	$18z - 2z = 16z$
$(6z - 3)(z + 2)$	$12z - 3z = 9z$
$(6z + 2)(z - 3)$	$-18z + 2z = -16z$
$(6z + 1)(z - 6)$	$-36z + z = -35z$
$(6z - 6)(z + 1)$	$6z - 6z = 0$

In the given factors, $\mathbf{(3z + 2)(2z - 3)}$ gives the sum of $-5z$, thus, making it as the factors of the trinomial $6z^2 - 5z - 36$.

How was inspection used in factoring? What do you think is the disadvantage of using it?

Factoring through inspection is a tedious and long process; thus, knowing another way of factoring trinomials would be very beneficial in studying this module.

Another way of factoring is through grouping or AC method. Closely look at the given steps and compare them with trial and error.

Factor $6z^2 - 5z - 6$

1. Find the product of the leading term and the last term.

   $\underline{6z^2} - 5z - \underline{6}$

   $(6z^2)(-6) = -36z^2$

2. Find the factors of $-36z^2$ whose sum is $-5z$.
   $-9z + 4z = -5z$
3. Rewrite the trinomial as a four-term expression by replacing the middle term with the sum of the factors.
   $6z^2 - 9z + 4z - 6$
4. Group terms with common factors.
   $(6z^2 - 9z) + (4z - 6)$
5. Factor the groups using greatest common monomial factor.
   $3z(2z - 3) + 2(2z - 3)$
6. Factor out the common binomial factor and write the remaining factor as a sum or difference of the common monomial factors.
   $(2z - 3)(3z + 2)$

Factor $2k^2 - 11k + 12$

1. Multiply the first and last terms.
   $(2k^2)(12) = 24k^2$
2. Find the factors of $24k^2$ whose sum is $-11k$.
   $(-3k) + (-8k) = -11k$
3. Rewrite the trinomial as four–term expressions by replacing the middle term by the sum factor.
   $2k^2 - 3k - 8k + 12$
4. Group the terms with a common factor.
   $(2k^2 - 8k) + (-3k + 12)$
5. Factor the groups using greatest common monomial factor.
   $2k(k - 4) - 3(k - 4)$
6. Factor out the common binomial and write the remaining factor as sum or difference of binomial.
   $(k - 4)(2k - 3)$

Factor $6h^2 - h - 2$

1. Multiply the first and last terms.
   $(6h^2)(-2) = -12h^2$
2. Find the factors of $12h^2$ whose sum is h.
   $(-4h) + (3h) = -h$
3. Rewrite the trinomial as a four–term expression by replacing the middle term with the sum of the factors.
   $6h^2 - 4h - 3h - 2$
4. Group the terms with a common factor.
   $(6h^2 - 3h) + (-4h - 2)$
5. Factor the groups using greatest common monomial factor.
   $3h(2h - 1) - 2(2h - 1)$
6. Factor out the common binomial factor and write the remaining factor as a sum or difference of the common monomial factors.
   $(3h - 2)(2h - 1)$

Activity 13 WE HAVE!

Description: This game will help you practice your factoring skills through a game.

Instruction: Form a group with 5 members. Your task as a group is to factor the trinomial that the other group will give. Raise a flaglet and shout "We have it!" If you have already factored the trinomial. The first group to get 10 correct answers wins the game.

QUESTIONS ?

1. Do you find difficulty in playing the game? Why?
2. What hindered you from finding the factors of the trinomial?
3. What plan do you have to address these difficulties?

Let's extend!!

We can use factoring by grouping technique in finding the factors of a polynomial with more than three terms.

Let's try factoring **$8mt - 12at - 10mh - 15ah$**

Solution:
1. Group the terms with a common factor.
   $(8mt - 12at) + (-10mh - 15ah)$
2. Factor out the greatest common monomial factor in each group.
   $4t(2m - 3a) - 5h(2m - 3a)$ ➔ Why?
3. Factor out the common binomial factor and write the remaining factor as a sum or difference of the common monomial factors.
   $(2m - 3a)(4t - 5h)$

Factor $\mathbf{18lv + 6le + 24ov + 8oe}$

Solution:

1. Group the terms with a common factor.
   $\mathbf{(18lv + 6le) + (24ov + 8oe)}$ ➔ Why?

2. Factor out the greatest common monomial factor in each group.
   $\mathbf{6l(3v + e) + 8o(3v + 3)}$ ➔ Why?

3. Factor out the common binomial factor and write the remaining factor as a sum or difference of the common monomial factors.
   $\mathbf{(3v + e)(6l + 8o)}$

Activity 14 FAMOUS FOUR WORDS

Description: This activity will reveal the most frequently used four-letter word (no letter is repeated) according to world-English.org through the use of factoring.

Instruction: With your groupmates, factor the following expressions by grouping and writing a four-letter word using the variable of the factors to reveal the 10 most frequently used four-letter words.

1. $4wt + 2wh + 6it + 3ih$
2. $15te - 12he + 10ty - 8hy$
3. $hv + av + he + ae$
4. $10ti - 8ts - 15hi + 12hs$
5. $88fo + 16ro - 99fm - 18rm$
6. $7s + 35om + 9se + 45oe$
7. $42wa + 54wt + 56ha + 72ht$
8. $36yu - 24ro + 12ou - 72yr$
9. $72he + 16we + 27hn + 6wh$
10. $26wr - 91or + 35od - 10wd$

Activity 15 TEACH ME HOW TO FACTOR (GROUP DISCUSSION /PEER MENTORING)

Description: This activity is intended to clear your queries about factoring with the help of your groupmates.

Direction: Together with your groupmates, discuss your thoughts and queries regarding factoring. Figure out the solution to each others' questions. You may ask other groups or your teacher for help.

1. What different types of factoring have you encountered?
2. What are your difficulties in each factoring technique?
3. Why did you face such difficulties?
4. How are you going to address these difficulties?

Activity 16 WITH A BLINK!

Description: This is a flash card drill activity to help you practice your factoring technique with speed and accuracy.

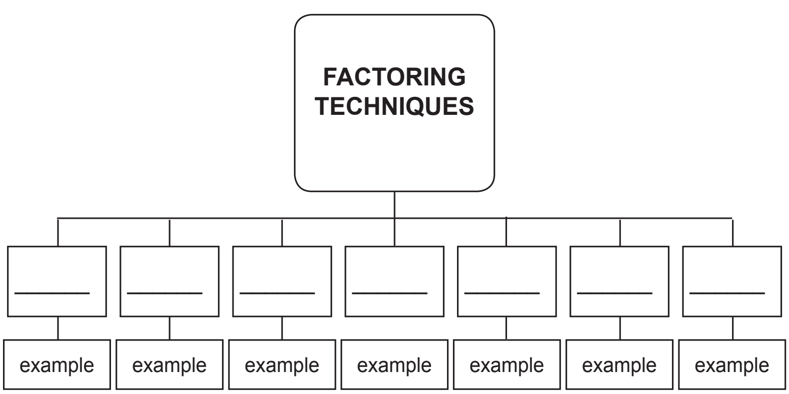
Instruction: As a group, you will factor the expressions that your teacher will show you. Each correct answer is given a point. The group with the most number of points wins the game.

1. What techniques did you use to answer the questions?
2. What things did you consider in factoring?
3. Did you find difficulty in the factoring the polynomials? Why?

Now that we have already discussed the different types of factoring, let us summarize our learning by completing the graphic organizer below.

Activity 17 GRAPHIC ORGANIZER

Description: To summarize the things you have learned, as a group, complete the chart below. You may add boxes if necessary.



HTTP://www.

Web – based learning (Video Watching)
Instruction: The following video clips contain the complete discussion of different types of factoring polynomials.

A. http://www.onlinemathlearning.com/algebra-factoring-2.html
B. http://www.youtube.com/watch?v=3RJlPvX-3vg
C. http://www.youtube.com/watch?v=8c7B-UaKl0U
D. http://www.youtube.com/watch?v=-hiGJwMNNsM

HTTP://www.

WEB – BASED LEARNING: LET'S PLAY!
Description: The links are interactive activities which will enhance your mastery on factoring polynomials. Perform all the exercises on the different types of factoring provided in these web sites.

Click the link below.
A. http://www.khanacademy.org/math/algebra/polynomials/e/factoring_polynomials_1
B. http://www.xpmath.com/forums/arcade.php?do=play&gameid=93
C. http://www.quia.com/rr/36611.html
D. http://www.coolmath.com/algebra/algebra-practice-polynomials.html (click only games for factoring)

Activity 18 IRF REVISIT

Revisit your IRF sheet and revise your answer by filling in column 2.

Initial	Revised	Final
	Express the following as products of factors. 1. $4x^2 - 12x =$ _______ 2. $9m^2 - 16n^2 =$ _______ 3. $4a^2 + 12a + 9 =$ _______ 4. $2x^2 + 9x - 5 =$ _______ 5. $27x^3 - 8y^3 =$ _______ 6. $a^3 + 125b^3 =$ _______ 7. $xm + hm - xn - hn =$ _______	

Now that you know the important ideas about this topic, let's go deeper by moving on to the next section.

REFLECTION

What I have learned so far...

What to Understand

Your goal in this section is to take a closer look at some aspects of the topic and to correct some misconceptions that might have developed.

The following activities will check your mastery in factoring polynomials.

Activity 19 SPOTTING ERRORS

Description: This activity will check how well you can associate the product and with its factors.

Instructions: Do as directed.

1. Your classmate asserted that $x^2 - 4x - 12$ and $12 - 4x - x^2$ has the same factors. Is your classmate correct? Prove by showing your solution.

2. Can the difference of two squares be applicable to $3x^3 - 12x$? If yes, how? If no, why?

3. Your classmate factored $x^2 + 36$ using the difference of two squares. How will you make him realize that his answer is not correct?

4. Make a generalization for the errors found in the following polynomials.

   a. $x^2 + 4 = (x + 2)(x + 2)$
   b. $1.6x^2 - 9 = (0.4x - 3)(0.4x + 3)$
   c. $4x^2y^5 - 12x^3y^6 + 2y^2 = 2y^2\ (2x^2y^3 - 6x^3y^4)$
   d. $3x^2 - 27$ is not factorable or prime

5. Are all polynomial expressions factorable? Cite examples to defend your answer.

Activity 20 IRF REVISIT

Revisit your IRF sheet and revise your answer by filling in column 3 under FINAL column.

Initial	Revised	Final
		Express the following as products of factors. 1. $4x^2 - 12x$ = ___ 2. $9m^2 - 16n^2$ = ___ 3. $4a^2 + 12a + 9$ = ___ 4. $2x^2 + 9x - 5$ = ___ 5. $27x^3 - 8y^3$ = ___ 6. $a^3 + 125b^3$ = ___ 7. $xm + hm - xn - hn$ = ___

QUESTIONS ?

1. What have you observed from your answers in the first column? Is there a big difference?
2. What realization have you made with regard to the relationship between special products and factors?

Activity 21 MATHEMAGIC! 2 = 1 POSSIBLE TO ME

Description: This activity will enable you to apply factoring to prove whether 2 = 1.

Instruction: Prove that 2 = 1 by applying your knowledge of factoring. You will need the guidance of your teacher in doing such.

If $a = b$, is 2 = 1?

a. Were you able to prove that 2 = 1?

b. What different factoring techniques did you use to arrive at the solution?

c. What error can you pinpoint to disprove that 2 = 1?

d. What was your realization in this activity?

Activity 22 JOURNAL WRITING

Description: This activity will enable you to reflect about the topic and activities you underwent.

Instruction: Reflect on the activities you have done in this lesson by completing the following statements. Write your answers on your journal notebook.

Reflect on your participation in doing all the activities in this lesson and complete the following statements:

- I learned that I...
- I was surprised that I...
- I noticed that I...
- I discovered that I...
- I was pleased that I...

Activity 23 LET'S SCALE TO DRAW!

Description: In this activity you will draw plane figures to help you do the final project after this module.

Directions: Using the skills you have acquired in the previous activity, follow your teacher's instruction.

1. Draw the following plane figures:
   a. a square with a side which measures 10 cm.
   b. a rectangle with a length 3 cm more than its width.
   c. any geometric figure whose dimensions are labelled algebraically.

2. A discussion on scale drawing will follow. After the discussion, the teacher will demonstrate the steps on how to do the following:
   a. A tree is five meters tall. Using a scale of 1m:2cm, draw the tree on paper.
   b. The school's flag pole is 10 m high. Using a scale of 2.5m:1dm, draw a smaller version of the flag pole. Give its height.

3. The teacher will demonstrate how a cube can be made using a square paper. Follow what your teacher did.

Activity 24 Model Making

Description: This activity involves the creation of a solid figure out of a given plane figure and expressing it in terms of factors of a polynomial.

Directions: Create a solid figure from the rectangular figure that was provided by following the steps given.

1. Cutout 2-in by 2-in squares in all edges of a 12 in by 6 in rectangle.
2. Fold all the sides upward.
3. Paste/tape the edges of the new figure.

QUESTIONS

a. What is the area of the original rectangle if its side is x units?
b. If the sides of the small squares is y, what expression represents its area?
c. How will you express the area of the new figure in terms of the variables stated in letters a and b?
d. What is the dimension of the new figure formed? How about the volume of the solid?
e. If the value of x = 4 cm and the value of y = 1 cm, what will be the dimension of the new figure? Its area? Its volume?
f. How did factoring help you find the dimensions of the new figure formed? The area? The volume?
g. What did you learn from this activity?

How can unknown quantities in geometric problems be solved?

What new realizations do you have about the topic? What new connections have you made for yourself?

Now that you have a deeper understanding of the topic, you are ready to do the tasks in the next section.

What to Transfer

Your goal in this section is to apply your learning to real-life situations. You will be given a practical task which will demonstrate your understanding in special products and factoring.

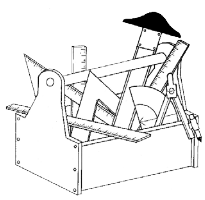
Activity 25 I BRING MY TRASH HOME

Directions: Perform the activity in preparation for your final output in this module.

In response to the school's environmental advocacy, you are required to make cylindrical containers for your trash. This is in support of the "I BRING MY TRASH HOME!" project of your school. You will present your output to your teacher and it will be graded according to the following criteria: explanation of the proposal, accuracy of computations, utilization of the resources, and appropriateness of the models.

Activity 26 PACKAGING ACTIVITY

Directions: This activity will showcase your learning in this module. You will assume the role of a member of a designing team that will present your proposal to a packaging company.



The RER packaging company is in search for the best packaging for a new dairy product that they will introduce to the market. You are a member of the design department of RER Packaging Company. Your company is tapped to create the best packaging box that will contain two identical cylindrical containers with the box's volume set at 100 in^3. The box has an open top. The cover will just be designed in reference to the box's dimensions. You are to present the design proposal for the box and cylinder to the Chief Executive Officer of the dairy company and head of the RER Packaging department. The design proposal is evaluated according to the following: explanation of the proposal, accuracy of computations, utilization of the resources, and appropriateness of the models.

The first commercial paperboard (not corrugated) box was produced in England in 1817

How did you find the performance task? How did the task help you see the real world application of the topic?

CRITERIA	Outstanding 4	Satisfactory 3	Developing 2	Beginning 1	RATING
Explanation of the Proposal (20%)	Explanations and presentation of the layout is detailed and clear.	Explanations and presentation of the layout is clear.	Explanations and presentation of the layout is a little difficult to understand but includes critical components.	Explanations and presentation of the layout is difficult to understand and is missing several components.	
Accuracy of Computations (30%)	The computations done are accurate and show understanding of the concepts of special products and factoring. There is an explanation for every computation made.	The computations done are accurate and show a wise use of the concepts of special products and factoring.	The computations done are erroneous and show some use of the concepts of special products and factoring.	The computations done are erroneous and do not show wise use of the concepts of special products and factoring.	
Utilization of Resources (20%)	Resources are efficiently utilized with less than 10% excess.	Resources are fully utilized with less than 10%-25% excess.	Resources are utilized but with a lot of excess.	Resources are not utilized properly.	
Appropriateness of the Model (30%)	The models are well-crafted and useful for understanding the design proposal. They showcase the desired product and are artistically done.	The models are well-crafted and useful for understanding the design proposal. They showcase the desired product.	The diagrams and models are less useful in understanding the design proposal	The diagrams and models are not useful in understanding the design proposal.	
				OVERALL RATING	

REFLECTION

In this lesson, I have understood that ____
____
____
____
____
____
____
____
____
____
____
____________________.

SUMMARY/SYNTHESIS/GENERALIZATION:

Now you have already completed this module, let's summarize what you have just learned. You have learned that product of some polynomials can be obtained using the different patterns, and these products are called **special products**. You also learned the different examples of special products, such as, perfect square trinomials, the difference of two squares, and the product when you raise a binomial to the third power.

This module also taught you to factor different products through the use of different patterns and rules. Factoring that you have learned are: (1) Factoring by greatest common monomial factor, (2) Factoring difference of two squares, (3) Factoring perfect square trinomials, (4) Factoring general trinomials, (5) Factoring the sum or difference of two cubes, and (6) Factoring by grouping.

You have learned that the special products and factoring can be applied to solve some real – life problems, as in the case of Punnet squares, packaging box-making, and even on tiles that can be found around us.

GLOSSARY OF TERMS:

AREA – the amount of surface contained in a figure expressed in square units

COMPOSITE FIGURE – a figure that is made from two or more geometric figures

FACTOR – an exact divisor of a number

GENETICS – the area of biological study concerned with heredity and with the variations between organisms that result from it

GEOMETRY – the branch of mathematics that deals with the nature of space and the size, shape, and other properties of figures as well as the transformations that preserve these properties

GREATEST COMMON MONOMIAL FACTOR – the greatest factor contained in every term of an algebraic expression

HETEROZYGOUS – refers to having two different alleles (group of genes) for a single trait

HOMOZYGOUS – refers to having identical alleles (group of genes) for a single trait

PATTERN – constitutes a set of numbers or objects in which all the members are related with each other by a specific rule

PERFECT SQUARE TRINOMIAL – result of squaring a binomial

PERIMETER – the distance around a polygon

POLYNOMIAL – a finite sum of terms each of which is a real number or the product of a numerical factor and one or more variable factors raised to a whole number power.

PRODUCT – the answer of multiplication

PUNNETT SQUARE - a diagram that is used to predict an outcome of a particular cross or breeding experiment used by biologists to determine the chance of an offspring's having a particular genotype.

SCALE DRAWING – a reduced or enlarged drawing whose shape is the same as the actual object that it represents

VOLUME – the measure of space occupied by a solid body

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2

RATIONAL ALGEBRAIC EXPRESSIONS AND ALGEBRAIC EXPRESSIONS WITH INTEGRAL EXPONENTS

I. INTRODUCTION AND FOCUS QUESTIONS

You have learned special products and factoring polynomials in Module 1. Your knowledge on these will help you better understand the lessons in this module.

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Have your ever asked yourself how many people are needed to complete a job? What are the bases for their wages? And how long can they finish the job? These questions may be answered using rational algebraic expressions which you will learn in this module.

After you finished the module, you should be able to answer the following questions:

a. What is a rational algebraic expression?
b. How will you simplify rational algebraic expressions?
c. How will you perform operations on rational algebraic expressions?
d. **How will you model rate–related problems?**

II. LESSONS AND COVERAGE

In this module, you will examine the above mentioned questions when you take the following lessons:

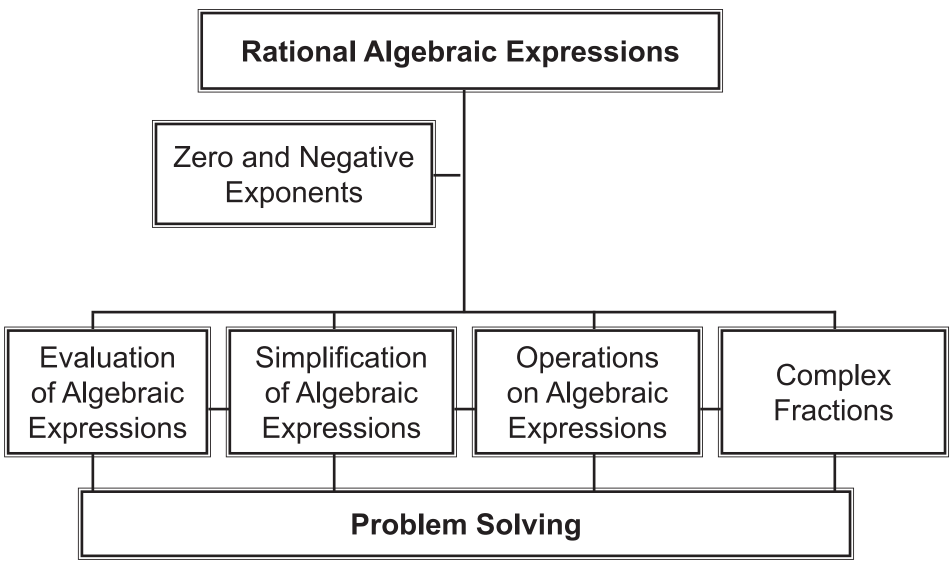
Lesson 1 – Rational Algebraic Expressions
Lesson 2 – Operations on Rational Algebraic Expressions

In these lessons, you will learn to:

Lesson 1	• describe and illustrate rational algebraic expressions; • interpret zero and negative exponents; • evaluate algebraic expressions involving integral exponents; and • simplify rational algebraic expressions.
Lesson 2	• multiply, divide, add, and subtract rational algebraic expressions; • simplify complex fractions; and • solve problems involving rational algebraic expressions.

Module Map

Here is a simple map of the lessons that will be covered in this module.



III. PRE-ASSESSMENT

Find out how much you already know about this module. Write the letter that you think is the best answer to each question on a sheet of paper. Answer all items. After taking and checking this short test, take note of the items that you were not able to answer correctly and look for the right answer as you go through in this module.

1. Which of the following expressions is a rational algebraic expression?

   a. $\frac{x}{\sqrt{3y}}$ c. $4y^{-2} + z^{-3}$

   b. $\frac{3c^{-3}}{\sqrt{(a+1)}}$ d. $\frac{a-b}{b+a}$

2. What is the value of a non–zero polynomial raised to 0?

   a. constant c. undefined

   b. zero d. cannot be determined

3. What will be the result when a and b are replaced by 2 and -1, respectively, in the expression $(-5a^{-2}b)(-2a^{-3}b^2)$?

   a. $\frac{27}{16}$ c. $\frac{3}{7}$

   b. $-\frac{5}{16}$ d. $-\frac{2}{7}$

4. What rational algebraic expression is the same as $\frac{x^{-2}-1}{x-1}$?

   a. $x + 1$ c. 1

   b. $x - 1$ d. -1

5. When $\frac{3}{x-5}$ is subtracted from a rational algebraic expression, the result is $\frac{-x-10}{x^2-5x}$. What is the other rational algebraic expression?

   a. $\frac{x}{4}$ c. $\frac{2}{x}$

   b. $\frac{x}{x-5}$ d. $\frac{-2}{x-5}$

6. Find the product of $\dfrac{a^2-9}{a^2+a-20}$ and $\dfrac{a^2-8a+16}{3a-9}$.

a. $\dfrac{a}{a-1}$

b. $\dfrac{a^2-1}{1-a}$

c. $\dfrac{a^2-a-12}{3a+15}$

d. $\dfrac{a^2-1}{a^2-a+1}$

7. What is the simplest form of $\dfrac{\frac{2}{b-3}}{\frac{2}{b-3}-1}$?

a. $\dfrac{2}{5-b}$

b. $\dfrac{b+5}{4}$

c. $\dfrac{1}{b-1}$

d. $\dfrac{1-b}{3}$

8. Perform the indicated operation: $\dfrac{x-2}{3}-\dfrac{x+2}{2}$.

a. $\dfrac{x+5}{6}$

b. $\dfrac{x+1}{6}$

c. $\dfrac{x-6}{6}$

d. $\dfrac{-x-10}{6}$

9. The volume of a certain gas will increase as the pressure applied to it decreases. This relationship can be modelled using the formula:

$$V_2 = \frac{V_1 P_1}{P_2}$$

where V_1 is the initial volume of the gas, P_1 is the initial pressure, P_2 is the final pressure, and the V_2 is the final volume of the gas. If the initial volume of the gas is 500 ml and the initial pressure is $\frac{1}{2}$ atm, what is the final volume of the gas if the final pressure is 5 atm?

a. 10ml b. 50ml c. 90ml d. 130ml

10. Angelo can complete his school project in x hours. What part of the job can be completed by Angelo after 3 hours?

a. $x+3$ b. $x-3$ c. $\dfrac{x}{3}$ d. $\dfrac{3}{x}$

11. If Maribel (Angelo's groupmate in number 10), can do the project in three hours, which expression below represents the rate of Angelo and Maribel working together?

12. Aaron was asked by his teacher to simplify $\frac{a^2 - 1}{a^2 - a}$ on the board. He wrote his solution on the board this way:

$$\frac{a^2 - 1}{a^2 - a} = \frac{(\cancel{a} + 1)\cancel{(a - 1)}}{\cancel{a}\cancel{(a - 1)}} = 1$$

Did he arrive at the correct answer?

a. Yes. The expressions that he crossed out are all common factors.

b. Yes. The LCD must be eliminated to simplify the expression.

c. No. a^2 must be cancelled out so that the answer is $\frac{1}{a}$.

d. No. a is not a common factor of the numerator.

13. Your friend multiplied $\frac{x - 1}{2 - x}$ and $\frac{1 + x}{1 - x}$. His solution is presented below:

$$\frac{x - 1}{2 - x} \bullet \frac{x + 1}{1 - x} = \frac{\cancel{(x - 1)}(x + 1)}{(2 - x)\cancel{(1 - x)}} = \frac{x + 1}{2 - x}$$

Is his solution correct?

a. No. There is no common factor to both numerator and denominator.
b. No. The multiplier must be reciprocated first before multiplying the expressions .
c. No. Common variables must be eliminated.
d. No. Dividing an expression by its multiplicative inverse is not equal to one.

14. Laiza added two rational algebraic expressions and her solution is presented below.

$$\frac{4x + 3}{2} + \frac{3x - 4}{3} = \frac{4x + 3 + 3x - 4}{2 + 3} = \frac{7x + 1}{5}$$

Is there something wrong in her solution?

a. Yes. Solve first the GCF before adding the rational algebraic expressions.
b. Yes. Cross multiply the numerator of the first expression to the denominator of the second expression.
c. Yes. She may express first the expressions as similar fractions.
d. Yes. $4x - 4$ is equal to x

15. Your father, a tricycle driver, asked you regarding the best motorcycle to buy. What will you do to help your father?

    a. Look for the fastest motorcycle.
    b. Canvass for the cheapest motorcycle.
    c. Find an imitated brand of motorcycle.
    d. Search for fuel – efficient type of motorcycle.

16. The manager of So – In Clothesline Corp. asked you, as the Human Resource Officer, to hire more tailors to meet the production target of the year. What will you consider in hiring a tailor?

    a. Speed and efficiency
    b. Speed and accuracy
    c. Time consciousness and personality
    d. Experience and personality

17. You own 3 hectares of land and you want to mow it for farming. What will you do to finish it at the very least time?

    a. Rent a small mower.
    b. Hire 3 efficient laborers.
    c. Do kaingin.
    d. Use germicide.

18. Your friend asked you to make a floor plan. As an engineer, what aspects should you consider in doing the plan?

    a. Precision
    b. Layout and cost
    c. Appropriateness
    d. Feasibility

19. Your SK Chairman planned to construct a basketball court. As a contractor, what will you do to realize the project?

    a. Show a budget proposal.
    b. Make a budget plan.
    c. Present a feasibility study.
    d. Give a financial statement.

20. As a contractor in number 19, what is the best action to do in order to complete the project on or before the deadline but still on the budget plan?

    a. All laborers must be trained workers.
    b. Rent more equipment and machines.
    c. Add more equipment and machines that are cheap.
    d. There must be equal number of trained and amateur workers.

IV. LEARNING GOALS AND TARGETS

As you finish this module, you will be able to demonstrate understanding of the key concepts of rational algebraic expressions and algebraic expressions with integral exponents. You must be able to present evidences of understanding and mastery of the competencies of this module. Activities must be accomplished before moving to the next topic and you must answer the questions and exercises correctly. Review the topic and ensure that answers are correct before moving to a new topic.

Your target in this module is to f**ormulate real-life problems involving rational algebraic expressions with integral exponents and solve these problems with utmost accuracy using variety of strategies**. You must present how you perform, apply, and transfer these concepts to real-life situations.

Lesson 1 Rational Algebraic Expressions

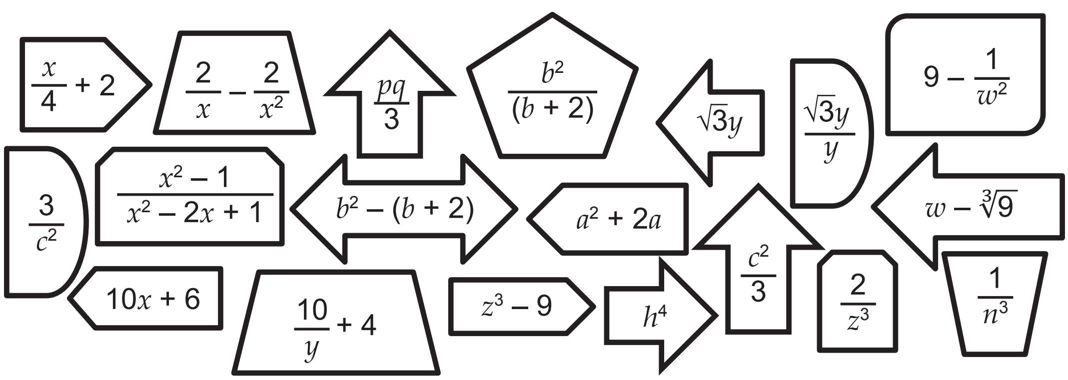
What to Know

Let's begin the lesson by reviewing some of the previous lessons and focusing your thoughts on the lesson.

Activity 1 MATCH IT TO ME

There are verbal phrases below. Look for the mathematical expression in the figures that corresponds to each verbal phrase.

1. The ratio of a number x and four added to two
2. The product of the square root of three and the number y
3. The square of a added to twice the a
4. The sum of b and two less than the square of b
5. The product of p and q divided by three
6. One–third of the square of c
7. Ten times a number y increased by six
8. The cube of the number z decreased by nine
9. The cube root of nine less than a number w
10. A number h raised to the fourth power



QUESTIONS ?

1. What did you feel while translating verbal phrases to mathematical expressions?
2. What must be considered in translating verbal phrases to mathematical phrases?
3. Will you consider these mathematical phrases as polynomial? Why or why not?
4. How will you describe a polynomial?

The previous activity deals with translating verbal phrases to polynomials. You also encountered some examples of non-polynomials. Such activity in translating verbal phrases to polynomials is one of the key concepts in answering word problems.

All polynomials are expressions but not all expressions are polynomials. In this lesson you will encounter some of these expressions that are not polynomials.

Activity 2 HOW FAST

Suppose you are to print a 40-page research paper. You observed that printer A in the internet shop finished printing it in two minutes.

a. How long do you think can printer A finish 100 pages?
b. How long will it take printer A to finish printing p pages?
c. If printer B can print x pages per minute, how long will printer B take to print p pages?

QUESTIONS ?

1. Can you answer the first question? If yes, how will you answer it? If no, what must you do to answer the question?
2. How will you describe the second and third questions?
3. How will you model the above problem?

Before moving to the lesson, you have to fill in the table on the next page regarding your ideas on rational algebraic expressions and algebraic expressions with integral exponents.

Activity 3 KWLH

Write your ideas on the rational algebraic expressions and algebraic expressions with integral exponents. Answer the unshaded portion of the table and submit it to your teacher.

What I Know	What I Want to Find Out	What I Learned	How I Can Learn More

You were engaged in some of the concepts in the lesson but there are questions in your mind. The next section will answer your queries and clarify your thoughts regarding the lesson.

What to Process

Your goal in this section is to learn and understand the key concepts on rational algebraic expressions and algebraic expressions with integral exponents.

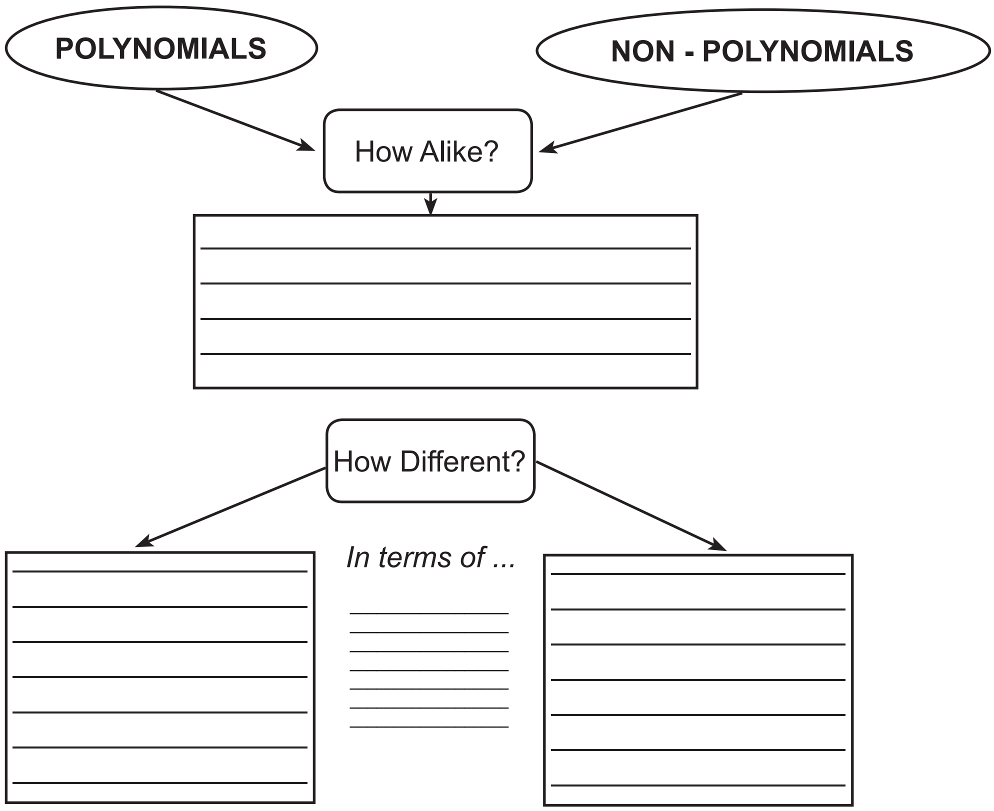
As the concepts on rational algebraic expressions and algebraic expressions with integral exponents become clear to you through the succeeding activities, do not forget to apply these concepts in real-life problems especially to rate-related problems.

Activity 4 MATCH IT TO ME – REVISITED (REFER TO ACTIVITY 1)

1. What are the polynomials in the activity "Match It to Me"? List these polynomials under set P.
2. Describe these polynomials.
3. In the activity, which are not polynomials? List these non-polynomials under set R.
4. How do these non-polynomials differ from the polynomials?
5. Describe these non-polynomials.

Activity 5 COMPARE AND CONTRAST

Use your answers in the activity “Match It to Me – Revisited” to complete the graphic organizer. Compare and contrast. Write the similarities and differences between polynomials and non-polynomials in the first activity.



*In the activity “Match It to Me”, the non–polynomials are called **rational algebraic expressions**. Your observations regarding the difference between polynomials and non – polynomials in activities 4 and 5 are the descriptions of rational expressions. Now, can you define rational algebraic expressions? Write your own definition about rational algebraic expressions in the chart on the next page.*

Activity 6 MY DEFINITION CHART

Write your initial definition of rational algebraic expressions in the appropriate box. Your final definition will be written after some activities.

My Initial Definition

My Final Definition

Try to firm up your own definition regarding the rational algebraic expressions by doing the next activity.

Activity 7 CLASSIFY ME

`

$\frac{m+2}{0}$ $\frac{k}{3k^2-6k}$ $\frac{1}{a^6}$ $\frac{1-m}{m^3}$ $\frac{y+2}{y-2}$ $\frac{a}{y^2-x^9}$ $\frac{c}{a-2}$ $\frac{c^4}{m-m}$

Rational Algebraic Expressions	Not Rational Algebraic Expressions

QUESTIONS ?

1. How many expressions did you place in the column of rational algebraic expressions?
2. How many expressions did you place under the column of not rational algebraic expression column?
3. How did you differentiate a rational algebraic expression from a not rational algebraic expression?
4. Were you able to place each expression in its appropriate column?
5. What difficulty did you encounter in classifying the expressions?

In the first few activities, you might have some confusions regarding rational algebraic expressions. However, this section firmed up your idea regarding rational algebraic expressions. Now, put into words your final definition of a rational algebraic expression.

Activity 8 MY DEFINITION CHART

Write your final definition of rational algebraic expressions in the appropriate box.

My Initial Definition

My Final Definition

Compare your initial definition with your final definition of rational algebraic expressions. Are you clarified with your conclusion by the final definition. How? Give at least three rational algebraic expressions different from those given by your classmate.

Remember:

A **rational algebraic expression** is a ratio of two polynomials provided that the denominator is not equal to zero. In symbols: $\frac{P}{Q}$, where P and Q are polynomials and $Q \neq 0$.

MATH DETECTIVE

Rational algebraic expression is a ratio of two polynomials where the denominator is not equal to zero. What will happen when the denominator of a fraction becomes zero?

Clue: Start investigating in

$\frac{4}{2} = 2 \gg\gg 4 = (2)(2)$ $\frac{4}{1} = 4 \gg\gg 4 = (1)(4)$

In the activities above, you had encountered rational algebraic expressions. You might encounter some algebraic expressions with negative or zero exponents. In the next activities, you will define the meaning of algebraic expressions with integral exponents including negative and zero exponents .

RECALL LAWS OF EXPONENTS

I – Product of Powers

If the expressions multiplied have the same base, add the exponents.

$x^a \cdot x^b = x^{a+b}$

II – Power of a Power

If the expression raised to a number is raised by another number, multiply the exponents.

$(x^a)^b = x^{ab}$

III – Power of a Product

If the multiplied expressions is raised by a number, multiply the exponents then multiply the expressions.

$(x^a y^b)^c = x^{ac} y^{bc}$ $(xy)^a = x^a y^a$

IV – Quotient of Power

If the ratio of two expressions is raised to a number, then

Case I. $\frac{x^a}{x^b} = x^{a-b}$, where $a > b$

Case II. $\frac{x^a}{x^b} = \frac{1}{x^{b-a}}$, where $a < b$

Activity 9

LET THE PATTERN ANSWER IT

Complete the table below and observe the pattern.

A	B	C	A	B	C	A	B	C	A	B
2•2•2•2•2	2^5	32	3•3•3•3•3	3^5	243	4•4•4•4•4	4^5	1,024	x•x•x•x•x	x^5
2•2•2•2			3•3•3•3			4•4•4•4			x•x•x•x	
2•2•2			3•3•3			4•4•4			x•x•x	
2•2			3•3			4•4			x•x	
2			3			4			x	

QUESTIONS

1. What do you observe as you answer column **B**?
2. What do you observe as you answer column **C**?
3. What happens to its value when the exponent decreases?
4. In the column **B**, how is the value in the each cell/box related to its upper or lower cell/box?

Use your observations in the activity above to complete the table below.

A	B	A	B	A	B	A	B
2^5	32	3^5	243	4^5	1,024	x^5	x•x•x•x•x
2^4		3^4		4^4		x^4	
2^3		3^3		4^3		x^3	
2^2		3^2		4^2		x^2	
2		3		4		x	
2^0		3^0		4^0		x^0	
2^{-1}		3^{-1}		4^{-1}		x^{-1}	
2^{-2}		3^{-2}		4^{-2}		x^{-2}	
2^{-3}		3^{-3}		4^{-3}		x^{-3}	

QUESTIONS

1. What did you observe as you answered column A? column B?
2. What happens to the value of the numerical expression when the exponent decreases?
3. In column A, how is the value in the each cell/box related to its upper or lower cell/box?
4. What do you observe when the number has zero exponent?
5. When a number is raised to a zero exponent, does it have the same value as another number raised to zero? Justify your answer.
6. What do you observe about the value of the number raised to a negative integral exponent?
7. What can you say about an expression with negative integral exponent?
8. Do you think it is true to all numbers? Cite some examples?

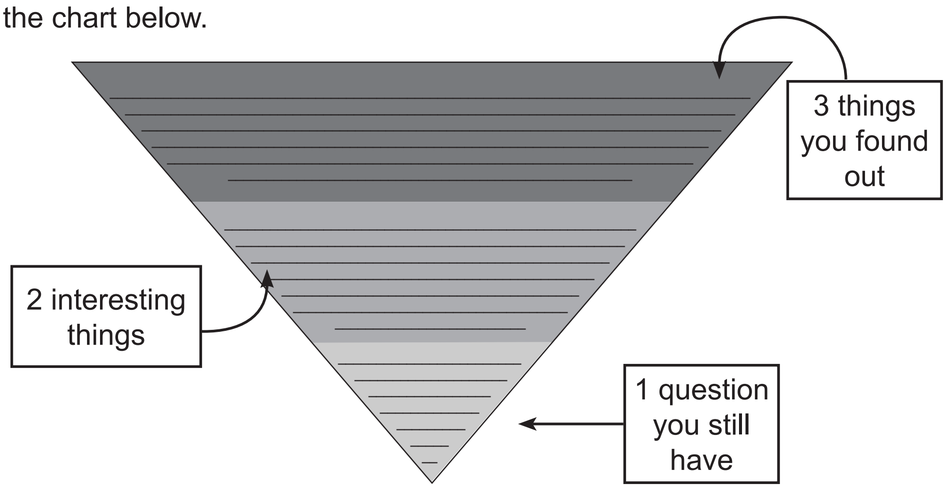
Exercises

Rewrite each item to expressions with positive exponents.

1. b^{-4}
2. $\dfrac{c^{-3}}{d^{-8}}$
3. $w^{-3}z^{-2}$
4. $n^{2}m^{-2}o$
5. $de^{-5}f$
6. $\dfrac{x + y}{(x - y)^{0}}$
7. $\left(\dfrac{a^{6}b^{8}c^{10}}{a^{5}b^{2}e^{8}}\right)^{0}$
8. $14t^{0}$
9. $\dfrac{l^{0}}{p^{0}}$
10. $\dfrac{2}{(a - b+c)^{0}}$

Activity 10 3 – 2 – 1 CHART

Complete the chart below.



Activity 11 WHO IS RIGHT?

Allan and Gina were asked to simplify $\frac{n^3}{n^{-4}}$. Their solutions are shown below together with their explanation.

Allan's Solution	Gina's Solution
$\frac{n^3}{n^{-4}} = n^{3-(-4)} = n^{3+4} = n^7$	$\frac{n^3}{n^{-4}} = \frac{n^3}{\frac{1}{n^{-4}}} = n^3\,\frac{n^4}{1} = n^7$
Quotient law was used in my solution.	I expressed the exponent of the denominator as positive integer, then followed the rules in dividing polynomials.

Who do you think is right? Write your explanation on a sheet of paper.

You have learned some concepts of rational algebraic expressions as you performed the previous activities. Now, let us try to use these concepts in a different context.

Activity 12 SPEEDY MARS

Mars finished the 15-meter dash within three seconds. Answer the questions below.

> **RECALL**
>
> **Speed** is the rate of moving object as it transfers from one point to another. The speed is the ratio between the distance and time travelled by the object.

1. How fast did Mars run?
2. At this rate, how far can Mars run after four seconds? five seconds? six seconds?
3. How many minutes can Mars run for 50 meters? 55 meters? 60 meters?

QUESTIONS ?

How did you come up with your answer? Justify your answer.

What you just did was evaluating the speed that Mars run. Substituting the value of the time to your speed, you come up with distance. When you substitute your distance to the formula of the speed, you get the time. This concept of evaluation is the same with evaluating algebraic expressions. Try to evaluate the following algebraic expressions in the next activity.

Activity 13 MY VALUE

Find the value of each expression below by evaluation.

My Expression	Value of a	Value of b	My solution	My Value
$a^2 + b^3$	2	3	Example: $a^2 + b^3 = 2^2 + 3^3$ $= 4 + 27$ $= 31$	31
	3	4		
	2	4		
$\frac{a^{-2}}{b^{-3}}$	-2	3	Example: $\frac{a^{-2}}{b^{-3}} = \frac{(-2)^{-2}}{3^{-3}}$ $= \frac{3^3}{(-2)^2}$ $= \frac{27}{4}$	$\frac{27}{4}$
$\frac{a^{-2}}{b^{-3}}$	3	2		
$a^{-1}b^0$	2	3		

QUESTIONS ?

1. What have you observed in the solution of the examples?
2. How did these examples help you find the value of the expression?
3. How did you find the value of the expression?

Exercises

Evaluate the following algebraic expressions.

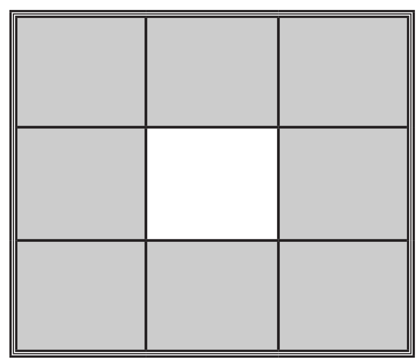
1. $40y^{-1}$, $y = 5$
2. $\dfrac{1}{m^{-2}(m + 4)}$, $m = -8$
3. $(p^2 - 3)^{-2}$, $p = 1$
4. $\dfrac{(x - 1)^{-2}}{(x + 1)^{-2}}$, $x = 2$
5. $y^{-3} - y^{-2}$, $y = 2$

Activity 14 BIN - GO

Make a 3 by 3 bingo card. Choose numbers to be placed in your bingo card from the numbers below. Your teacher will give an algebraic expression with integral exponents and the value of its variable. The first student who forms a frame wins the game.

1	$\frac{17}{4}$	2	$-\frac{31}{8}$	$\frac{1}{15}$
1	$\frac{2}{9}$	$\frac{3}{4}$	$\frac{37}{4}$	25
$\frac{1}{11}$	$\frac{1}{3}$	$\frac{3}{2}$	32	2
$\frac{1}{5}$	5	0	$\frac{23}{4}$	$\frac{4}{3}$
$\frac{1}{4}$	9	0	$\frac{126}{5}$	6

The frame card must be like this:



Activity 15 QUIZ CONSTRUCTOR

Be like a quiz constructor. Write on a one-half crosswise piece of paper three algebraic expressions with integral exponents in at least two variables and decide what values to be assigned to the variables. Show how to evaluate your algebraic expressions. Your algebraic expressions must be different from your classmates'.

Activity 16
CONNECT TO MY EQUIVALENT

Match column A to its equivalent simplest fraction in column B.

A	B
$\frac{5}{20}$	$\frac{1}{3}$
$\frac{8}{12}$	$\frac{1}{4}$
$\frac{4}{8}$	$\frac{3}{4}$
$\frac{5}{15}$	$\frac{1}{2}$
$\frac{6}{8}$	$\frac{2}{3}$

QUESTIONS ?

1. How did you find the equivalent fractions in column A?
2. Do you think you can apply the same concept in simplifying a rational algebraic expression?

You might wonder how to answer the last question but the key concept of simplifying rational algebraic expressions is the concept of reducing a fraction to its simplest form.

Examine and analyze the following examples.

Illustrative example: Simplify the following rational algebraic expressions.

1. $\dfrac{4a + 8b}{12}$

Solution

$$\frac{4a + 8b}{12} = \frac{4(a + 2b)}{4 \cdot 3}$$

$$= \frac{a + 2b}{3}$$

? What factoring method is used in this step?

2. $\dfrac{15c^3d^4e}{12c^2d^5w}$

Solution

$$\frac{15c^3d^4e}{12c^2d^5w} = \frac{\cancel{3}\bullet 5\cancel{c^2}c\cancel{d^4}e}{\cancel{3}\bullet 4\cancel{c^2}\cancel{d^4}dw}$$

$$= \frac{5ce}{4dw}$$

? What factoring method is used in this step?

3. $\dfrac{x^2 + 3x + 2}{x^2 - 1}$

Solution

$$\frac{x^2 + 3x + 2}{x^2 - 1} = \frac{\cancel{(x + 1)}(x + 2)}{\cancel{(x + 1)}(x - 1)}$$

$$= \frac{x + 2}{x - 1}$$

? What factoring method is used in this step?

QUESTIONS

Based on the above examples:
1. What is the first step in simplifying rational algebraic expressions?
2. What happens to the common factors in the numerator and the denominator?

Web Based Booster
click on this web site below to watch videos in simplifying rational algebraic expressions

http://mathvids.com/lesson/mathhelp/845-rational-expressions-2---simplifying

Exercises

Simplify the following rational algebraic expressions.

1. $\dfrac{y^2 + 5x + 4}{y^2 - 3x - 4}$
2. $\dfrac{-21a^2b^2}{28a^3b^3}$
3. $\dfrac{x^2 - 9}{x^2 - 7x + 12}$
4. $\dfrac{m^2 + 6m + 5}{m^2 - m - 2}$
5. $\dfrac{x^2 - 5x - 14}{x^2 + 4x + 4}$

Activity 17 MATCH IT DOWN

Match each rational algebraic expression to its equivalent simplified expression from choices A to E. Write the rational expression in the appropriate column. If the equivalent is not among the choices, write it in column F.

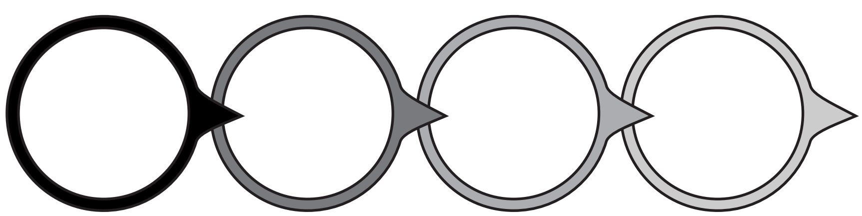
A. -1 B. 1 C. $a + 5$ D. $3a$ D. $\frac{a}{3}$

$\frac{a^2 + 6a + 5}{a + 1}$	$\frac{a^3 + 2a^2 + a}{3a^2 + 6a + 3}$	$\frac{3a^2 - 6a}{a - 2}$	$\frac{a - 1}{1 - a}$
$\frac{(3a + 2)(a + 1)}{3a^2 + 5a + 2}$	$\frac{3a^3 - 27a}{(a + 3)(a - 3)}$	$\frac{a^3 + 125}{a^2 - 25}$	$\frac{a - 8}{-a + 8}$
$\frac{18a^2 - 3a}{-1 + 6a}$	$\frac{3a - 1}{1 - 3a}$	$\frac{3a + 1}{1 + 3a}$	$\frac{a^2 + 10a + 25}{a + 5}$

A	B	C	D	E	F

Activity 18 CIRCLE PROCESS

In each circle write the steps in simplifying rational algebraic expressions. You can add or delete circles if necessary.



In this section, the discussions are introduction to rational algebraic expressions. How much of your initial ideas are found in the discussion? Which ideas are different and need revision? Try to move a little further in this topic through the next activities.

REFLECTION

What I have learned so far...

What to Understand

Your goal in this section is to relate the operations of rational expressions to real-life problems, especially rate problems.

Work problems are one of the rate-related problems and usually deal with persons or machines working at different rates or speed. The first step in solving these problems involves determining how much of the work an individual or machine can do in a given unit of time called the **rate**.

Illustrative example:

A. Nimfa can paint the wall in five hours. What part of the wall is painted in three hours?

Solution:

Since Nimfa can paint in five hours, then in one hour, she can paint $\frac{1}{5}$ of the wall. Her rate of work is $\frac{1}{5}$ of the wall each hour. The rate of work is the part of a task that is completed in 1 unit of time.

Therefore, in three hours, she will be able to paint $3 \bullet \frac{1}{5} = \frac{3}{5}$ of the wall.

You can also solve the problem by using a table. Examine the table below.

Rate of work (wall painted per hour)	Time worked	Work done (Wall painted)
$\frac{1}{5}$	1 hour	$\frac{1}{5}$
$\frac{1}{5}$	next 1 hour	$\frac{2}{5}$
$\frac{1}{5}$	another next 1 hour	$\frac{3}{5}$

You can also illustrate the problem.

1st hour	2nd hour	3rd hour	4th hour	5th hour
$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$

So after three hours, Nimfa only finished painting $\frac{3}{5}$ of the wall.

B. Pipe A can fill a tank in 40 minutes. Pipe B can fill the tank in x minutes. What part of the tank is filled if either of the pipes is opened in ten minutes?

Solution:

Pipe A fills $\frac{1}{40}$ of the tank in 1 minute. Therefore, the rate is $\frac{1}{40}$ of the tank per minute. So after 10 minutes,

$10 \bullet \frac{1}{40} = \frac{1}{4}$ of the tank is full.

Pipe B fills $\frac{1}{x}$ of the tank in x minutes. Therefore, the rate is $\frac{1}{x}$ of the tank per minute. So after x minutes,

$10 \bullet \frac{1}{x} = \frac{10}{x}$ of the tank is full.

In summary, the basic equation that is used to solve work problem is:

Rate of work • time worked = work done. [r • t = w]

Activity 19
HOWS FAST 2

Complete the table on the next page and answer questions that follow.

You printed your 40–page reaction paper. You observed that printer A in the internet shop finished printing in two minutes. How long will it take printer A to print 150 pages? How long will it take printer A to print p pages? If printer B can print x pages per minute, how long will it take to print p pages? The rate of each printer is constant.

Printer	Pages	Time	Rate
Printer A	40 pages	2 minutes	
	45 pages		
	150 pages		
	p pages		
Printer B	p pages		x ppm
	30 pages		
	35 pages		
	40 pages		

QUESTIONS ?

1. How did you solve the rate of each printer?
2. How did you compute the time of each printer?
3. What will happen if the rate of the printer increases?
4. How do time and number of pages affect the rate of the printer?

The concepts on rational algebraic expressions were used to answer the situation above. The situation above gives you a picture how these were used in solving rate-related problems.

What new realizations do you have about the topic? What new connections have you made for yourself? What questions do you still have? Fill-in the **Learned, Affirmed,** and **Challenged cards** given below.

Learned

What new realizations and learnings do you have about the topic?

Affirmed

What new connections have you made? Which of your old ideas have been confirmed or affirmed?

Challenged

What questions do you still have? Which areas seem difficult for you? Which do you want to explore?

What to Transfer

Your goal in this section is to apply your learning in real-life situations. You will be given a practical task which will demonstrate your understanding.

Activity 20
HOURS AND PRINTS

The JOB Printing Press has two photocopying machines. P1 can print a box of bookpaper in three hours while P2 can print a box of bookpaper in $3x + 20$ hours.

a. How many boxes of bookpaper are printed by P1 in 10 hours? In 25 hours? in 65 hours?

b. How many boxes of bookpaper can P2 print in 10 hours? in $120x + 160$ hours? in $30x^2 + 40x$ hours?

You will show your output to your teacher. Your work will be graded according to mathematical reasoning and accuracy.

Rubrics for your output

CRITERIA	Outstanding 4	Satisfactory 3	Developing 2	Beginning 1	RATING
Mathematical reasoning	Explanation shows thorough reasoning and insightful justifications.	Explanation shows substantial reasoning.	Explanation shows gaps in reasoning.	Explanation shows illogical reasoning.	
Accuracy	All computations are correct and shown in detail.	All computations are correct.	Most of the computations are correct.	Some the computations are correct.	
				OVERALL RATING	

REFLECTION

In this lesson, I have understood that ____
____
____
____
____
____
____
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____
______________.

Lesson 2 Operations on Rational Algebraic Expressions

What to Know

In the first lesson, you learned that a rational algebraic expression is a ratio of two polynomials where the denominator is not equal to zero. In this lesson, you will be able to perform operations on rational algebraic expressions. Before moving to the new lesson, let's review the concepts that you have learned that are essential to this lesson.

In the previous mathematics lesson, your teacher taught you how to add and subtract fractions. What mathematical concept plays a vital role in adding and subtracting fractions? You may think of LCD or least common denominator. Now, let us take another perspective in adding or subtracting fractions. Ancient Egyptians had special rules on fractions. If they have five loaves for eight persons, they would not divide them immediately by eight instead, they would use the concept of unit fraction. A unit fraction is a fraction with one as numerator. Egyptian fractions used unit fractions without repetition except $\frac{2}{3}$. To be able to divide five loaves among eight persons, they had to cut the four loaves into two and the last one would be cut into eight parts. In short:

$$\frac{5}{8} = \frac{1}{2} + \frac{1}{8}$$

Activity 1 EGYPTIAN FRACTION

Now, be like an Ancient Egyptian. Give the unit fractions in Ancient Egyptian way.

1. $\frac{7}{10}$ using two unit fractions.
2. $\frac{8}{15}$ using two unit fractions.
3. $\frac{3}{4}$ using two unit fractions.
4. $\frac{11}{30}$ using two unit fractions.
5. $\frac{7}{12}$ using two unit fractions.
6. $\frac{13}{12}$ using three unit fractions.
7. $\frac{11}{12}$ using three unit fractions.
8. $\frac{31}{30}$ using three unit fractions.
9. $\frac{19}{20}$ using three unit fractions.
10. $\frac{25}{28}$ using three unit fractions.

QUESTIONS ?

1. What did you do in getting the unit fraction?
2. How did you feel while getting the unit fractions?
3. What difficulties did you encounter in giving the unit fraction?
4. What would you do to overcome these difficulties?

Activity 2 ANTICIPATION GUIDE

There are sets of rational algebraic expressions in the table below. Check the column Agree if the entry in column I is equivalent to the entry in column II and check the column Disagree if the entries in the two columns are not equivalent.

I	II	Agree	Disagree
$\frac{x^2 - xy}{x^2 - y^2} \cdot \frac{x + y}{x^2 - xy}$	$x^{-1} - y^{-1}$		
$\frac{6y - 30}{y^2 + 2y + 1} \div \frac{3y - 15}{y^2 + y}$	$\frac{2y}{y + 1}$		
$\frac{5}{4x^2} + \frac{7}{6x}$	$\frac{15 + 14x}{12x^2}$		
$\frac{a}{b - a} - \frac{b}{a - b}$	$\frac{a + b}{b - a}$		
$\dfrac{\frac{a + b}{b} - \frac{b}{a + b}}{\frac{1}{b} + \frac{2}{a}}$	$\frac{a^2}{a + b}$		

Activity 3 PICTURE ANALYSIS

Take a closer look at this picture. Describe what you see.

http://www.portlandground.com/archives/2004/05/volunteers_buil_1.php

QUESTIONS

1. What would happen if one of the men in the picture would not do his job?
2. What will happen when there are more people working together?
3. How does the rate of each worker affect the entire work?
4. How will you model the rate-related problem?

The picture shows how the operations on rational algebraic expressions can be applied to real-life scenario. You'll get to learn more rate-related problems and how operations on rational algebraic expressions relate to them.

What to Process

Your goal in this section is to learn and understand key concepts on the operations on rational algebraic expressions.

As these become clear to you through the succeeding activities, do not forget to think about how to apply these concepts in solving real-life problems especially rate-related problems.

REVIEW

Perform the operation on the following fractions.

1. $\frac{1}{2} \cdot \frac{4}{3}$
2. $\frac{3}{4} \cdot \frac{2}{3}$
3. $\frac{8}{11} \cdot \frac{33}{40}$
4. $\frac{1}{4} \cdot \frac{3}{2}$
5. $\frac{1}{6} \cdot \frac{2}{9}$

Activity 4 MULTIPLYING RATIONAL ALGEBRAIC EXPRESSIONS

Examine and analyze the illustrative examples below. Pause once in a while to answer the checkup questions.

The product of two rational expressions is the product of the numerators divided by the product of the denominators. In symbols,

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd},\ bd \neq 0$$

Illustrative example 1: Find the product of $\frac{5t}{8}$ and $\frac{4}{3t^2}$.

$$\frac{5t}{8} \cdot \frac{4}{3t^2} = \frac{5t}{2^3} \cdot \frac{2^2}{3t^2}$$

$$= \frac{(5)\cancel{(t)}\cancel{(2^2)}}{\cancel{(2^2)}(2)(3t)\cancel{t}}$$

Express the numerators and denominators into prime factors if possible.

$= \frac{5}{(2)(3t)}$

$= \frac{5}{6t}$

Simplify rational expressions using laws of exponents.

Illustrative example 2: Multiply $\frac{4x}{3y}$ and $\frac{3x^2y^2}{10}$.

$$\frac{4x}{3y} \cdot \frac{3x^2y^2}{10} = \frac{(2^2)x}{3y} \cdot \frac{3x^2y^2}{(2)(5)}$$

$$= \frac{\cancel{(2)}(2)(x)\cancel{(3)}(x^2)(y)\cancel{(y)}}{\cancel{(3)}\cancel{(y)}\cancel{(2)}(5)}$$

$$= \frac{(2)(x^3)(y)}{(5)}$$

$$= \frac{2x^3y}{5}$$

What laws of exponents were used in these steps?

Illustrative example 3: What is the product of $\frac{x-5}{4x^2-9}$ and $\frac{4x^2+12x+9}{2x^2-11x+5}$?

$$\frac{x-5}{4x^2-9} \cdot \frac{4x^2+12x+9}{2x^2-11x+5} = \frac{x-5}{(2x-3)(2x+3)} \cdot \frac{(2x+3)^2}{(2x-1)(x-5)}$$

$$= \frac{\cancel{(x-5)}\cancel{(2x+3)}(2x+3)}{(2x-3)\cancel{(2x+3)}(2x-1)\cancel{(x-5)}}$$

$$= \frac{2x+3}{(2x-3)(2x-1)}$$

$$= \frac{2x+3}{4x^2-8x+4}$$

What factoring methods were used in this step?

What are the rational algebraic expressions equivalent to 1 in this step?

QUESTIONS

1. What are the steps in multiplying rational algebraic expressions?
2. What do you observe from each step in multiplying rational algebraic expressions?

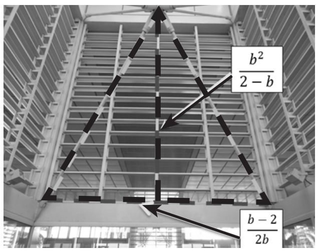
Exercises

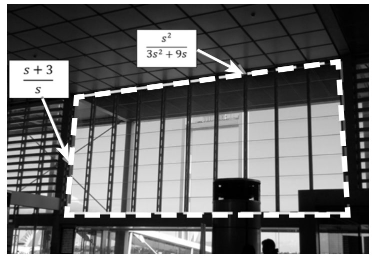
Find the product of the following rational algebraic expressions.

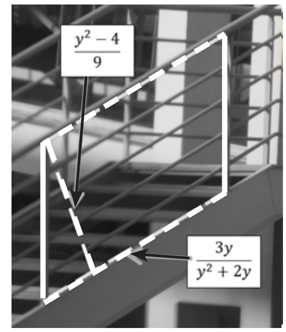
1. $\frac{10uv^2}{3xy^2} \cdot \frac{6x^2y^2}{5u^2v^2}$
2. $\frac{a^2-b^2}{2ab} \cdot \frac{a^2}{a-b}$
3. $\frac{x^2-3x}{x^2+3x-10} \cdot \frac{x^2-4}{x^2-x-6}$
4. $\frac{x^2+2x+1}{y^2-2y+1} \cdot \frac{y^2-1}{x^2-1}$
5. $\frac{a^2-2ab+b^2}{a^2-1} \cdot \frac{a-1}{a-b}$

Activity 5 WHAT'S MY AREA?

Find the area of the plane figures below.

a. 

b. 

c. 

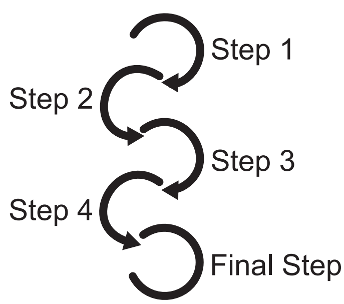
QUESTIONS ?

1. How did you find the area of the figures?
2. What are your steps in finding the area of the figures?

Activity 6 THE CIRCLE ARROW PROCESS

Based on the steps that you made in the previous activity, make a conceptual map on the steps in multiplying rational algebraic expressions. Write the procedure and other important concepts in every step inside the circle. If necessary, add a new circle.

Web – based Booster:
Watch the videos in this websites for more examples. http://www.onlinemathlearning.com/multiplying-rational-expressions-help.html



QUESTIONS ?

1. Does every step have a mathematical concept involved?
2. What makes that mathematical concept important to every step?
3. Can the mathematical concepts used in every step be interchanged? How?
4. Can you give another method in multiplying rational algebraic expressions?

REVIEW

Perform the operation of the following fractions.

1. $\frac{1}{2} \div \frac{3}{4}$
2. $\frac{5}{2} \div \frac{9}{4}$
3. $\frac{9}{2} \div \frac{3}{4}$
4. $\frac{10}{16} \div \frac{5}{4}$
5. $\frac{1}{2} \div \frac{1}{4}$

Activity 7

DIVIDING RATIONAL ALGEBRAIC EXPRESSIONS

Examine and analyze the illustrative examples below. Pause once in a while to answer the checkup questions.

> *The quotient of two rational algebraic expressions is the product of the dividend and the reciprocal of the divisor. In symbols,*
>
> $$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \bullet \frac{d}{c} = \frac{ad}{bc},\ b, c, d \neq 0$$

Illustrative example 4: Find the quotient of $\frac{6ab^2}{4cd}$ and $\frac{9a^2b^2}{8dc^2}$.

$$\frac{6ab^2}{4cd} \div \frac{9a^2b^2}{8dc^2} = \frac{6ab^2}{4cd} \div \frac{8dc^2}{9a^2b^2}$$

Multiply the dividend by the reciprocal of the divisor.

$$= \frac{(2)(3)ab^2}{(2)^2cd} \div \frac{(2^3)dc^2}{(3^2)a^2b^2}$$

$$= \frac{\cancel{(2^2)}(2^2)\cancel{(3)}\cancel{a}\cancel{b^2}\cancel{d}\cancel{c}c}{\cancel{(2^2)}(3)\cancel{(3)}\cancel{c}\cancel{d}a\cancel{a}\cancel{b^2}}$$

$$= \frac{(2)^2c}{(3)a}$$

$$= \frac{4c}{3a}$$

Perform the steps in multiplying rational algebraic expressions.

Illustrative example 5: Divide $\frac{2x^2 + x - 6}{2x^2 + 7x + 5}$ by $\frac{x^2 - 2x - 8}{2x^2 - 3x - 20}$.

$$\frac{2x^2 + x - 6}{2x^2 + 7x + 5} \div \frac{x^2 - 2x - 8}{2x^2 - 3x - 20}$$

$$= \frac{2x^2 + x - 6}{2x^2 + 7x + 5} \bullet \frac{2x^2 - 3x - 20}{x^2 - 2x - 8}$$

$$= \frac{(2x - 3)(x + 2)}{(2x + 5)(x + 1)} \bullet \frac{(x - 4)(2x + 5)}{(x + 2)(x - 4)}$$

Why do we need to factor out the numerators and denominators?

$$= \frac{(2x - 3)\cancel{(x + 2)}\cancel{(x - 4)}\cancel{(2x + 5)}}{\cancel{(2x + 5)}(x + 1)\cancel{(x + 2)}\cancel{(x - 4)}}$$

$$= \frac{(2x - 3)}{(x + 1)}$$

What happens to the common factors between numerator and denominator?

$$= \frac{2x - 3}{x + 1}$$

Exercises

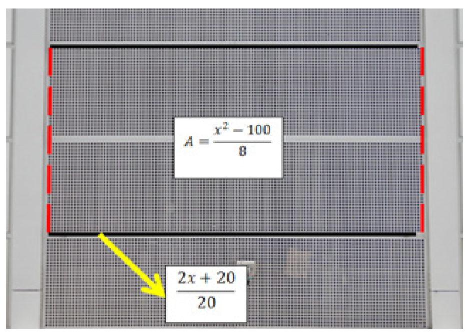
Find the quotient of the following rational algebraic expressions.

1. $\dfrac{81xz^3}{36y} \div \dfrac{27x^2z^2}{12xy}$
2. $\dfrac{2a + 2b}{a^2 + ab} \div \dfrac{4}{a}$
3. $\dfrac{16x^2 - 9}{6 - 5x - 4x^2} \div \dfrac{16x^2 + 24x + 9}{4x^2 + 11x + 6}$
4. $\dfrac{x^2 + 2x + 1}{x^2 + 4x + 3} \div \dfrac{x^2 - 1}{x^2 + 2x + 1}$
5. $\dfrac{x - 1}{x + 1} \div \dfrac{1 - x}{x^2 + 2x + 1}$

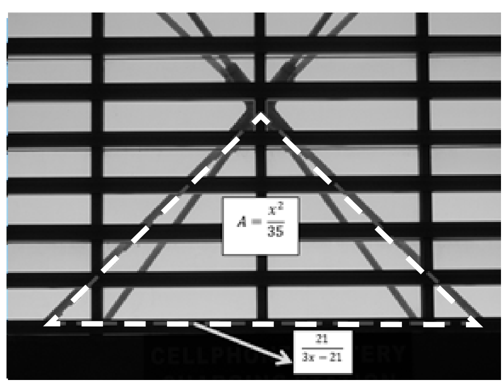
Activity 8 MISSING DIMENSION

Find the missing length of the figures.

1. The area of the rectangle is $\dfrac{x^2 - 100}{8}$ while the length is $\dfrac{2x + 20}{20}$. Find the height of the rectangle.



2. The base of the triangle is $\dfrac{21}{3x - 21}$ and the area is $\dfrac{x^2}{35}$. Find the height of the triangle.



QUESTIONS ?

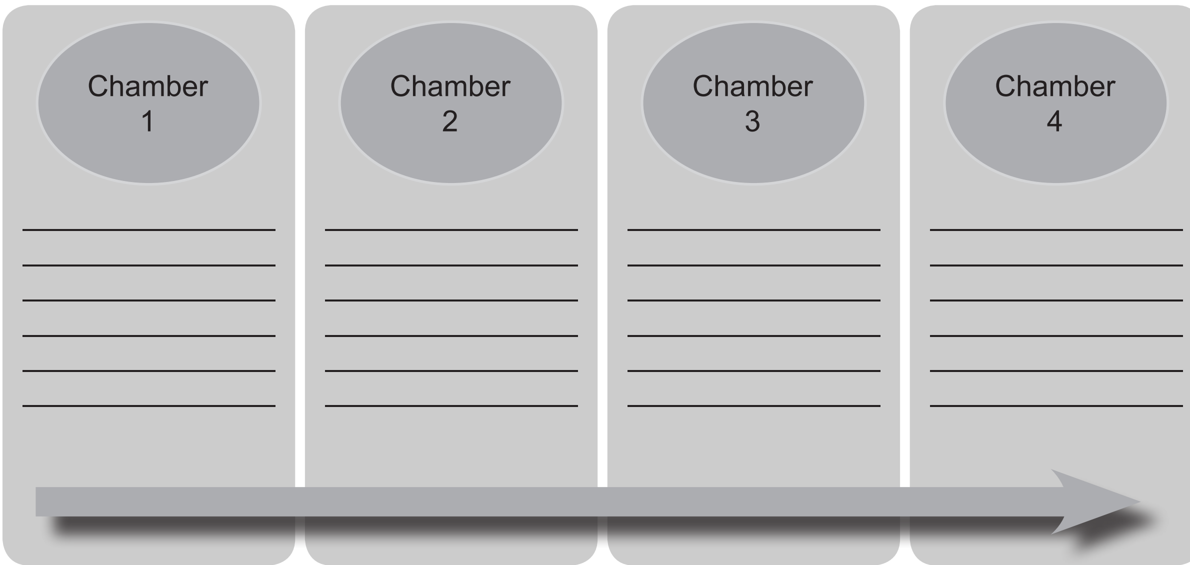
1. How did you find the missing dimension of the figures?
2. Enumerate the steps in solving the problems.

Activity 9 CHAIN REACTION

Web Based Booster
Click on this website below to watch videos in dividing rational algebraic expressions

http://www.onlinemathlearning.com/dividing-rational-expressions-help.html

Use the Chain Reaction Chart to sequence your steps in dividing rational algebraic expressions. Write the process or mathematical concepts used in each step in the chamber. Add another chamber, if necessary.



QUESTIONS

1. Does every step have a mathematical concept involved?
2. What makes that mathematical concept important to every step?
3. Can mathematical concept in every step be interchanged? How?
4. Can you make another method in dividing rational algebraic expressions? How?

Activity 10 ADDING AND SUBTRACTING SIMILAR RATIONAL ALGEBRAIC EXPRESSIONS

Examine and analyze the following illustrative examples on the next page. Answer the check-up questions.

In adding or subtracting similar rational expressions, add or subtract the numerators and write the answer in the numerator of the result over the common denominator. In symbols,

$$\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b},\ b \neq 0$$

REVIEW

Perform the operation on the following fractions.

1. $\frac{1}{2} + \frac{3}{2}$
2. $\frac{5}{4} + \frac{9}{4}$
3. $\frac{9}{5} + \frac{3}{5}$
4. $\frac{10}{13} - \frac{5}{13}$
5. $\frac{5}{4} - \frac{1}{4}$

Illustrative example 6: Add $\dfrac{x^2 - 2x - 7}{x^2 - 9}$ and $\dfrac{3x + 1}{x^2 - 9}$.

$$\frac{x^2 - 2x - 7}{x^2 - 9} + \frac{3x + 1}{x^2 - 9} = \frac{x^2 - 2x + 3x - 7 + 1}{x^2 - 9}$$

$$= \frac{x^2 + x - 6}{x^2 - 9}$$

Combine like terms in the numerator.

$$= \frac{\cancel{(x + 3)}(x - 2)}{(x - 3)\cancel{(x + 3)}}$$

Factor out the numerator and denominator.

$$= \frac{(x - 2)}{(x + 3)}$$

$$= \frac{x - 2}{x + 3}$$

Do we always factor out the numerator and denominator? Explain your answer.

Illustrative example 7: Subtract $\dfrac{-10 - 6x - 5x^2}{3x^2 + x - 2}$ from $\dfrac{x^2 + 5x - 20}{3x^2 + x - 2}$.

$$\frac{x^2 + 5x^2 - 20}{3x^2 + x - 2} - \frac{-10 - 6x - 5x^2}{3x^2 + x - 3} = \frac{x^2 + 5x^2 - 20 - (-10 - 6x - 5x^2)}{3x^2 + x - 2}$$

$$= \frac{x^2 + 5x - 20 + 10 + 6x + 5x^2}{3x^2 + x - 2}$$

Why do we need to multiply the subtrahend by – 1 in the numerator?

$$= \frac{x^2 + 5x^2 + 5x + 6x - 20 + 10}{3x^2 + x - 2}$$

$$= \frac{6x^2 + 11x - 10}{3x^2 + x - 2}$$

$$= \frac{\cancel{(3x - 2)}(2x + 5)}{\cancel{(3x - 2)}(x + 1)}$$

Factor out the numerator and denominator.

$$= \frac{2x + 5}{x + 1}$$

Exercises

Perform the indicated operation. Express your answer in simplest form.

1. $\dfrac{6}{a - 5} + \dfrac{4}{a - 5}$
2. $\dfrac{x^2 + 3x - 2}{x^2 - 4} + \dfrac{x^2 - 2x + 4}{x^2 - 4}$
3. $\dfrac{7}{4x - 1} - \dfrac{5}{4x - 1}$
4. $\dfrac{x^2 + 3x + 2}{x^2 - 2x + 1} - \dfrac{3x + 3}{x^2 - 2x + 1}$
5. $\dfrac{x - 2}{x - 1} + \dfrac{x - 2}{x - 1}$

Activity 11 ADDING AND SUBTRACTING DISSIMILAR RATIONAL ALGEBRAIC EXPRESSIONS

Examine and analyze the following illustrative examples below. Answer the checkup questions.

> *In adding or subtracting dissimilar rational expressions, change the rational algebraic expressions into similar rational algebraic expressions using the least common denominator or LCD and proceed as in adding similar fractions.*

REVIEW

Perform the operation on the following fractions.

1. $\frac{1}{2}+\frac{4}{3}$
2. $\frac{3}{4}+\frac{2}{3}$
3. $\frac{3}{4}+\frac{1}{8}$
4. $\frac{1}{4}-\frac{3}{2}$
5. $\frac{1}{6}-\frac{2}{9}$

Illustrative example 8: Find the sum of $\frac{5}{18a^4b}$ and $\frac{2}{27a^3b^2c}$.

$$\frac{5}{18a^4b}+\frac{2}{27a^3b^2c}=\frac{5}{(3^2)(2)a^4b}+\frac{2}{(3^3)a^3b^2c}$$

Express the denominators as prime factors.

LCD of $\frac{5}{(3^2)(2)a^4b}$ and $\frac{2}{(3^3)a^3b^2c}$

$(3^2)(2)a^4b$ and $(3^3)a^3b^2c$

Denominators of the rational algebraic expressions

The LCD is $(3^3)(2)(a^4)(b^2)(c)$

Take the factors of the denominators. When the same factor is present in more than one denominator, take the factor with the highest exponent. The product of these factors is the **LCD**.

$$=\frac{5}{(3^2)(2)a^4b}\cdot\frac{3bc}{3bc}+\frac{2}{(3^3)a^3b^2c}\cdot\frac{2a}{2a}$$

$$=\frac{(5)(3)bc}{(3^3)(2)a^4b^2c}+\frac{(2^2)a}{(3^3)(2)a^4b^2c}$$

$$=\frac{15bc}{54a^4b^2c}+\frac{4a}{54a^4b^2c}$$

$$=\frac{15bc+4a}{54a^4b^2c}$$

Find a number equivalent to 1 that should be multiplied to the rational algebraic expressions so that the denominators are the same with the **LCD**.

Illustrative example 9: Subtract $\dfrac{t+3}{t^2-6t+9}$ and $\dfrac{8t-24}{t^2-9}$.

$$\frac{t+3}{t^2-6t+9}-\frac{8t-24}{t^2-9}=\frac{t+3}{(t-3)^2}-\frac{8t-24}{(t-3)(t+3)}$$

Express the denominators as prime factors.

LCD of $\dfrac{t+3}{(t-3t)^2}$ and $\dfrac{8t-24}{(t-3)(t+3)}$

$(t-3)^2$ and $(t-3)(t+3)$

The LCD is $(t-3)^2(t+3)$

$$=\frac{t+3}{(t-3)^2}\bullet\frac{t+3}{t+3}-\frac{(8t-24)}{(t-3)^2(t+3)}\bullet\frac{t-3}{t-3}$$

What property of equality is illustrated in this step?

$$=\frac{(t+3)(t+3)}{(t-3)^2(t+3)}-\frac{(8t-24)}{(t-3)(t+3)}$$

$$=\frac{t^2+6t+9}{t^3-9t^2+27t-27}-\frac{8t-48t+72}{t^3-9t^2+27t-27}$$

What special products are illustrated in this step?

$$=\frac{t^2+6t+9-(8t^2-48t+72)}{t^3-9t^2+27t-27}$$

$$=\frac{t^2+6t+9-8t^2+48t-72}{t^3-9t^2+27t-27}$$

$$=\frac{-7t^2+54t-63}{t^3-9t^2+27t-27}$$

Illustrative example 10: Find the sum of $\dfrac{2x}{x^2+4x+3}$ and $\dfrac{3x-6}{x^2+5x+6}$.

$$\frac{2x}{x^2+4x+3}+\frac{3x-6}{x^2+5x+6}=\frac{2x}{(x+3)(x+1)}+\frac{3x-6}{(x+3)(x+2)}$$

What special products are illustrated in this step?

LCD of $\dfrac{2x}{(x+3)(x+1)}$ and $\dfrac{3x-6}{(x+3)(x+2)}$

$(x+3)(x+1)$ and $(x+3)(x+2)$

The LCD is $(x+3)\,(x+1)\,(x+2)$.

What property of equality was used in this step?

$$=\frac{2x}{(x+3)(x+1)}\bullet\frac{(x+2)}{(x+2)}+\frac{(3x-6)}{(x+3)(x+2)}\bullet\frac{(x+1)}{(x+1)}$$

$$=\frac{(2x)(x+2)}{(x+3)(x+1)(x+2)}+\frac{(3x-6)(x+1)}{(x+3)(x+2)(x+1)}$$

$$=\frac{2x^2+4x}{x^3+6x^2+11x+6}+\frac{3x^2-3x-6}{x^3+6x^2+11x+6}$$

$$= \frac{2x^2 + 3x^2 + 4x - 3x - 6}{x^3 + 6x^2 + 11x + 6}$$

$$= \frac{5x^2 + x - 6}{x^3 + 6x^2 + 11x + 6}$$

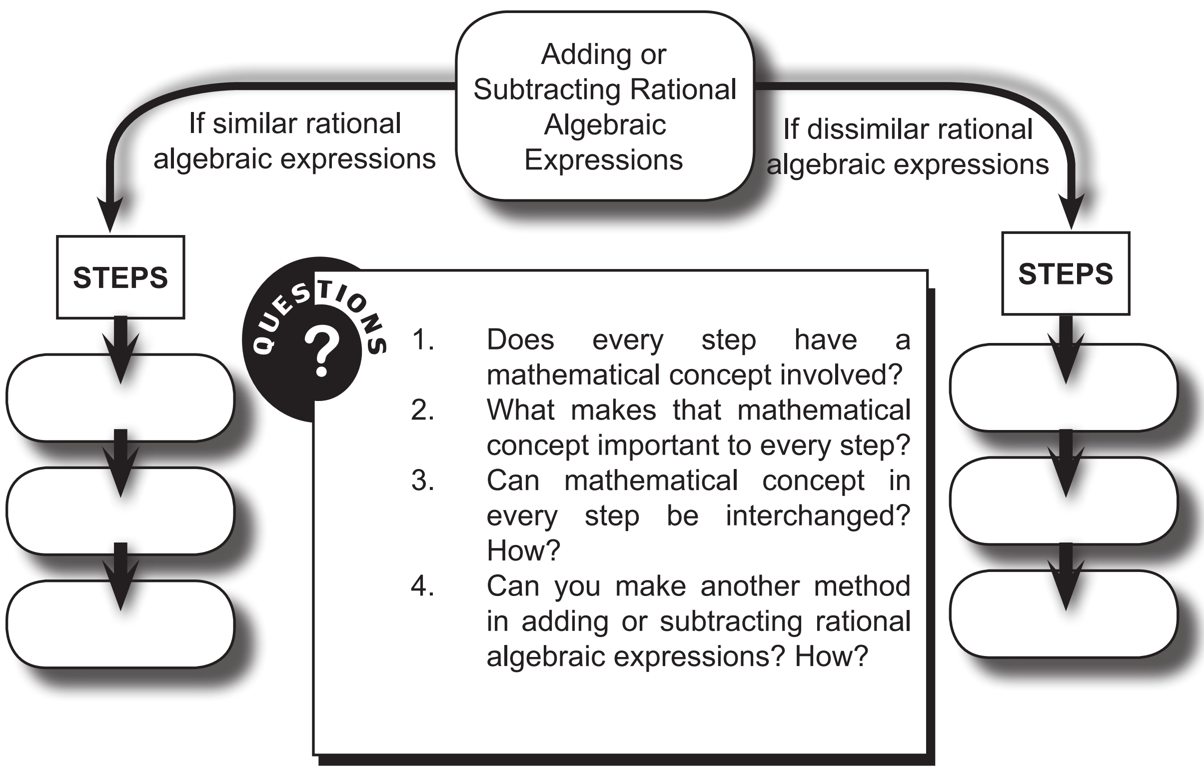
Exercises:

Perform the indicated operation. Express your answers in simplest form.

1. $\dfrac{3}{x + 1} + \dfrac{4}{x}$
2. $\dfrac{x + 8}{x^2 - 4x + 4} + \dfrac{3x - 2}{x^2 - 4}$
3. $\dfrac{2x}{x^2 - 9} - \dfrac{3}{x - 3}$
4. $\dfrac{3}{x^2 - x - 2} - \dfrac{2}{x^2 - 5x + 6}$
5. $\dfrac{x + 2}{x} - \dfrac{x + 2}{2}$

Activity 12 FLOW CHART

Now that you have learned adding and subtracting rational algebraic expressions, you are now able to fill in the graphic organizer below. Write each step in adding or subtracting rational algebraic expressions in each box below.



1. Does every step have a mathematical concept involved?
2. What makes that mathematical concept important to every step?
3. Can mathematical concept in every step be interchanged? How?
4. Can you make another method in adding or subtracting rational algebraic expressions? How?

Web – based Booster:
Watch the videos in these websites for more examples. http://www.onlinemathlearning.com/adding-rational-expressions-help.html http://www.onlinemathlearning.com/subtracting-rational-expressions-help.html

Activity 13 WHAT IS WRONG WITH ME?

Rewrite the solution in the first box. Write your solution in the second box. In the third box, write your explanation on how your solution corrects the original one .

Original	My Solution	My Explanation
$\frac{2}{36-x^2} - \frac{1}{x^2-6x} = \frac{2}{(6-x)(6-x)} - \frac{1}{x(x+6)}$ $= \frac{2}{(x-6)(x+6)} - \frac{1}{x(x+6)}$ $= \frac{2}{(x-6)(x+6)} \bullet \frac{x}{x} - \frac{1}{x(x+6)} \bullet \frac{x-6}{x-6}$ $= \frac{2x}{x(x-6)(x+6)} - \frac{1(x-6)}{x(x+6)(x-6)}$ $= \frac{2x-(x-6)}{x(x-6)(x+6)}$ $= \frac{2x-x+6}{x(x-6)(x+6)}$ $= \frac{x+6}{x(x-6)(x+6)}$ $= \frac{1}{x(x-6)}$ $= \frac{1}{x^2-6x}$		
$\frac{2}{a-5} - \frac{3}{a} = \frac{2}{a-5} \bullet \frac{a}{a} - \frac{3}{a} \bullet \frac{a-5}{a-5}$ $= \frac{2a}{a-5(a)} - \frac{3(a-5)}{a(a-5)}$ $= \frac{2a}{a-5(a)} - \frac{3a-15}{a(a-5)}$ $= \frac{2a-3a-15}{a(a-5)}$ $= \frac{-a-15}{a^2-5a}$		

$\frac{3x}{2x-3}+\frac{9}{3-2x}=\frac{3x}{2x-3}+\frac{9}{(-1)(2x-3)}$ $=\frac{3x}{2x-3}-\frac{9}{2x-3}$ $=\frac{3x-9}{2x-3}$ $=\frac{3(x-3)}{2x-3}$ $=\frac{x-3}{2x}$		
$\frac{4}{b-2}+\frac{b^2-4b}{b-2}=\frac{b^2-4b+4}{b-2}$ $=\frac{(b-2)(b+2)}{b-2}$ $=b+2$		

QUESTIONS ?

1. What did you feel while answering the activity?
2. Did you encounter difficulties in answering the activity?
3. How did you overcome these difficulties?

The previous activities deal with the fundamental operations on rational expressions. Let us try these concepts in a different context.

Activity 14 COMPLEX RATIONAL ALGEBRAIC EXPRESSIONS

Examine and analyze the following illustrative examples on the next page. Answer the checkup questions.

A rational algebraic expression is said to be in its simplest form when the numerator and denominator are polynomials with no common factors other than 1. If the numerator or denominator, or both numerator and denominator of a rational algebraic expression is also a rational algebraic expression, it is called a ***complex rational algebraic expression****. Simplifying complex rational expressions is transforming it into a simple rational expression. You need all the concepts learned previously to simplify complex rational expressions.*

REVIEW

Perform the operation on the following fractions.

1. $\dfrac{\frac{1}{2}+\frac{4}{3}}{1-\frac{2}{3}}$
2. $\dfrac{\frac{1}{2}-\frac{4}{3}}{\frac{3}{4}-\frac{2}{3}}$
3. $\dfrac{\frac{5}{2}-\frac{4}{3}}{\frac{2}{3}+2}$
4. $\dfrac{\frac{1}{2}+\frac{5}{4}}{\frac{4}{3}-\frac{2}{3}}$
5. $\dfrac{\frac{5}{9}+\frac{4}{3}}{1+\frac{2}{3}}$

Illustrative example 11: Simplify $\dfrac{\frac{2}{a}-\frac{3}{b}}{\frac{5}{b}+\frac{6}{a^2}}$.

$$\frac{\frac{2}{a}-\frac{3}{b}}{\frac{5}{b}+\frac{6}{a^2}}$$

Main fraction bar (——) is a line that separates the main numerator and the main denominator.

$$=\frac{\left(\frac{2}{a}\cdot\frac{b}{b}\right)-\left(\frac{3}{b}\cdot\frac{a}{a}\right)}{\left(\frac{5}{b}\cdot\frac{a^2}{a^2}\right)+\left(\frac{6}{a^2}\cdot\frac{b}{b}\right)}$$

Where did $\frac{b}{b}$ and $\frac{a}{a}$ in the main numerator and the $\frac{a^2}{a^2}$ and $\frac{b}{b}$ in the main denominator come from?

$$=\frac{\left(\frac{2b}{ab}\right)-\left(\frac{3a}{ab}\right)}{\left(\frac{5a^2}{a^2b}\right)+\left(\frac{6b}{a^2b}\right)}$$

$$=\frac{\frac{2b-3a}{ab}}{\frac{5a^2+6b}{a^2b}}$$

What happens to the main numerator and the main denominator?

$$=\frac{2b-3a}{ab}\div\frac{5a^2+6b}{a^2b}$$

What principle is used in this step?

$$=\frac{2b-3a}{ab}\bullet\frac{a^2b}{5a^2+6b}$$

$$=\frac{(2b-3a)a\cancel{a}\cancel{b}}{(5a^2+6b)\cancel{a}\cancel{b}}$$

Simplify the rational algebraic expression.

$$=\frac{(2b-3a)a}{5a^2+6b}$$

What laws of exponents are used in this step?

$$=\frac{2ab-3a^2}{5a^2+6b}$$

Illustrative example 12: Simplify $\dfrac{\frac{c}{c^2-4}-\frac{c}{c-2}}{1+\frac{1}{c+2}}$.

$$\frac{\frac{c}{c^2-4}-\frac{c}{c-2}}{1+\frac{1}{c+2}}=\frac{\frac{c}{(c-2)(c+2)}-\frac{c}{c-2}}{1+\frac{1}{c+2}}$$

$$=\frac{\frac{c}{(c-2)(c+2)}-\frac{c}{c-2}\bullet\frac{(c+2)}{(c+2)}}{1\bullet\frac{c+2}{c+2}+\frac{1}{c+2}}$$

$$= \frac{\dfrac{c}{(c-2)(c+2)} - \dfrac{c(c+2)}{(c-2)\,(c+2)}}{\dfrac{c+2}{c+2} + \dfrac{1}{(c+2)}}$$

$$= \frac{\dfrac{c}{(c-2)(c+2)} - \dfrac{c^2+2c}{(c-2)\,(c+2)}}{\dfrac{c+2}{c+2} + \dfrac{1}{(c+2)}}$$

$$= \frac{\dfrac{c-(c^2+2c)}{(c-2)(c+2)}}{\dfrac{c+2+1}{c+2}}$$

$$= \frac{\dfrac{-c^2-2c+c}{(c-2)(c+2)}}{\dfrac{c+2+1}{c+2}}$$

$$= \frac{\dfrac{-c^2-c}{(c-2)(c+2)}}{\dfrac{c+3}{c+2}}$$

$$= \frac{-c^2-c}{(c-2)(c+2)} \div \frac{c+3}{c+2}$$

$$= \frac{-c^2-c}{(c-2)(c+2)} \bullet \frac{c+2}{c+3}$$

$$= \frac{(-c^2-c)\cancel{(c+2)}}{(c-2)\cancel{(c+2)}\,(c+3)}$$

$$= \frac{-c^2-c}{(c-2)(c+3)}$$

$$= \frac{-c^2-c}{c^2+c-6}$$

Exercises

Simplify the following complex rational expressions.

1. $\dfrac{\dfrac{1}{x} - \dfrac{1}{y}}{\dfrac{1}{x^2} + \dfrac{1}{y^2}}$

2. $\dfrac{\dfrac{x-y}{x+y} - \dfrac{y}{x}}{\dfrac{x}{y} + \dfrac{x-y}{x+y}}$

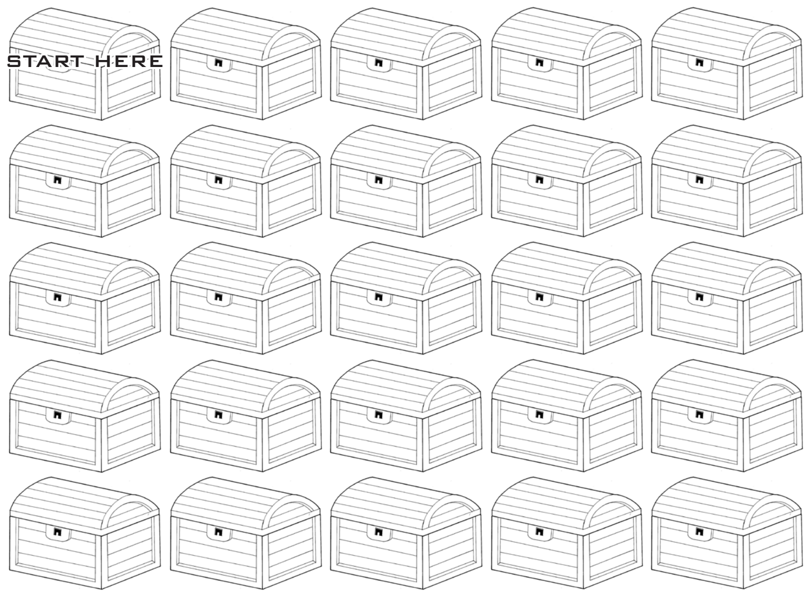
3. $\dfrac{\dfrac{b}{b-1} - \dfrac{2b}{b-2}}{\dfrac{2b}{b-2} - \dfrac{3b}{b-3}}$

4. $\dfrac{\dfrac{1}{a-2} - \dfrac{3}{a-1}}{\dfrac{5}{a-2} + \dfrac{2}{a-1}}$

5. $\dfrac{4 - \dfrac{4}{y^2}}{2 + \dfrac{2}{y}}$

Activity 15 TREASURE HUNTING

Directions: Find the box that contains the treasure by simplifying the rational expressions below. Find the answer of each expression in the hub. Each answer contains a direction. The correct direction will lead you to the treasure. Go hunting now.



1. $\dfrac{x^2 - \dfrac{4}{x^2}}{x + \dfrac{2}{x}}$ 2. $\dfrac{\dfrac{x}{2} + \dfrac{x}{3}}{\dfrac{1}{2}}$ 3. $\dfrac{\dfrac{3}{x^2 + 3x + 2}}{\dfrac{x}{x + 2}}$

THE HUB

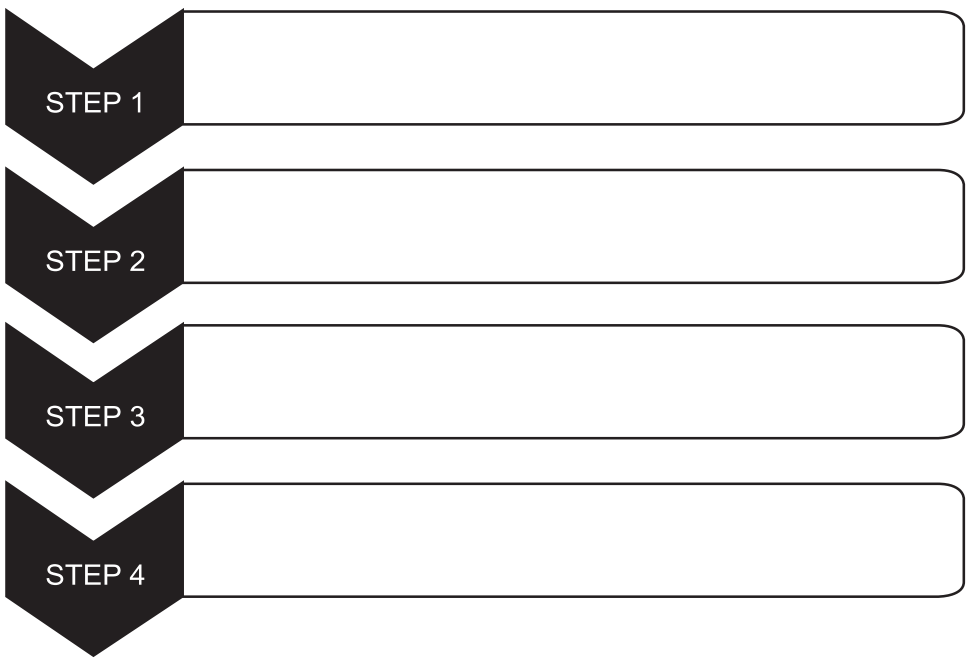
$\dfrac{5x}{3}$	$\dfrac{x^2 - 2}{x}$	$\dfrac{1}{x - 1}$	$\dfrac{x^2 + 2}{x^2 + x - 6}$	$\dfrac{3}{x^2 + x}$
2 steps to the right	Down 4 steps	3 steps to the left	4 steps to the right	Up 3 steps

QUESTIONS ? Based on the above activity, what are your steps in simplifying complex rational algebraic expressions?

Activity 16 VERTICAL CHEVRON LIST

Web – based Booster:
Watch the videos in these websites for more examples
http://www.wtamu.edu/academic/anns/mps/math/mathlab/col_algebra/col_alg_tut11_complexrat.htm
http://www.youtube.com/watch?v=-jli9PP_4HA
http://spot.pcc.edu/~kkling/Mth_95/SectionIII_Rational_Expressions_Equations_and_Functions/Module4/Module4_Complex_Rational_Expressions.pdf

Directions: Make a conceptual map in simplifying complex rational expressions using a vertical chevron list. Write the procedure or important concepts in every step inside the box. If necessary, add another chevron to complete your conceptual map.



Activity 17 REACTION GUIDE

Directions: Revisit the second activity. There are sets of rational algebraic expressions in the following table. Check agree if column I is the same as column II and check disagree if the two columns are not the same.

I	II	Agree	Disagree
$\frac{x^2 - xy}{x^2 - y^2} \cdot \frac{x + y}{x^2 - xy}$	$x^{-1} - y^{-1}$		
$\frac{6y - 30}{y^2 + 2y + 1} \div \frac{3y - 15}{y^2 + y}$	$\frac{2y}{y + 1}$		

$\dfrac{5}{4x^2} + \dfrac{7}{6x}$	$\dfrac{15 + 14x}{12x^2}$		
$\dfrac{a}{b - a} - \dfrac{b}{a - b}$	$\dfrac{a + b}{b - a}$		
$\dfrac{\dfrac{a + b}{b} - \dfrac{b}{a + b}}{\dfrac{1}{b} + \dfrac{2}{a}}$	$\dfrac{a^2}{a + b}$		

QUESTIONS

?

Compare your answer in the anticipation guide to your answer in the reaction guide. Do they differ from each other? Why?

In this section, the discussion is all about operations on rational algebraic expressions. How much of your initial ideas were discussed? Which ideas are different and need revision? The skills in performing the operations on rational algebraic expressions is one of the key concepts in solving rate-related problems.

REFLECTION

What I have learned so far...

What to Understand

Your goal in this section is to relate the operations on rational expressions to real-life problems, especially the rate problems.

Activity 18 WORD PROBLEM

Read the problems below and answer the questions that follow.

1. Two vehicles travelled $(x + 4)$ kilometers. The first vehicle travelled for $(x^2 - 16)$ hours while the second travelled for $\frac{2}{x-4}$ hours.

   a. Complete the table below.

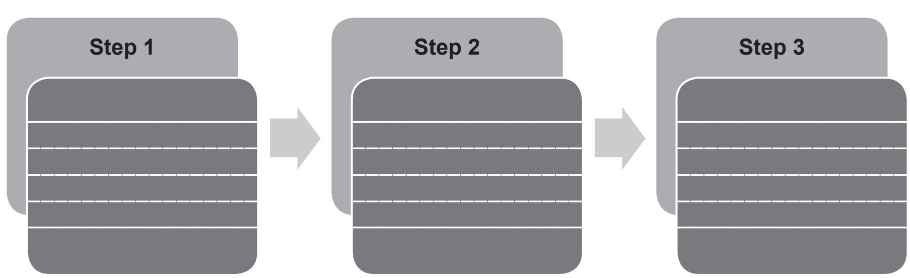
Vehicles	Distance	Time	Speed
Vehicle A			
Vehicle B			

   b. How did you compute the speed of the two vehicles?

2. Pancho and Bruce were asked to fill the tank with water. Pancho can fill the tank in x minutes alone, while Bruce is slower by two minutes compared to Pancho.
   a. What part of the job can Pancho finish in one minute?
   b. What part of the job can Bruce finish in one minute?
   c. Pancho and Bruce can finish filling the tank together within y minutes. How will you represent algebraically, in simplest form, the job done by the two if they worked together?

Activity 19 ACCENT PROCESS

List down the concepts and principles in solving problems involving operations on rational algebraic expressions in every step. You can add a box if necessary.



Activity 20 PRESENTATION

Present and discuss to the class the process of answering the questions below. Your output will be graded according to reasoning, accuracy, and presentation.

Alex can pour concrete on a walkway in x hours alone while Andy can pour concrete on the same walkway in two more hours than Alex.

a. How fast can they pour concrete on the walkway if they work together?

b. If Emman can pour concrete on the same walkway in one more hour than Alex, and Roger can pour the same walkway in one hour less than Andy, who must work together to finish the job with the least time?

Rubrics for your output

CRITERIA	**Outstanding 4**	**Satisfactory 3**	**Developing 2**	**Beginning 1**
Mathematical reasoning	Explanation shows thorough reasoning and insightful justifications.	Explanation shows substantial reasoning	Explanation shows gaps in reasoning.	Explanation shows illogical reasoning.
Accuracy	All computations are correct and shown in detail.	All computations are correct.	Most of the computations are correct.	Some of the computations are correct.
Presentation	The presentation is delivered in a very convincing manner. Appropriate and creative visual materials used.	The presentation is delivered in a clear manner. Appropriate visual materials used.	The presentation is delivered in a disorganized manner. Some visual materials used.	The pres-entation is delivered in a clear manner. It does not use any visual materials.

In this section, the discussion is about application of operations on rational algebraic expressions. It gives you a general picture of relation between operations on rational algebraic expressions and rate–related problems.

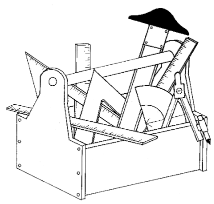
What new realizations do you have about the topic? What new connections have you made for yourself? What questions do you still have? Copy the ***Learned, Affirmed,*** *and* ***Challenged*** *cards in your journal notebook and complete each.*

Learned	**Affirmed**	**Challenge**
What new realizations and learning do you have about the topic?	What new connections have you made? Which of your old ideas have been confirmed/affirmed	What questions do you still have? Which areas seem difficult for you? Which do you want to explore

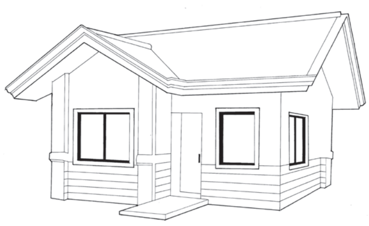
What to Transfer

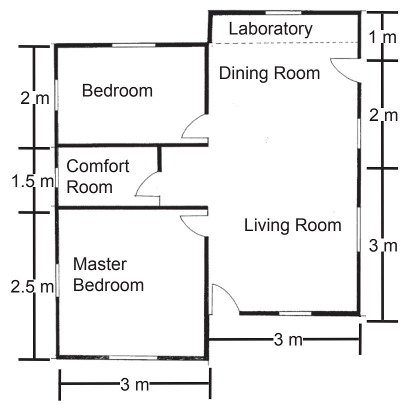
Your goal in this section is to apply your learning in real-life situations. You will be given a practical task which will demonstrate your understanding.

Activity 21 PRESENTATION



A newly-wed couple plans to construct a house. The couple has already a house plan made by their engineer friend. The plan of the house is illustrated below:





As a foreman of the project, you are tasked to prepare a manpower plan to be presented to the couple. The plan includes the number of workers needed to complete the project, their daily wage, the duration of the project, and the budget. The manpower plan will be evaluated based on reasoning, accuracy, presentation, practicality, and efficiency.

Rubrics for your output

CRITERIA	Outstanding 4	Satisfactory 3	Developing 2	Beginning 1
Reasoning	Explanation shows thorough reasoning and insightful justifications.	Explanation shows substantial reasoning.	Explanation shows gaps in reasoning.	Explanation shows illogical reasoning.
Accuracy	All computations are correct and shown in detail.	All computations are correct.	Most of the computations are correct.	Some of the computations are correct.
Presentation	The presentation is delivered in a very convincing manner. Appropriate and creative visual materials are used.	The presentation is delivered in a clear manner. Appropriate visual materials are used.	The presentation is delivered in a disorganized manner. Some visual materials are used.	The presentation is delivered in a clear manner. It does not use any visual materials.
Practicality	The proposed plan will be completed at the least time.	The proposed plan will be completed in lesser time.	The proposed project will be completed with greater number of days.	The proposed plan will be completed with the most number of days.
Efficiency	The cost of the plan is minimal.	The cost of the plan is reasonable.	The cost of the plan is expensive.	The cost of the plan is very expensive.

REFLECTION

In this lesson, I have understood that ____
____
____
____
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____________________.

SUMMARY/SYNTHESIS/GENERALIZATION:

Now that you have completed this module, let us summarize what have you learned:

1. Rate–related problems can be modelled using rational algebraic expressions.
2. A rational algebraic expression is a ratio of two polynomials where the denominator is not equal to one.
3. Any expression not equal to zero raised to a zero exponent is always equal to one.
4. When an expression is raised to a negative integeral exponent, it is the same as the multiplicative inverse of the expression.
5. A rational algebraic expression is in its simplest form if there is no common prime factor in the numerator and the denominator except 1.
6. To multiply rational algebraic expressions, multiply the numerator and the denominator, then simplify.
7. To divide rational algebraic expressions, multiply the dividend by the reciprocal of the divisor, then multiply.
8. To add/subtract similar rational algebraic expressions, add/subtract the numerators, and copy the common denominator.
9. To add/subtract dissimilar rational algebraic expressions, express each with similar denominator, then add/subtract the numerators and copy the common denominator.
10. A complex rational algebraic expression is an expression where the numerator or denominator, or both the numerator and the denominator, are rational algebraic expressions.

GLOSSARY OF TERMS:

Complex rational algebraic expression – an expression where the numerator or denominator or both the numerator and the denominator are rational algebraic expressions.

LCD – also known as least common denominator is the least common multiple of the denominators.

Manpower plan – a plan where the number of workers needed to complete the project, wages of each worker in a day, how many days can workers finish the job and how much can be spend on the workers for the entire project.

Rate-related problems – problems involving rates (e.g., speed, percentage, ratio, work)

Rational algebraic expression – a ratio of two polynomials where the denominator is not equal to one.

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http://www.onlinemathlearning.com/dividing-rational-expressions-help.html

http://www.onlinemathlearning.com/multiplying-rational-expressions-help.html

http://spot.pcc.edu/~kkling/Mth_95/SectionIII_Rational_Expressions_Equations_and_Functions/Module4/Module4_Complex_Rational_Expressions.pdf

Images credit:

http://www.portlandground.com/archives/2004/05/volunteers_buil_1.php